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ANALYSIS AND SYNTHESIS OF DIGITAL CURRENT CONTROLLER OF D.C MOTOR FED BY THYRISTOR CHOPPER

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ABSTRACT:

The purpose of this article is to present the application of a new modelling method to determine the parameters of a digital proportional - integral controller for the current of a separately excited D.C. motor fed by d.c. chopper. Three modes of operation of chopper are considered, duty, commutation and freewheeling. A non-linear discrete model is obtained from which a linearised discrete model is deduced. The characteristic equation of the system is then determined. The expressions of controller parameters as a fanction of the required poles is given. To check the results a program of simulation is made to get the current response due to step change in current reference.

1. INTRODUCTION:

The systems including static converters are usually non-linear and sampled data but, often are approximated by linear continuous models [1]. In these models the non -

3. MODELLING PRINCIPLE][2]:

The operation of any static converter can be considered as repetitive cycles each of which may be devided into several intervals called modes. The equations of the system and its state variables differ from one mode to another. The problem is firstly how to determine recurrence equations representing the different modes which is valied over the whole mode. Secondly how to determine the joining relations e.g. which can provid a solution at the end of one mode and the begining of the next mode. Then we can determine a recurrence equation for a basic cycle of the system.

Let us consider a mode K of period $T_K = (t_n , t'_n)$ and the state variable is X_K. The limit states are the initial $[X_K(t_n), t_n]$ and the final $[X_K(t'_n), t'_n]$

The deferential equation of this mode is

$$F_K \frac{d\underline{X}_K}{dt} + G_K \underline{X}_K = \underline{V}_K(t) \qquad t \in (t_n , t'_n) \quad \ldots(1)$$

and its state equation is:

$$\frac{d\underline{X}_K}{dt} = A_K \underline{X}_K + \underline{B}_K (t) \qquad \ldots\ldots(2)$$

with

$$A_K = - F_K^{-1} G_K \quad \text{and} \quad \underline{B}_K = F_K^{-1} \underline{V}_K$$

The solution of equation (2) is given by :

$$X_K(t'_n) = e^{(t'_n - t_n)A_k} \underline{X}_K(t_n) + \int_{t_n}^{t'_n} e^{(t'_n - t)A_K} B_K(t)\,dt \quad \ldots(3)$$

multiplying both sides of equation (3) by $e^{-t'_n A_K}$ we get

$$\underline{E}_K\,(X_K(t_n')\,,\,t_n') = \underline{E}_K\,(X_K\,(t_n)\,,\,t_n) \quad \ldots (4)$$

with

$$\underline{E}_K\,(X_K\,,\,t) = e^{-t\,A_K}\,\underline{X}_K - \int_0^t e^{-\tau A_K}\,B(\tau)\,d\tau \quad \ldots (5)$$

The E function is invariant on the trajectory $(X(t),\ t)$ its variation verifies

$$\delta\,\underline{E}_K'\,(n) = \delta\,\underline{E}_K\,(n) \quad \ldots (6)$$

with

$$\delta\,\underline{E}_K = e^{-t\,A_K}\,[\,\delta\,X_K - Y_K\,\delta t\,] \quad \ldots (7)$$

where

$$\underline{Y}_K\,(t) = A_K\,X_K\,(t) + B_K\,(t)$$

4. THE SYSTEM EQUATIONS:

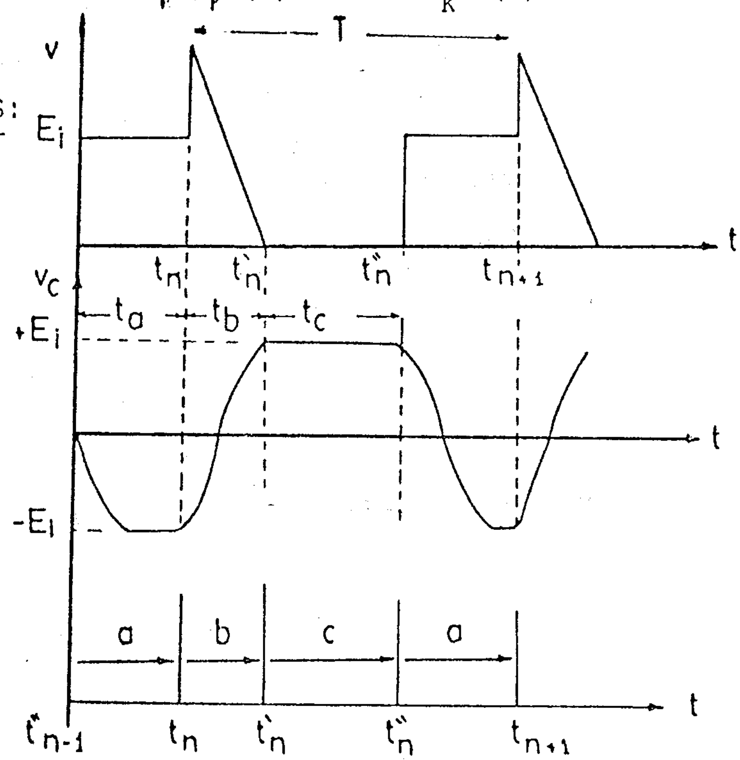

Figure (2): The wave forms of the armature and capacitor voltages.

Figure (2) shows the wave forms of the armature voltage and the voltage across the capacitor of the chopper. Also it shows the considered three modes, duty (a), comutation (b) and free-wheeling (c).

4.1. THE MOTOR AND CHOPPER EQUATIONS:

MODE A: (duty)

The normal conduction mode of duration $t_a = t_n - t''_{n-1}$. We do not consider the oscillation of the voltage V_c which has no effect on the system. From figure (3) we can write.

$$\ell \frac{di}{dt} + ri = (E_i - E_o) \quad \ldots (8)$$

$$\frac{di}{dt} = -\frac{r}{\ell} + \frac{(E_i - E_o)}{\ell} \quad \ldots (9)$$

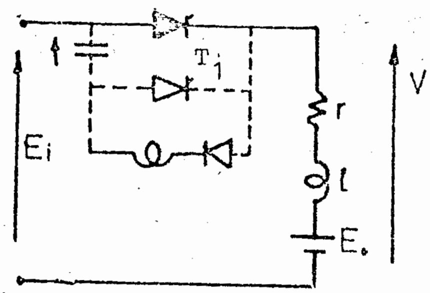

Figure (3): The equivalent circuit of chopper in mode a.

The state variable $X_a = (i)$ and

$$A_a = (\frac{-r}{\ell}) \quad , \quad B_a = \frac{(E_i - E_o)}{\ell}$$

$$F_a = (\ell) \quad , \quad G_a = (r) \quad , \quad V_a = (E_i - E_o)$$

MODE B:

The commutation interval of duration $t_b = t'_n - t_n$. From Fig. (4) : The equations in this mode are:

$$\ell \frac{di}{dt} + ri + V_c = E_i - E_o$$

$$c \frac{dV_c}{dt} - i = o$$

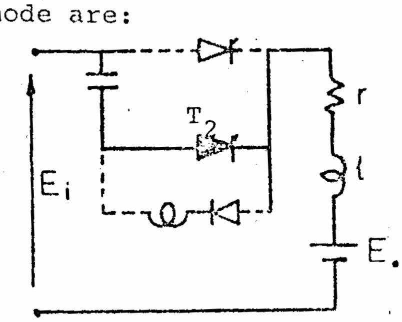

Figure (4): The equivalent circuit of chopper in mode b.

Or in matrix form,

$$\begin{bmatrix} 1 & o \\ o & c \end{bmatrix} \begin{bmatrix} di/di \\ dV_c/di \end{bmatrix} + \begin{bmatrix} r & 1 \\ -1 & o \end{bmatrix} \begin{bmatrix} i \\ V_c \end{bmatrix} = \begin{bmatrix} E_i - E_o \\ o \end{bmatrix} \qquad \ldots(10)$$

Equation (10) can be writen in state equation form:

$$\begin{bmatrix} \dfrac{di}{dt} \\[2ex] \dfrac{dV_c}{d_t} \end{bmatrix} = \begin{bmatrix} \dfrac{-r}{\ell} & \dfrac{-1}{\ell} \\[2ex] \dfrac{1}{c} & o \end{bmatrix} \begin{bmatrix} i \\ V_c \end{bmatrix} + \begin{bmatrix} \dfrac{E_i - E_o}{\ell} \\[2ex] o \end{bmatrix} \qquad \ldots \quad (11)$$

The state vector is $X_b = \begin{bmatrix} i \\ V_c \end{bmatrix}$

$$A_b = \begin{bmatrix} \dfrac{-r}{\ell} & \dfrac{-1}{\ell} \\[2ex] \dfrac{1}{C} & o \end{bmatrix} \qquad B_b = \begin{bmatrix} \dfrac{E_i - E_o}{\ell} \\[2ex] o \end{bmatrix}$$

$$F_b = \begin{bmatrix} I & o \\ o & C \end{bmatrix} \quad G_b = \begin{bmatrix} r & 1 \\ -1 & o \end{bmatrix} \text{ and } V_b = \begin{bmatrix} E_i - E_o \\ o \end{bmatrix}$$

THE MODE C:

The freewheeling interval of duration $t_c = t''_n - t'_n$

From fig. (5) the equations are:

$$\ell \frac{di}{dt} + r\,i = -E_o \qquad \ldots \; (12)$$

$$\frac{di}{dt} = -\frac{r}{\ell} i - \frac{E_o}{\ell} \qquad \ldots \; (13)$$

with

$X_c = (i)$

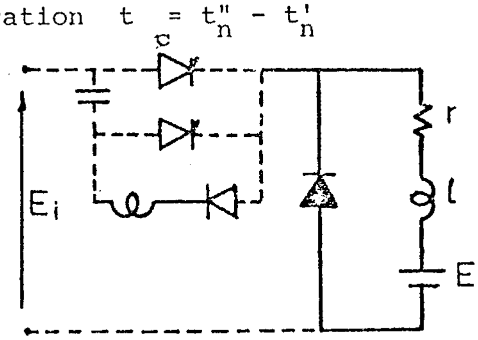

Figure (5): The equivalent circuit of of chopper c .

linear effects are negligible and the band width is not too large. For the cases where these aassumptions are not applicable, previous works [2] and [3], presented a systematic method to establish the rigorous non-linear model and the linearised model about a steady state operating point. Using this method, calculations have been carried out to determine the parameters of a digital current and speed controllers for d.c. motor fed by thyristor bridge [4], [5]. In this paper the problem is different we try to adapte the general method in [2] to the case where the motor is fed by thyristor chopper and not thyristor bridge. The model of the motor and chopper is deduced in the form of recurrence equations where the digital controller is expressed directcly in recurrence form. A linearised model of the system is obtained by linearising the non-linear model around a steady state operating point. The characteristic equation of the system is then obtained, from which the controller parameters are calculated for the desired response. A complete program of simulation is developed to study the behaviour of the system, taking into consideration all the discontinuities which may be occur in the system.

2. DISCRIPTION OF THE SYSTEM:

The system is shown in Figure [1]. It consists of a separately excited d.c. motor fed by chopper. The firing pulses for the auxiliary and main thyristor are generated by digital impulse generator. The input to the impulse generator is the output of a digital P.I controller.

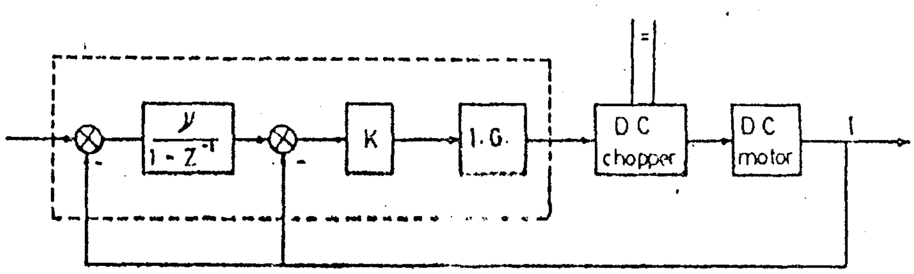

Figure (1): The block diagram of the system.

$, A_c = (\frac{-r}{\ell})$ $\qquad B_c = (\frac{-E_o}{\ell})$

$F_c = (\ell).$ $\qquad G_c = (r)$ and $V_c = (-E_o)$

The three modes are separated by three events, two external events, the firing of the thyristors and an internal event, the turning on of the freewheeling diode (Figure 2).

The recurrence equations of d.c motor and the chopper are:

$$E_a(X''_a, t''_{n-1}) = E_a(X_a, t_n) \quad \dots (14)$$

$$E_b(X_b, t_n) = E_b(X'_b, t'_n) \quad \dots (15)$$

$$E_c(X'_c, t'_n) = E_c(X''_c, t''_n) \quad \dots (16)$$

4.2: THE CONTROLLER EQUATIONS:

From Figure (1) the controller equations can be written directly in the following discrete form.

$$u(t''_n) - u(t''_{n-1}) = \vartheta\,[\text{Ref}(I) - i(t''_{n-1})] \quad \dots (17)$$

$$t''_{n-1} = K\,[u(t''_{n-1}) - i(t''_{n-1})] \quad \dots (18)$$

4.3. THE LINEARISED MODEL:

Equations (14 to 16) can be linearised around a steady state operating point defined by:

$$i(t''_n) = i(t''_{n-1}) = i_\infty$$

$$t''_n = t''_{n-1} = t_\infty$$

and may be written as follows:

$$R_b \longrightarrow \quad \delta E_b(t_n) = \delta E_b(t'_n) \quad \dots\dots (19)$$

$$R_c \longrightarrow \quad \delta E_c(t'_n) = \delta E_c(t''_n) \quad \dots\dots (20)$$

$$R_a \longrightarrow \quad \delta E_a(t''_{n-1}) = \delta E_a(t_n) \quad \dots\dots (21)$$

The linearised model of the d.c motor and the chopper will be the equations (19), (20) and (21). The joining relations R_{ca}, R_{ab} and R_{bc} necessary to describe the global point maping:

$$\delta X''_{n-1} \;\rightsquigarrow\!\xrightarrow{\quad R \quad}\; \delta X''_{n}$$

(i) R_{bc} (at the instant t'_n):

Assuming that the relation between the state vector at the end of mode b and the state vector at the begining of mode c is:

$$\underline{X}_c = C_{cb}\,\underline{X}_b$$

$$\text{Or} \quad \underline{X}_b = C_{bc}\,\underline{X}_c$$

we can write also

$$\underline{V}_c = D_{cb}\,\underline{V}_b$$

with

$$C_{cb} = D_{cb} = C_{bc}^T = (\,1 \quad o\,)$$

and

$$F_c = D_{cb}\,F_b\,C_{bc} \quad , \quad G_c = D_{cb}\,G_b\,C_{bc}$$

Using the above relations and the defintions of $\delta E_c\,(t'_n)$ and $\delta E_b\,(t'_n)$ from eauation (7) the following joing relation R_{bc} can be written as:

$$F_c\; e^{t'_n \Lambda_c}\;\delta E_c\,(t'_n) = C_{cb}\,F_b\; e^{t'_n \Lambda_b}\quad \delta E_b\,(t'_n) \qquad \ldots(22)$$

(ii) R_{ab} (at the instant t_n):

The commutation mode begins at a fixed instant t_n (i.e. δt_n=o), Then using eauation (7) for modes (a) and (b) :

$$\delta E_a\,(t_n) = e^{-t_n \Lambda_a}\quad \delta X_a\,(t_n) \qquad \ldots \quad \ldots (23)$$

$$\delta E_b\,(t_n) = e^{-t_n \Lambda_b}\quad \delta X_b\,(t_n) \qquad \ldots \quad \ldots (24)$$

Now suppose that the state vector $\underline{X}_b$ is related by the following equation with the state vector $\underline{X}_a$:

$$\underline{X}_b = H_{ba}\,\underline{X}_a + H_b\;E_i \qquad \ldots \quad \ldots \quad \ldots (25)$$

with

$$H_{ba} = \begin{pmatrix} 1 \\ o \end{pmatrix} \quad , \quad H_b = \begin{pmatrix} o \\ -1 \end{pmatrix}$$

The variation of eauation (25) around the steady state point is:

$$\delta \underline{X}_b = H_{ba}\quad \delta \underline{X}_a \qquad \ldots \quad \ldots \quad \ldots (26).$$

from equation (23), (24) and (26) we can write the joing relation R_{ab}

$$\delta E_b\ (t_n) = e^{-t_n A_b} H_{ba}\, e^{t_n A_a}\ \delta E_o\ (t_n) \quad \ldots \quad \ldots \quad (27)$$

<u>(iii) R_{ca} at the instent $(t''_n - 1)$:</u>

The relations between state vectors $X_a\ (t''_n - 1)$ and $X_c\ (t''_n - 1)$ can be written as:

$$X_a\ (t''_n - 1) = H_{ac}\ X_c\ (t''_n - 1) \quad \text{with } H_{ac} = (1)$$

$$\delta X_a\ (t''_n - 1) = H_{ac}\ \delta X_c\ (t''_n - 1) \quad \ldots \quad \ldots \quad (28)$$

4.4. DETERMINATION OF THE LINEARISED MODEL FOR THE D.C. MOTOR AND CHOPPER:

The joining recurrence R_{bc} equation (22) can be written in the following from using equations (19) and (20).

$$\delta E_c\ (t''_n) = e^{-t'_n A_c} F_c^{-1}\ C_{cb}\ F_b\ e^{t'_n A_b}\ \delta E_b\ (t_n) \quad \ldots \quad (29)$$

eleminating $\delta E_b\ (t_n)$ from equation (29) using equation (27), we get:

$$\delta E_c (t''_n) = e^{-t'_n A_c} F_c^{-1}\ C_{cb} F_b e^{-(t'_n - t_n) A_b} H_{ba} e^{t_n A_b}\ \delta E_a (t_n) \quad \ldots \quad \ldots (30)$$

using the recurrence equation of mode a, R_a (eauation 21) and the relation (28) the linearised recurrence equantion of the system (motor - chopper) can be obtained:

$$\delta X_c (t''_n) - Y_c (t''_\infty)\ \delta t''_n = M_R H_{ac}\ \ X_c (t''_n - 1) - M_R\ Y_a (t''_\infty)\ \delta t''_n - 1 \ \ldots (31)$$

where

$$M_R = e^{t_c \Lambda_c} F_c^{-1}\ \ C_{cb}\ F_b\ e^{t_b \Lambda_b}\ H_{ba}\ e^{t_b \Lambda_b}\ H_{ba}\ e^{t_a A_a} \quad \ldots \quad (32)$$

In equation (31) $\underline{X}_c$ is replaced by $\underline{i}$ and the general form of the linearsed recurrence equation is given by:

$$\delta i_a\ (t''_n) - A\ \ \delta t''_n = B\ \ \delta\ i\ \ (t''_{n-1}) - C\, \delta\, (t''_{n-1}) \ldots (33)$$

A, B, C are given in appendix.

The linearised equations of the controller is deduced directly from (17) and (18) as follows:

$$\delta u\ (t''_n) = \delta u(t''_{n-1}) - \nu\ \delta\ i\ (t''_{n-1}) \quad \dots (34)$$

$$\delta u\ (t''_{n-1}) - \delta i\ (t''_{n-1}) = {}^1/_k\ \delta(t''_{n-1}) \quad \dots (35)$$

4.5. THE CHARACTERISTIC EQUATION OF THE SYSTEM:

Using the Z transform in equations (33 to 35), the charactrised equation of the system (motor - chopper - controller) may be written as:

$$Z^2(1-KA)+[1+B+(C+A+A\ \nu)K\ ]\ Z - B+KC\ (1-\nu\ ) = o \quad \dots (36)$$

assuming that the desired poles of the system are Z_1 and Z_2 .

Then the controller parameters can be written as a function of the desired poles:

$$K = \frac{A\ (Z_1Z_2 + B) + C\ [\ 1 + B + (Z_1 + Z_2)]}{A\ (C+ A\ Z_1Z_2\ ) + C\ [\ A(Z_1 + Z_2)-C(C+A)]} \quad \dots\ \dots (37)$$

$$\nu K = \frac{Z_1Z_2 + B}{C} - K\ (\frac{C + A\ Z_1Z_2}{C}) \quad \dots\ \dots\ \dots .(39)$$

for fast response we will determine the controller parameter for the deadbeat response ie. $Z_1 = Z_2 = o$, then

$$K = \frac{A\ B + (1 + B\ )}{2\ A\ C - C^2} \quad \dots\ \dots\ \dots\ \dots .(39)$$

$$\nu\ K = \frac{B}{C} - K \quad \dots\ \dots\ \dots\ \dots\ \dots (40)$$

5. SIMULATION:

A program of simulation is written based on Runge kutta numerical method to solve the differential equations of each interval. The digital controller equations are used at the end of commutation interval to calculate the firing instant of thyristor T, (the begining of the duty interval). The process of simulation is presented by the following flow chart (Figure 6):

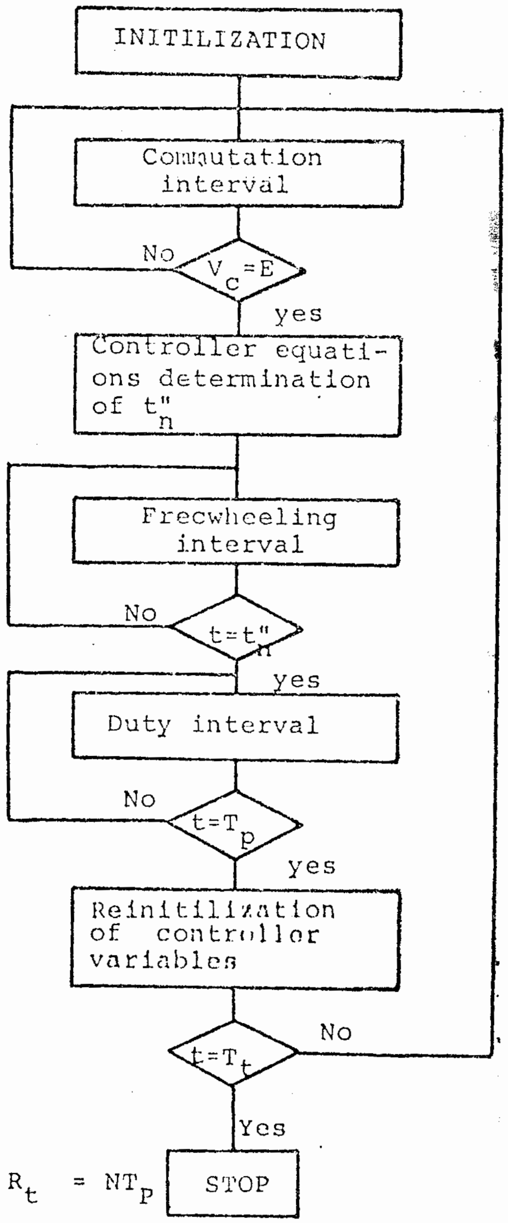

Figure 6: Flow chart of simulation program

6. RESULTS:

The d.c motor has the following characteristics

r = 8 ohm , $l1$ = 33.6 mH , K_i = 1 Volt/Amp.

and the chopper has the following characteristics

c = 10 μ.F , E_i = 100 Volt.

The parameters of the digital controller are calculated for an operating point given by $i(t_n'') = I_o = 2.812$ A and their values are $K = 0.022$ $\nu = 0.81$ for deadbeat response.

The simulation results are given by Figure (7) which illustrates the motor armature current due to step change in current reference.

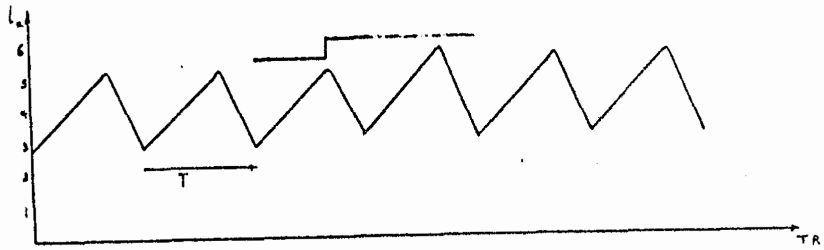

Fig. (7): The motor current response of the d,c. chopper for step change in current reference.

7. CONCLUSION:

The use of Microprocesser necessitates sophisticated model of the control systems.

This paper presents a new application of a general method of modelling for control system including DC - DC converters. The formula of digital controller parameters are given as a function of the desired poles of the system. The results of simulation carried out on digital computer satisfies the theoritical results.

8. LIST OF SYMPOLES:

i	=	Instantaneous value of the armature current.
K_i	=	Gain of current transducer.
K, ν	=	Controller parameters.
Ref(I)	=	Current reference.

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[10] APPENDIX:

$$A = Y_c\,(t''_n) = -\left[\frac{E_o}{L} + \frac{R}{L}\, i\,(t''_n)\right]$$

$$B = \phi_{11}\, e^{-(R/L)}\,(t_c + t_a)$$

$$C = B\left[\frac{E_i - E_o}{L} - \frac{R}{L}\, i\,(t''_{n-1})\right]$$

$$\phi_{11} = e^{-Bt_c}\,\sin(wt_c + \phi) / \sin\phi$$