STUDIES IN TIDAL PHENOMENA

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ABSTRACT

As a result of complexity of the tidal current and water level prediction problem; before the computer time, reliance has mainly been placed on field measurements and tidal model studies to obtain a reliable picture of water levels and tidal currents in inlets, bays, and estuaries. The expense connected with either of these two methods has greatly restricted to amount of tidal current and tidal range informations.

At the present, due to the increasing capacity of high speed digital computer and the different numerical techniques for solving the governing equations of motion, the use of mathematical models for tidal flow computation is quite well established. The main objective of this present study is to investigate the influence of boundary resistance on tidal wave propagating along an estuary or inlet and also on the tidal currents. An increase of boundary resistance will decrease the tidal range and the corresponding tidal current. Resonance could occur at low value of boundary resistance, which will influence both the characteristics of tidal wave and tidal current. The influence of fresh water discharge in conjunction with boundary resistance on the tidal phenomena is also investigated.

INTRODUCTION

The aim of studies of tides could be classified into three main purposes: (i) scientific interest; (ii) navigation to predict water level and/or current for a given place; (iii) hydraulic engineering to predict the effect of changing the conditions by hydraulic structures or by

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natural causes as an example is a prediction of tidal current in planned canal connecting two different tidal regimes such as Suez Canal and Panama Canal. The main objective of this work is to investigate the influence of boundary resistance and/or river flow on the tidal wave and tidal current. Analytical studies were used, before the computer time under simplified assumptions Ref.(4).

The mathematical model has been used since comprehensive field measurements are expensive and time consuming, and the physical models suffer serious scale effects.

Tidal Constituents:

Orbits of earth around the sun and of moon around earth are not circles but ellipses, that is distance sun-earth and earth-moon vary periodically so the gravitational force of attraction exhibits a maximum and a minimum value during each orbit. The orbital plane of revolution of earth round the sun is inclined to an angle to the sun axis. The axis of the moon is also inclined to the plane of its orbit round the earth consequently the gravitational tide producing force at a given point on the earth varies in a complex, but a predictable manner. The largest component of this force is due to the moon for its proximity from the earth and has a period of about 12 h 25 min.

The lunar tidal force reaches its maximum value once in 28 days when the moon is nearest to the earth, when the moon is furthest, the lunar tidal force is about 2/3 its maximum value. The total force due to combined action of sun and moon is greatest when they act together that is when the sun and moon are as nearly in line with the earth as possible. This happens twice a month, when the moon on the side of earth as the sun, i.e. during a full moon and a new moon. When this happens spring tides occur, having arange of movement bigger than the average. When sun and moon in quadraturc with the earth their effect gives rise to smaller range than the average when this happens neap tide occure which is also twice a month.

A list of components can be made from the harmonic analysis of the tide generating force Ref.(10). Bogren (1883) gave a list of 29 partial tides. A list in the Admiralty Mannual of Tide by Harris Conteins 23. The British Admiralty Manual of tide gave 20.Some of these partial tides half a period, up to half year, S_{sa} component due to the declination of sun with the equator. The hydraulic engineer is not concerned directly with tide generating force, but with thair effects on rise and fall of water level. He is not usually interested with partial tide having long periods, as he wants to be informed about a very high or very low,high water level and low water level, or the maximum velocity of the current at a place where he is going to work.For that reason it is not necessary to consider all the partial tides in the hydraulic engineering, the following table gives the principal harmonic components which account for 83% of the total tide generating force Ref.(8).

Table (1) Principal Harmonic Component

Name of Component	Symbol	Period solar hours	Amplitude ratio	
Principal lunar	M_2	12.42	100	semi diurnal
Principal solar	S_2	12.00	46.6	
Larger lunar elliptic	N_2	12.66	19.2	
Luni-solar semidiurnal	K_2	11.97	12.7	
Luni-solar diurnal	K_1	93.93	58.4	diurnal
Principal lunar diurnal	O_1	25.82	41.5	
Principal solar diurnal	P_1	24.07	19.4	

It is seen that the larger part of the tidal variation is due to moon and the principal effects are due to variation of phase of moon, distance and declination. The diurnal tide could be found on the Coast of China, Alaska, and the gulf of Mexico. The semi diurnal tide on the shore of Atlantic ocean and the mixed tide on the pacific Coast of the North America.

NUMERICAL MODEL

Mathematical models of tidal flow computations are based on the equations of motion, the continuity and dynamic equations;

$$\frac{\partial Q}{\partial x} + b\frac{\partial h}{\partial t} = 0 \qquad \text{.....(1)}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = S_o - S_f \qquad \text{.....(2)}$$

in which;

Q = water discharge;
b = instantaneous surface breadth;
u = water velocity;
h = water surface elevation;
S_o = bed slope; and
S_f = friction slope.

The numerical techniques for the solution of these two governing equations of motion are based on three main approaches: (i) method of characteristics; (ii) finite difference method; (iii) finite element method. The characteristic method has the advantages of transformation of tidal wave as it proceed along the estuary. Convergence of forward characteristics indicates the steepening of the wave. Their intersection indicates the formation of a bore Fig.(2). Figure (3) demonstrates the steepening of

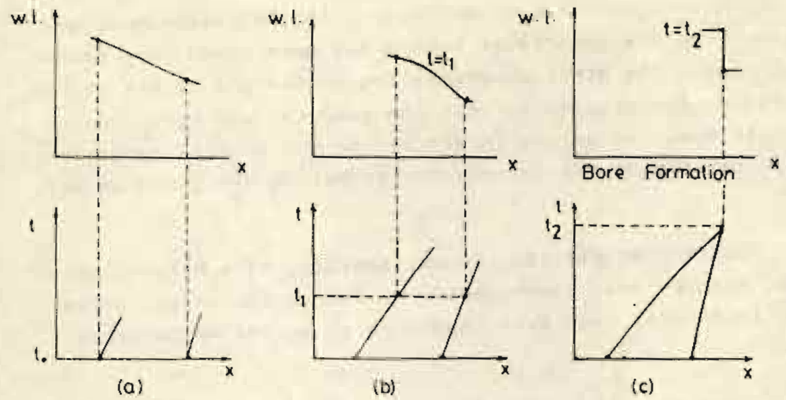

Fig (2) Convergence of the forward characteristics indicates the steepening of the wave form

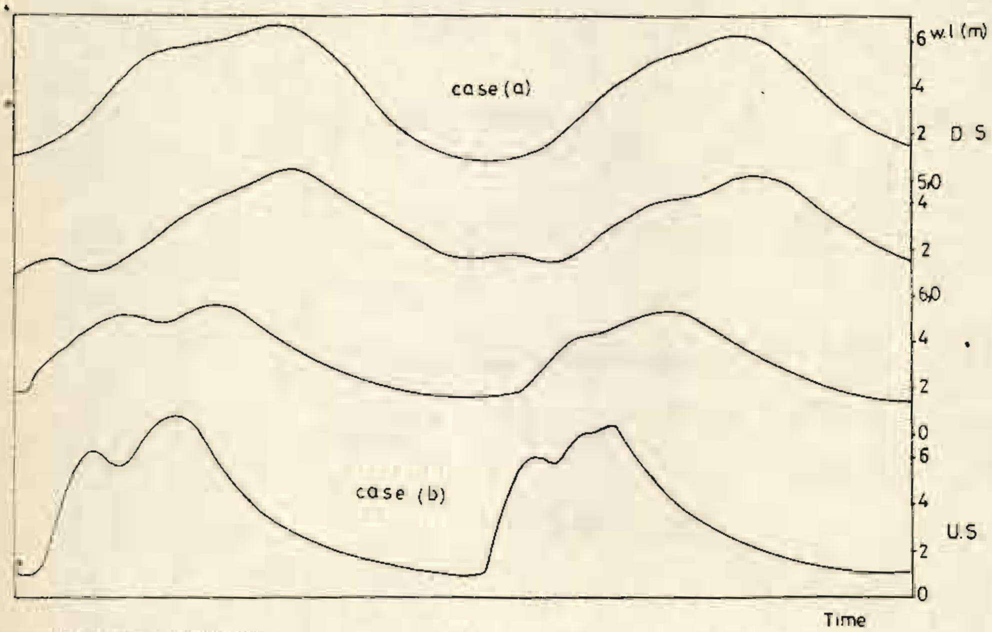

Fig (3) Steepening of the Wave Form

flood tide when the wave propagates in the upstream direction. The characteristic method has been found more accurate among the other numerical techniques and it has proved a faster convergence between the computed and actual water levels even for errors in estimating the initial conditions. More details of the characteristic method are given in Ref. (1).

Section properties, schematization, of a natural estuary, such as mean depth depth, wetted perimeter and cross sectional area have been incorporated in the mathematical model.

Equations (1) and (2) yield the two following characteristic equations Fig.(1);

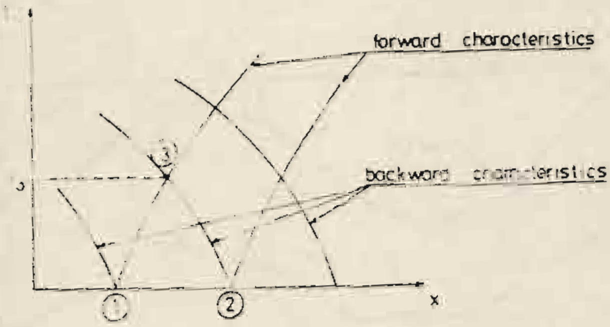

Fig.(1)

The forward characteristic equation:

$$u_3 + 2c_3 = u_1 + 2c_1 - \int_0^{t_3} \left[g \frac{\partial z}{\partial x} + g \frac{u|u|}{C^2 . R} + \frac{u.c}{b} \frac{\partial b}{\partial x} \right] dt \qquad .....(3)$$

The backward characteristic equation:

$$u_3 - 2c_3 = u_2 - 2c_2 - \int_0^{t_3} \left[g \frac{\partial z}{\partial x} + g \frac{u|u|}{C^2.R} - \frac{u.c}{b} \frac{\partial b}{\partial x} \right] dt \qquad .....(4)$$

The two equations confirms the effect of estuary shape on wave speed and water current. This is small if $u.c \dfrac{\partial \ln(b)}{\partial x}$ is less than $\dfrac{g\,u|u|}{C^2.d}$.

Some estuaries have appreciable value of bed slope with respect to section slope such as the Thames of England and the Houghly of India other esturies have very flat bed slope ($\frac{dz}{\partial x} = 0$) such as the Delaware estuary of the United State, where the friction term is three to four times the slope term. This flat bed slope could help in the analytical solution based on the incident and reflected wave.

It seems desirable to develop characteristic models which have the ability to choose the most efficient and stable computation for a given set of conditions of time increment, space increment, water velocity and celerity of the vave at a particular time and place Ref.(4).

Boundary Resistance

Friction parameters for use in a numerical model can be obtained in several ways. The final choice may be modified during the process of calibration and will depend on the feature of the channel , they are used as a compensating factors for the lack of interpolation technique and represention of the cross sectional properties along the estuary in the mathematical model. The roughness coefficients computed by Manning's equation $n = \dfrac{AR^{2/3}\sqrt{S}}{Q}$

or Chezy coefficient $C = \dfrac{Q}{A.\sqrt{RS}}$, or any similar equation are not real roughness, but numerical coefficient, which relate to the section properties such as hydraulic radius, and cross sectional area, to the discharge and energy slope. In lined prismatic channels, these roughness values can be considered to represent physical roughness. In a composite natural section contains zones of different bed roughness and/or over bank flow areas, these coefficients are numerical values, any procedure which attempt to use these values as a true roughness is absolutely wrong Ref.(3).

The numerical Manning's coefficients 0.01, 0.022, and 0.05 have been used in the model.

Boundary Conditions

Estuaries are controlled by the tidal action at sea end and by the river flow. They are two main independent variables. Some estuaries have negligible fresh water flow, and others have lost all contact with river that formed them, all estuaries were formed by the combined action of tide and river flow.

All numerical models require boundary conditions at the landward end the seaward end. At the landward end, from river flow. When tide propagates along an estuary of finite length and negligible river flow, assuming a complete reflection at the estuary head, the flow is zero and this could be applied in the case of a gulf.

Two values of river flow have been incorported into the mathematical model for hypothetical estuaries under investigation, normal river flow (50 m^3/Sec) and flood water flow (1000 m^3/Sec.). At the seaward end tidal gauge records have been used.

ANALYTICAL STUDY

The governing equations of motion, (1) and (2), reduce to the classical wave equation, for estuaries of constant depth and cross sectional shape, and negligible friction effect.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \qquad \text{.....(5)}$$

$$\frac{\partial^2 \zeta}{\partial t^2} = c^2 \frac{\partial^2 \zeta}{\partial t^2} \qquad \text{.....(6)}$$

in which:

u = tidal current;

c = wave celerity;

ζ = water surface elevation elative to the mean depth.

The two foregoing equations could be satisfied by a single harmonic function

$$\zeta = \zeta_{max} \text{ Cos } (ft - kx) \qquad \text{.....(7)}$$

$$u = \frac{\zeta_{max}\, c}{d} \text{ Cos } (ft - kx) \qquad \text{.....(8)}$$

in which:

f = tidal wave frequency = $2\pi/T$;

k = wave number = $2\pi/L$;

L = wave length;

T = wave period;

ζ_m = maximum amplitude;

d = mean water depth.

In short frictionless estuary the tidal wave will be reflected at the head of the estuary. For a complete, reflection the reflected wave is given by

$$\zeta = \zeta_{max} \text{Cos} (ft + kx) \qquad .....(9)$$

and the corresponding current is given by

$$u = \frac{\zeta_{max} c}{d} \text{Cos} (ft + kx) \qquad .....(10)$$

superposition of the incident and reflected wave gives

$$= 2 \zeta_{max} \text{Cos} (ft). \text{Cos} (kx) \qquad .....(11)$$

The corresponding current is

$$u = \frac{2\zeta_{max} c}{d} \text{Sin} (ft) \text{Sin} (kx) \qquad .....(12)$$

which represents a case of standing oscillation maximum velocities occur at time of mean water level that is at half tide.

The tidal elevation and tidal currents in real systems can be considered to be a combination of large number of incident and reflected tidal constituents, each have its own amplitude, wave length and frequency.

The reflection occurs at the head of the estuarg is not a complete reflection, mainly due to the presence of fresh water dischargeand boundary resistance. The incident wave will suffer frictional dissipation and convergence of the estuary cross section, while the reflected wave surffers divergence of the channel cross section and frictional dissipation of energy as well. For these reasons the value of maximum tidal current may not occur exactly at half tides Fig.(13), and the ebb period will be larger than the corresponding flood period.

RESULTS AND ANALYSES

The total energy, which consists of kinetic energy and potential energy, of a wave could be given by, Ref.(6)

$$E = \frac{1}{8} \cdot \gamma \cdot H^2 . L \qquad .....(13)$$

in which; E is the total energy, H = wave height, and L is the wave length.

Wave not only possesses energy but also transmit this energy in the direction of motion. This is given by the equation:

$$\frac{dE}{dt} = \frac{1}{8} \gamma . H^2 . c \qquad .....(14)$$

in which; c is the wave celerity.

When the wave propagates in the upstream direction of an estuary or a gulf, refraction could occur, and the wave height will increase because of a fixed amount of energy per unit length is being confined within an increasing narrow passage.On the other hand, as the wave travels in the upstream direction frictional dissipation of energy will occur and this could decrease the tidal range. Another complicated problem will exist for an estuary of finite length, reflection at the head of the estuary which could increase the wave amplitude in the upriver.

For reasons mentioned before, it seems difficult to predict what will happen to a tidal wave when propagating in the upstream direction. Fig.(4) gives the longitudinal hourly intervals for spring tide, normal discharge and normal friction coefficient. The figure demonstrates the decrease of tidal range when going in the upstream direction, steepening is noticed in the flood tide. As the river flow increases, damping of the wave increases, resulting in smaller amplitude of wave Fig.(5). More distortion is also noticed especially in the upriver.

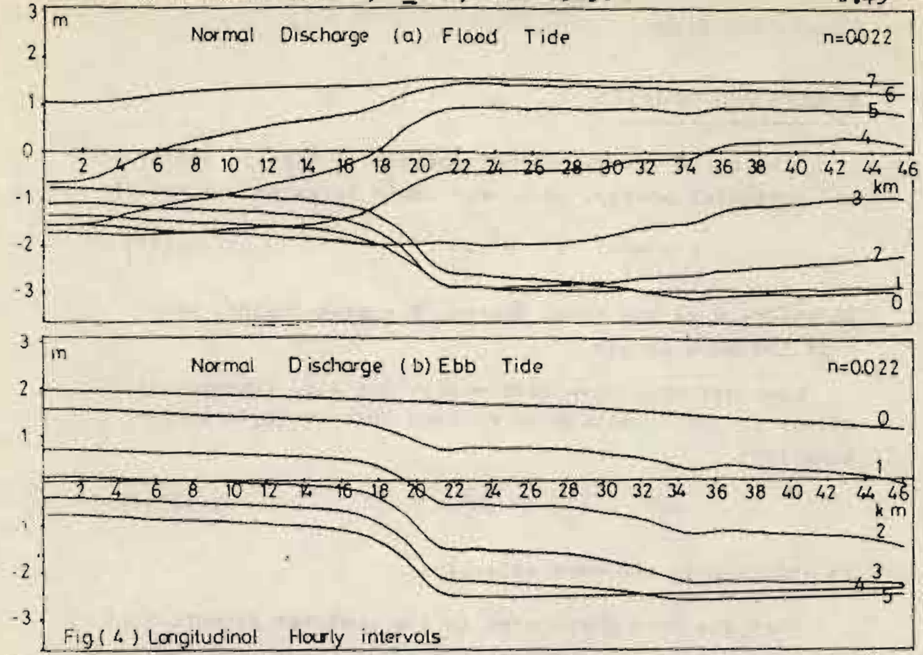

Fig.(4) Longitudinal Hourly intervals

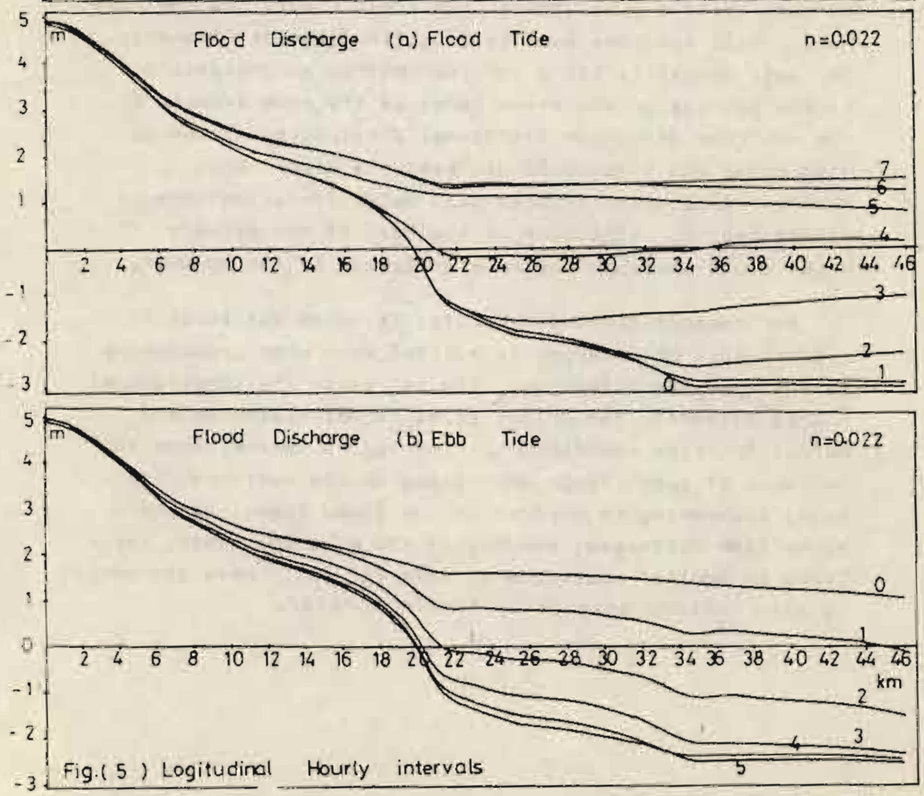

Fig.(5) Logitudinal Hourly intervals

When the friction coefficient increases the tidal range decreasee, Fig.(6), due to more dissipation of the wave energy. In the condition of flood discharge more damping of tidal wave will occur resulting in a more decrease of the tidal amplitude which reaches a negligible value in the upriver.

Amflification of wave amplitude could occur in the up river for lower value of friction coefficient, Fig. (8). This amplification becomes infinite if the tedal period of any tidal component approaches the value $4l/\sqrt{gd}$. in which; l is the estuary length. The phenomenon is called resonance. As the river flow water increases damping will occur which prevent such resonance. Fig. (9).The decrease of boundary resistance will have an effect of shape of tide, Fig.(10) show the size of tidal hump is bigger. for n = 0.01 than the corresponding size for n = 0.01 especially at the upstream stations. Low boundary resistence will change the amplitude, phase, and frequency of the tidal consitudents resulting in the shape of wave given in figure.

Comparison between Fig.(11) and Fig.(12) shows the resonance is more effective at upstream station, as the reflected wave at the head of the estuary, will suffer frictional dissipation of energy and a channel divergence which decrease its amplitude in going downstream.

The resonance frequency is equidistant i.e. the difference between adjacent frequencies is constant and is about 2 hours Fig.(12). In the abscence of damping factor, which is difficult to obtain, it may be said that the semi diurnal tide is responsible for this phenomenon. The resonance frequency of tidal current, Fig.(12), is then same as for the tide wave,which could mean the corresponding current for that partial tide is responsible for the irregularities of the tidal current.

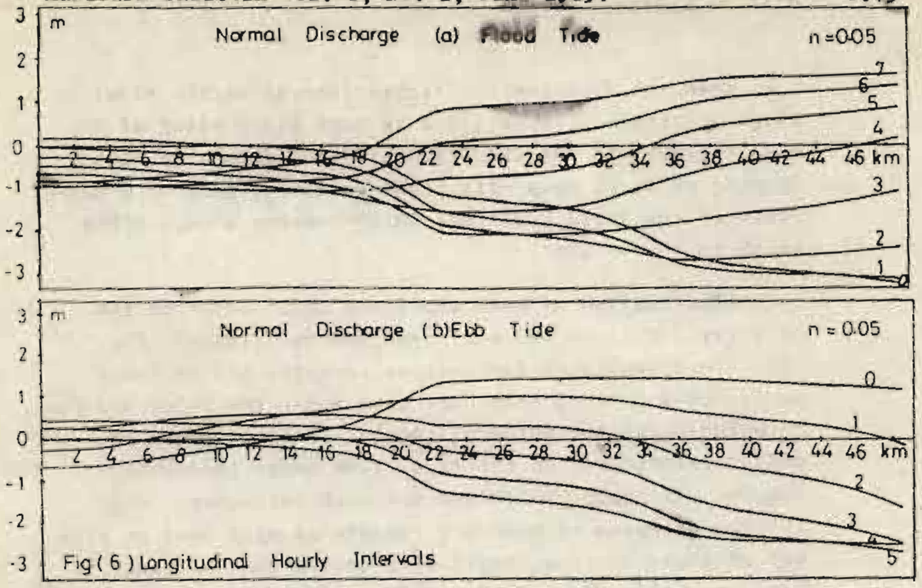

Fig (6) Longitudinal Hourly Intervals

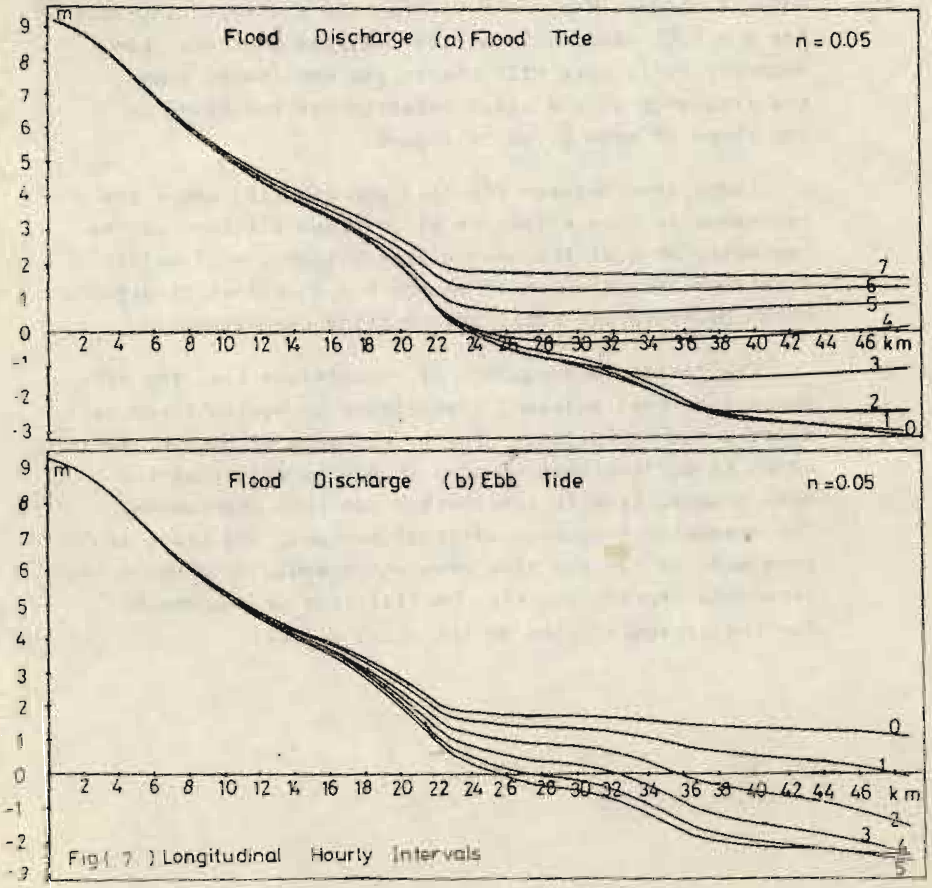

Fig (7) Longitudinal Hourly Intervals

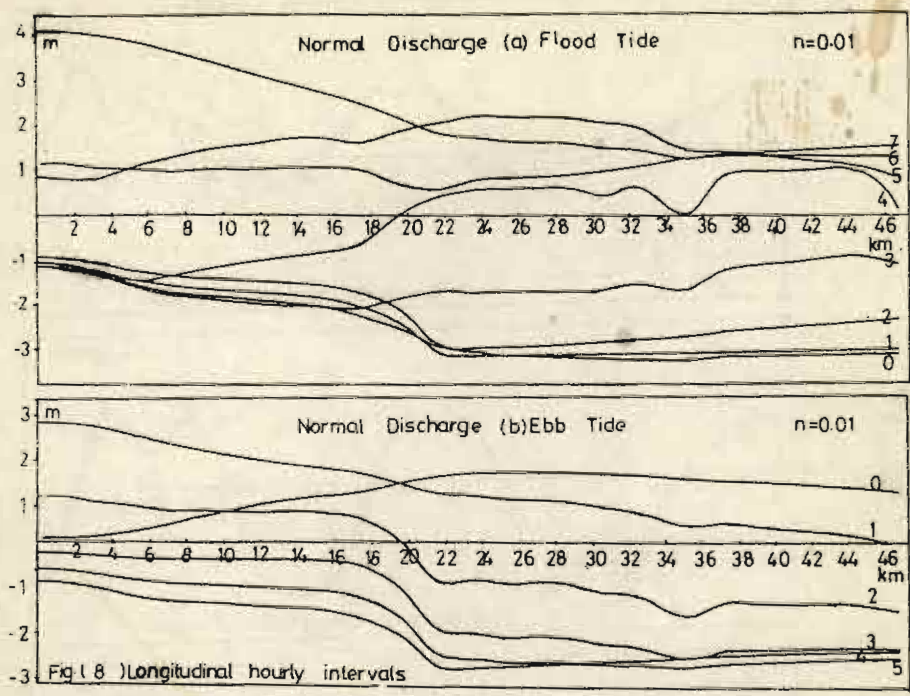

Fig (8)Longitudinal hourly intervals

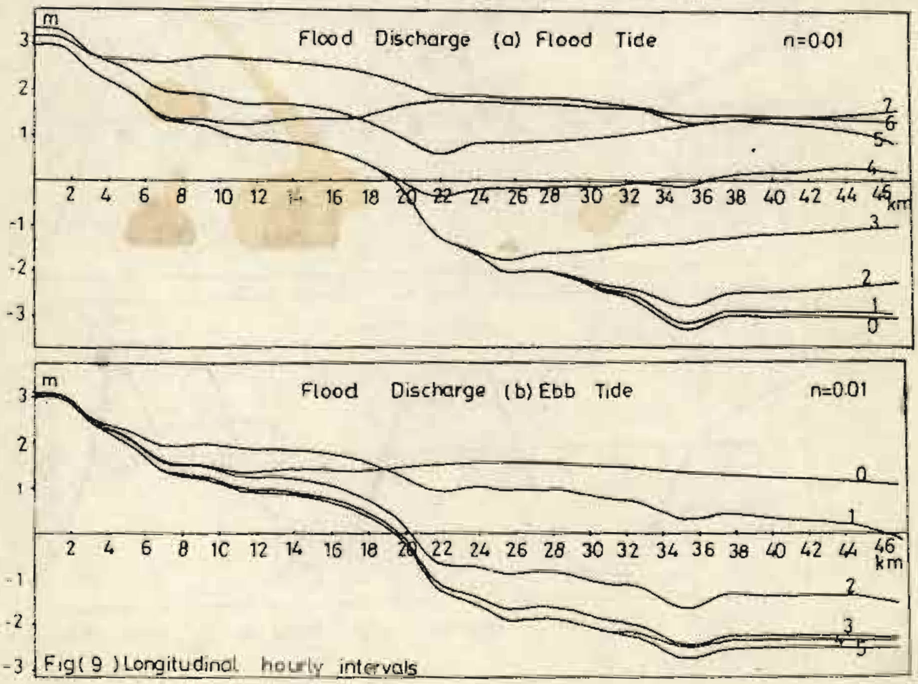

Fig (9)Longitudinal hourly intervals

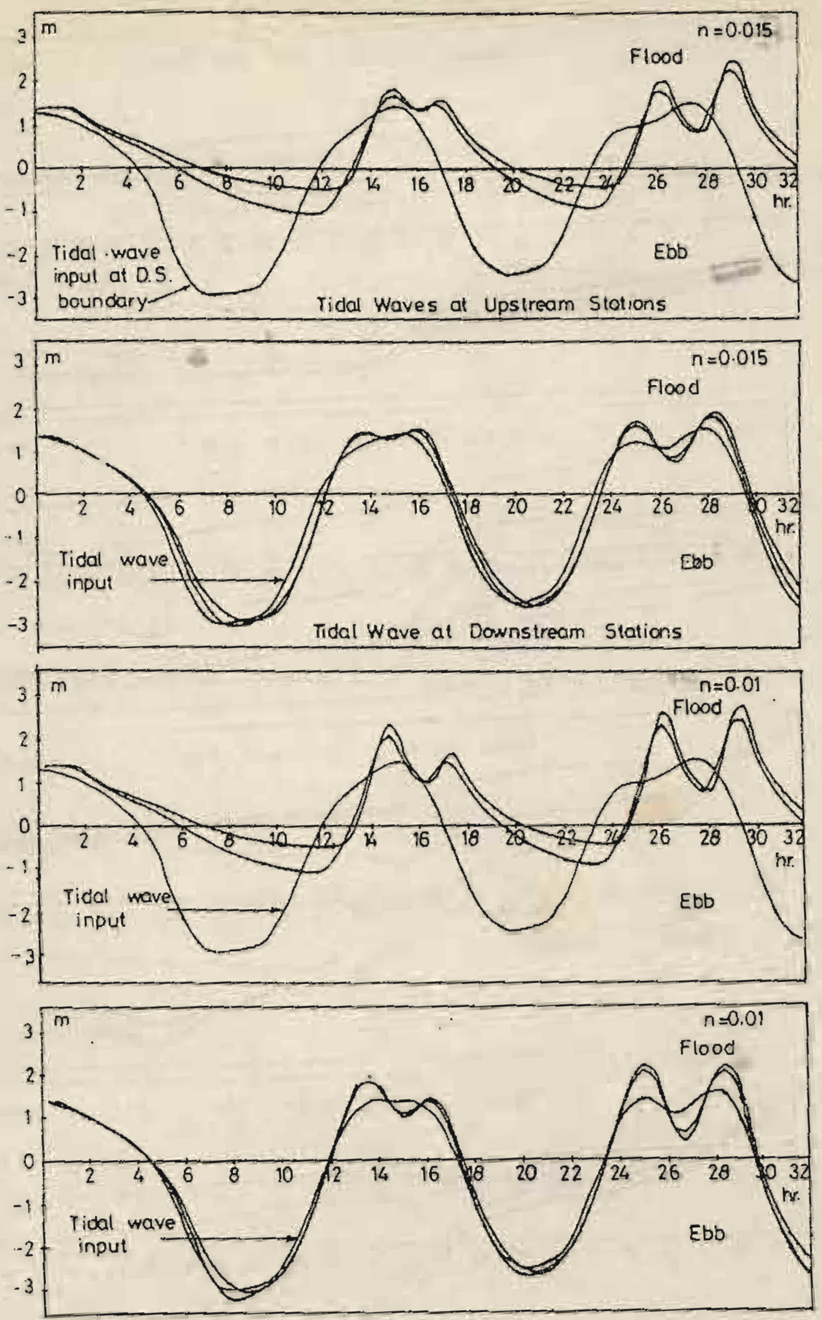

Fig.(10) Influence of Boundary Resistance on the Shape of Wave

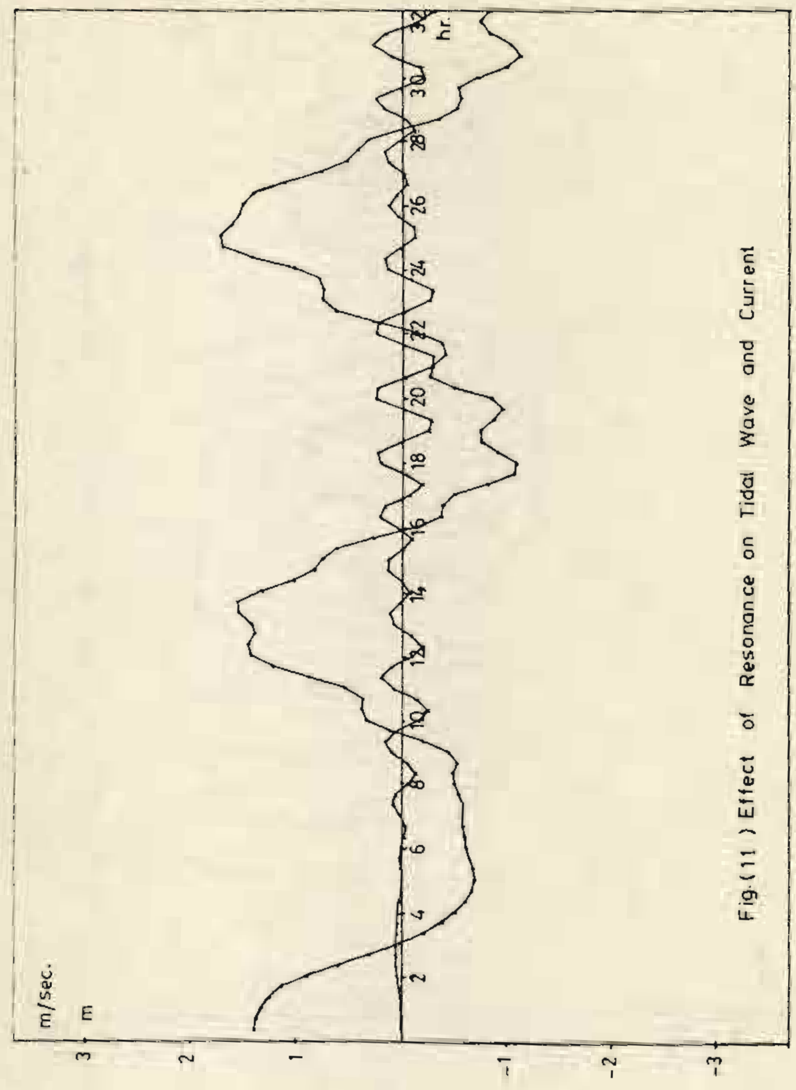

Fig.(11) Effect of Resonance on Tidal Wave and Current

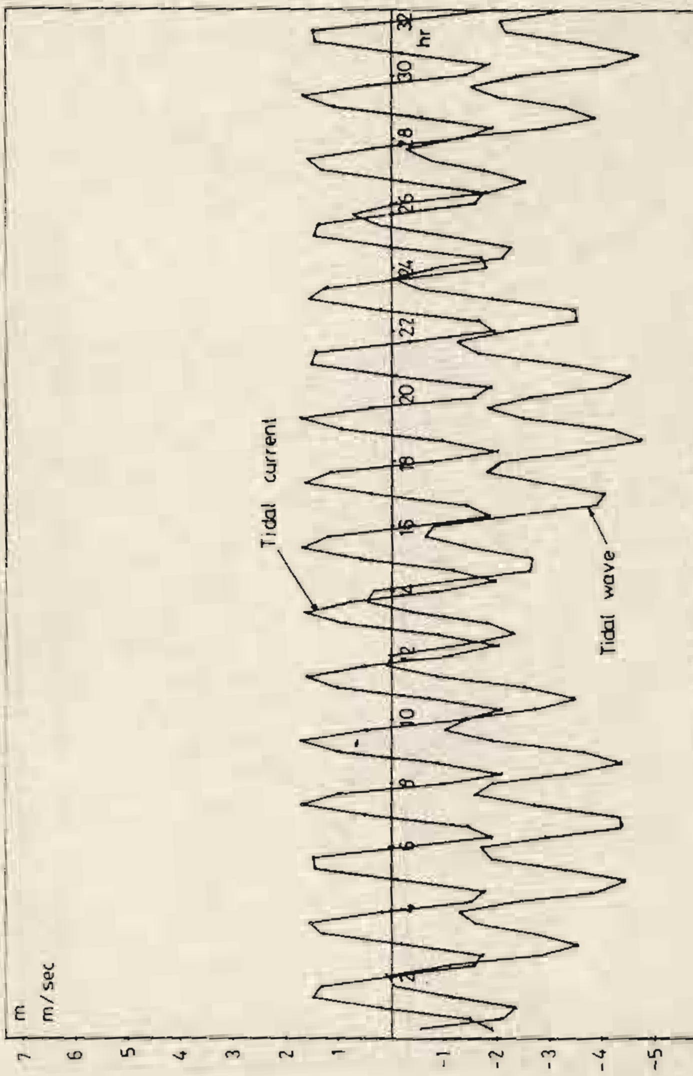

Fig.(12) Effect of Resonance on Tidal Wave and Current

Due to coonplexity of tidal current, no theory of tidal current has been advanced which has found general acceptance by the hydraulic engineers. Stevenson (1872) stated the tidal wave and tidal current are two seperate phenomena, he considered that the tidal current is entirely due to the slope of water surface. West (1973) introduced an approximate linear relationship between tidal current and tidal range in the Tay estuary of Scotland Ref.(2). Considering the strength of the ebb and flood currents, the ebb flow will drain off flats tends to concetrate in the main channel as a consequence generally to be faster as in the Tay estuary Ref.(7). This method could be reversed in some estuaries i.e. the strength of flood will be stronger than the corresponding ebb current, due to the topographic feature of the estuary as in the river Houghly of India Ref.(8).

Deepening of an estuary could increase the flood duration and decrease the ebb period. In the Lune estuary of England, deepening the main channel by about 1.0 m over a length 7Km. The duration of flood tide increased from 3h 20 min to 3h 45 min. The ebb tide was shortened by a curresponding amount. Extensive deepending can have a measurable effect on tidal propagation, reduces the maximum flood tide velocities and increases the ebb tide velocities Ref.(8).

The hydraulic investigation of the river Forth of Scotland has revealed that at the mouth of the estuary strength of tidal current is proportional to the tidal range and when the wave travels in the upstream direction, the current is a function of the cross sectional area, frictional dissipation of energy, reflection of the tidal wave and type of tide Ref.(11).

The tidal wave has a sinusoidal shape in the deep water, for $L/d \lesssim 2$. When the wave propagate in a shollow water distortion for both tidal wave and tidal current will occur. Fig.(13) gives the shape of both tide wave and tidal current

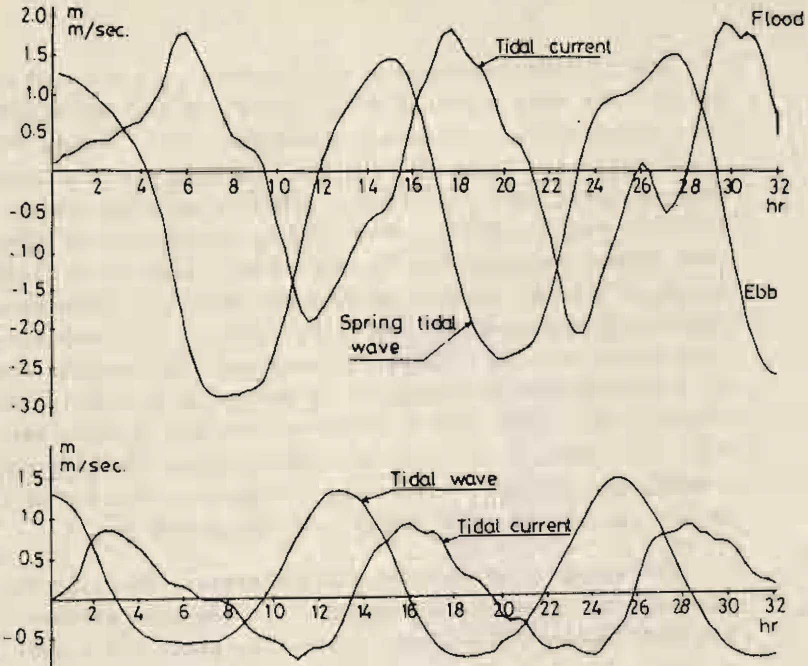

Fig.(13)

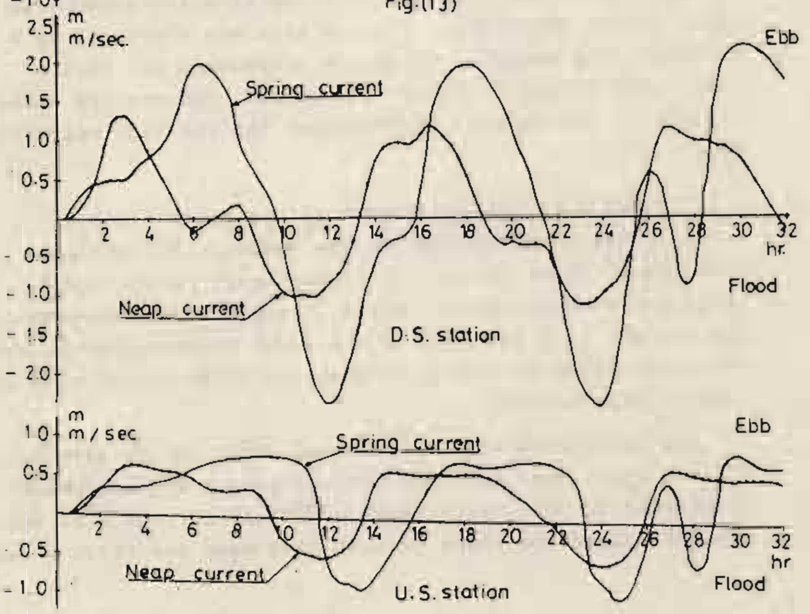

Fig.(14) Distortion of tidal current

for spring and neap tides, the profiles of both tide wave and tide current are more or less sinusoidal but as the wave propagate in the upstream direction distortion will occur for both tidal wave and tidal current Fig.(14).

CONCLUSION

Tidal motion in any estuary is a turbulent unsteady non uniform flow which is mainly governed by the boundary resistance, river flow and downstream tide wave. Difficulties lie in predicting the water surface elevation or tidal current in any analytical way. The accuracy of mathematical tidal flow model is as good, if not better than the tradiditional physical model, provided care is taken to minimise numerical schematization error and the friction parameters are determined from field results.

Bed slope, friction effect and change in channel geometry have a direct effect on both the tidal current and tidal wave. A configuration between estuary mean depth, boundary resistance and estuary length may lead to the phenomenon of resonance. This will effect both the tidal current and tidal wave. An increase of fresh water discharge and/or boundary resistance will dampen the tidal amplitude and prevent the phenomenon to occur. The tidal current has been found more sensitive to this resonance than the tidal wave.

Dredging of an estuary or any tidal water way will decrease the boundary resistance according to the formula $g \frac{u|u|}{C^2 R}$ or $\frac{g\, u|u|}{C^2 d}$ for wide channel. This increases the tidal range, increases the flood duration, decreases the ebb period,and the tidol wave will be less distorted. Further and extensive deepening could lead to a resonance.

The increase of boundary resistance and/or river flow will distort the shape of tidal wave, further increase of one of these two parameters could result in changing the character of the wave from being oscillatory to a progressive type.

Propagation of the tide wave along the river could result in steepening of the flood tide. Once a steep fronted wave is formed vertical accelerations become large and may lead to the formation of highly turbulent travelling surge or a bore. The dynamic equation satisfies quite well the conditions upstream and downstream the bore, but it is not applicable in the bore itself. The equation has to be modified to cope with such situation.

APPENDIX (I) REFERENCES

(1) Abbott, M.B.,"Computational Hydraulics" Elements of Theory of Free surface Flow, Pitman Advanced Publishing Program, London, 1980.

(2) Charlton et al., "Tidal and Fresh Water Induced Circulation in the Tay Estuary" Proceeding of the Royal Society of Edinburgh (B), 75, 2, 1974/1975.

(3) Cunge J.A., et al., "Practical Aspect of Computational Hydraulics", pitman Advanced publishing program, London, 1980.

(4) Einstein and Fuchs, "Computation of Tides and Tidal Currents", United State Practice, Proc. A.S.C.E. Journal of the Hyd. Div., June 1955.

(5) Ellis, J., "Unsteady Open Channel Flow" Annual Meeting of Hydraulic Specialists, Portsmouth Polytechnic, September 1979.

(6) Henderson, F.M., "Open Channel Flow", Macmillan Co. Ltd, London, 1966.

(7) Huthance, J.M., "Tidal Current Asymmetric over the Norfolk Sandbanks", Estuarine and Coastal Marine Science, 1973, 1, pp. 89-99.

(8) MacDowell, D.M., and O'Conner, B.A. "Hydraulics Behaviour of Estuaries" Macmillan Press Ltd, London, 1980.

(9) Proudman, J., "The Effect of Friction on a Progressive Wave of Tide and Surge in an Estuary" Proc. Roy. Soc., London, A., 233, 407.

(10) Thijisse, J.T.H., "International Course in Hydraulic Engineering" Delft, Netherland, 1965.

(11) Zidan, A.R., "A Hydraulic Investigation of the River Forth", Ph.D. Thesis, Strathclyde University, Scotland.

APPENDIX (II) NOTATION

The following symbols are used in this paper

A = water area;
b = instantaneous top width;
C = Chezy coefficient;
c = wave celerity;
d = mean water depth;
E = wave energy;
f = wave frequency = $\frac{2\pi}{T}$;
g = acceleration due to gravity;
H = wave amplitude;
h = water surface elevation;
k = wave number;
L = wave length;
l = estuary length;
n = Manning's coefficient,
R = hydraulic radius;
S_o = bed slope;
S_f = friction slope;
T = wave period;
u = water velocity;
W.L. = water level;
δx = space increment;
δt =time increment;
ζ = water surface elevation relative to mean depth, and
ζ_{max} = maximum amplitude of wave.