APPLICATION OF ENERGY LOOPS TO STUDY THE STABILITY OF SYNCHRONOUS COMPENSATOR COUPLED WITH RIGIDLY-IMPULSED INVERTER TO SUPPLY ISOLATED AC NETWORKS

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ABSTRACT

The combination of a rigid-impulsed inverter and a synchronous compensator for supplying an isolated AC load, is one of the moet economical solutions for adapting the raw electric energy supplied from a DC link. This system has an oscillating nature due to the lack of synchronisation between the air-gap rotating field and the rotor field inside the compensator machine. The resulting oscillations are actually too small but may lead to system instability or inverter-failures. They are affected by the system paramters and the applied type of control.

This paper presents the energy-loops as a helpful tool in predicting wheather a proposed system of such a structure will maintain stability or not. Thereby an optimal dimensioning of the system parameters can be ensured.

0.0 LIST OF SYMBOLS

V := infinite bus-bar voltage ;
V_o := common terminal voltage in the proposed system ;
E := voltage behind the compensator stator leakage reactance ;
E_f := AC excitation voltage ;
f := infinite bus-bar frequency ;
f_o := rigid inverter frequency ;
I_{in} := AC inverter current ;
I_s := compensator stator current ;
I_L := load receiver current ;
R_s := compensator stator resistance ;
X_s := compensator stator synchronous reactance ;
X'_s := compensator subtransient-reactance ;
X_e := subtransient-reactance of series winding with the syn. m/c ;
Z_s := compensator synchronous-impedance ;
J := inertia-coefficient of compensator-rotor ;
D := damping-coefficient due to friction and windage losses ;
T_{DO} := damping torque ($=D.w_o$) , due to P_{f+w} ;
Δw_r := speed difference between the rotor speed w_r and the reference angular frequency w_o , in elec. rads. ;
α = arctan (R_s/X_s) ;
β := advance angle of the inverter , in elec. rads. ;
δ := load angle of the compensator , in elec. rads. ;
γ := commutation angle , in elec. rads.

All the above variables and constants are considered in SI units.

1.0 INTRODUCTION

Electric power generated by special projects, such as windmills, shaft-driven alternators , or solar energy plants ; usually needs to be adapted to suit the specifications of isolated AC networks. In particular, the frequency and the terminal voltage must be constant.
One of the most economical solutions for this problem is the use of a rigidly impulsed inverter, Fig.(1), to receive the generated power from a DC-link, which will be supplied in its adapted form to the load. In this case a synchronous compensator will be needed across the inverter AC-side to ensure natural commutation and to supply the reactive power required by both load and inverter. Triggering impulses of the inverter must be generated independently and its frequency must be equal to the load frequency.

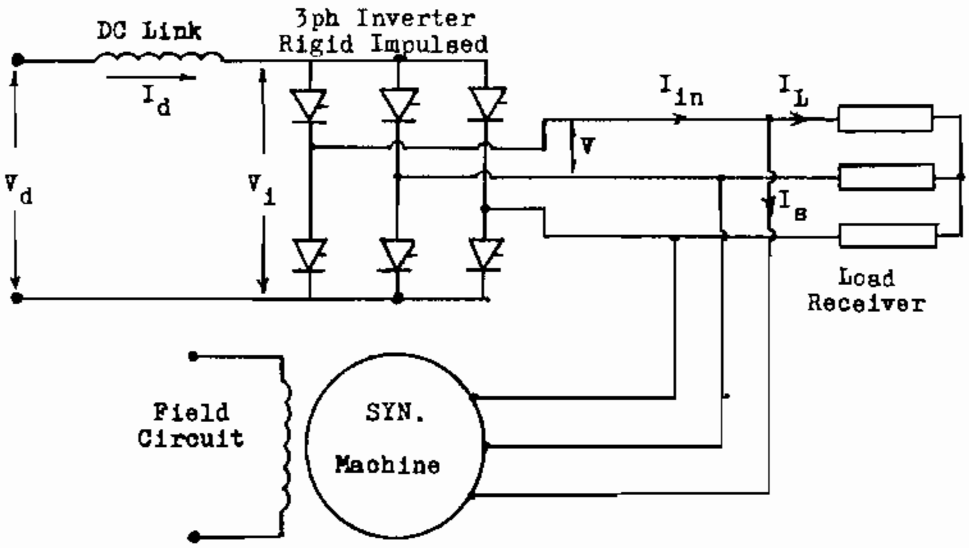


Fig.(1): Schematic Diagram.

Stability of such a system is affected to a great extent by the its parameters and the applied type of control. If these parameters are not dimensioned precisely, it is expected that the compensator machine will operate under oscillating conditions.
The suggestion of feed-back control between the rotor angular-position and the impulser frequency , to prevent these oscillations, must be avoided; in order to hold the load frequency constant. Therefore, the system may be assisted by one or both following conventional controls ;

i. control of compensator excitation ;
ii. control of voltage behind the smoothing reactor.

The first type of control is suggested to maintain the terminal voltage constant. The second type aims to hold the advance angle β of the inverter at a proper constant value.
Introducing both controls improve the overall system stability. Accordingly, it seems necessary to study the stability of such a system in order to ensure a higher degree of reliability,

avoiding inverter failures, and minimising the overall capital costs [5].
In this paper, a stability study of synchronous compensator operating across a rigidly impulsed inverter will be carried out. It presents the energy loops as a suitable method of investigating the stability of these systems.

2.0 PRINCIPLE OF ENERGY LOOPS

In a synchronous machine under normal running condition, the relative velocity between stator and rotor fields is zero. Whenever this relative velocity is disturbed, synchronising restoring torques tending to maintain this equality come into play. This tendency of the rotor to attain synchronous speed after a slight disturbance, is the basic reason for the analysis of synchronous machine dynamics. This analysis is based on the electromechanical or dynamic equation which describes the synchronous machine behaviour under oscillating conditions. This equation in its primitive form is

$$T_e + T_m = T_j \qquad (1)$$

where T_e , T_m and T_j are the electrical developed torque, the mechanical torque and the inertia-torque, respectively.
A synchronous compensator is actually an over excited synchronous motor operating at no-load. Therefore, a better understood of the compensator behaviour, when operating with infinite bus-bars under both steady and oscillating conditions, can be gained when the same behaviour of the motor is firstly recalled.

2.1 Synchronous Motor Operating with Infinite Bus-bars :

The above dynamic equation is the base of the proposed analysis. In this equation, the developed electrical torque, T_e, consists mainly of two components: the synchronous torque T_s and the asynchronous torque T_{as}. The first component is due to the angular position or load angle δ between the machine and the bus-bars. In other words, it is due to the phase difference between the excitation voltage E_f and the terminal voltage V, Fig.(2-a).
This torque can be determined by :

$$T_s = (p/w_o).E_f.I_s.\cos(\varphi_s+\delta) \qquad (2\text{-a})$$

or

$$T_s = (p/w_o).\ V.E_f.\sin(\delta+\alpha)/Z_s - (E_f/Z_s)^2.R_s \qquad (2\text{-b})$$

For simplicity, the following relation can be also used :

$$T_s = (p/w_o).(V.E_f.\sin\delta)/X_s \qquad (2\text{-c})$$

The second component, T_{as}, is due to any difference in the angular velocities of the air-gap rotating field, established by the stator currents, and the rotor field. This slip speed gives rise to the production of asynchronous torque ,by the machine damping circuits, in such a direction to minimise the difference in the two angular velocities. This torque can be determined by [4] :

$$T_{as} = (p/w_o.R_k).(E.X_s'/(X_e+X_s'))^2.(1-w_r/w_o) \qquad (3)$$

It depends on the air-gap voltage E and the subtransient reactance X_s''. Naturally, the equivalent resistance for the damping circuits R_k plays a great role in the reeulting damping effects. Also, if the mentioned difference in the angular velocities is varying with time ; inertia torque T_j will be estabilshed. It can be determined by :

$$T_j = (J/p) \cdot dw_r/dt$$

or

$$T_j = (J.w_o/p) \cdot d(w_r/w_o)/dt \qquad (4)$$

Consequently, and as $T_m = -T_{sh}$ in synchronous motor operation, equation (1) becomes :

$$T_s + T_{as} = T_j + T_{sh} \qquad (5)$$

Under steady-state, the synchronous torque will be equal to the shaft load torque, $T_s = T_{sh}$, and the operating point will depend on the angular position δ. This point moves along the ordinary torque-angle curve of synchronous machine connected to infinite bus-bars. Here and due to exsisting mechanical output, a complete active energy conversion process takes place, and such a curve is actually available.
Distubances in motor operation may be caused by sudden change of load. A sudden increasing ,for example, in the shaft load must reduce the motor speed momentarily so that load angle δ inorease to supply the additional load. Before the rotor settles down to a new value of load angle, the rotor oscillates about its final equilibrium position. As a result of it, the rotor speed fluctuates around synchronous speed. The nature of these oscillations, either large or small, decides the method by which the electromechanical equation will be solved. This equation is a non-linear equation in δ and it can be linearised for both small and large oscillations [2]. Stability creterions, such as the equal area creterion, had been derived in order to predict wheather the machine will maintain stability or not.

2.2 Synchronous Compensator Operation with Infinite Bus-bars :

2.2.1 Steady-state Operation

In case of operating the synchronous machine as a compensator, the mechanical torque T_m is only to overcome the friction and windage torques T_D inside the machine. Therefore, equation 5 can be rewritten as :

$$T_s + T_{as} = T_j + T_D \qquad (6)$$

Under steady-state conditions ,$w_r = w_o$, both T_s and T_{as} will be equal zero and equation (6) yields that :

$$T_s' = 0 \qquad (7)$$

where

$$T_s' = T_s - T_D \qquad (8)$$

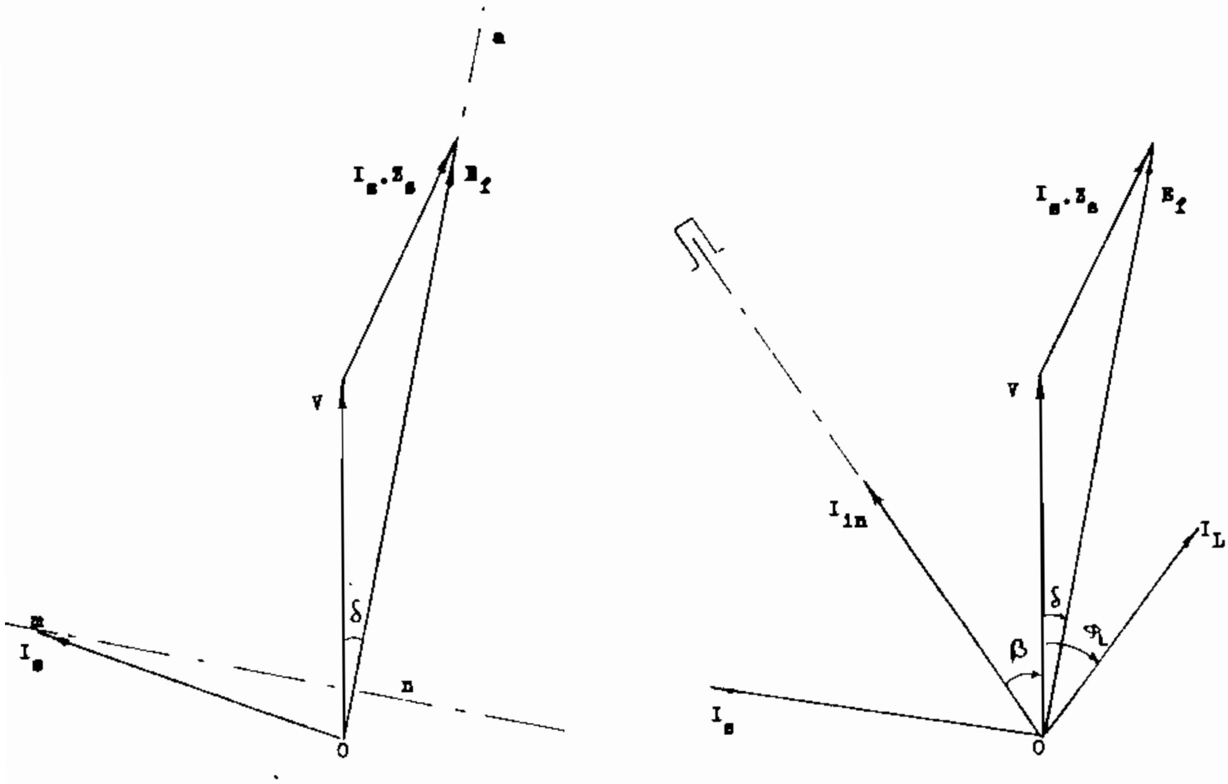
In a synchronous compensator the relation between I_s and E_f is represented by the V-curve of synchronous motor operating at no-load.

This relation depends mainly on the excitation level. Increasing the excitation, for example, results in increasing E_f and, in turn, in increasing both I_s and φ_s. In this case the machine will operate at a more leading power factor and delivers more reactive power. In accordance with the synchronous torque developed, the increase in both E_f and I_s will be compensated by the increase in φ_s, Eq.(2-a), and it will be constant to a great extent. Variations in the load-angle due to varying excitation level are negligible, and δ will be constant as long as w_r is constant. The corresponding complexor phasor diagram is given in Fig.(2-a). It is shown that oa and mn will be the loci of the excitation voltage E_f and the stator current I_s, respectively. Thus, for compensator operation under steady conditions, a Torque-Angle curve is not excisting. The machine has a unique operating point δ_o, at which T_s=T_D.

2.2.2. Oscillating Operation

Assuming the compensator is running at synchronous speed, i.e., the slip speed, $\Delta w_r = w_o - w_r$, is equal to zero. Any small oscillations in the angular position of the compensator rotor with respect to the rigid reference V, may be happen due to sudden variations in the excitation level, or due to lack of natural damping. In this case the angle δ will oscillate around its nominal value, δ_o , which is assumed to be constant. Assume that, as the rotor starts to oscillate, at $\delta = \delta_{a1}$, the speed difference Δw_r starts increasing from zero in opposite direction to rotor speed. The resultant speed will be less than the synchronous speed. The rotor will decelerates to minimum speed at δ_{b1} and then accelerates to synchronous speed at δ_{a2} as shown in Fig.(3). As Δw_r increases from zero at δ_{a1} , it has its maximum value at δ_{b1} and then decreases to zero at δ_{a2}. For $\delta_{a1} \leq \delta < \delta_{b1}$, the synchronous torque T_s is less than T_D ,i.e. T_s' is negative, and the corresponding power P_s is not able to supply the higher damping losses P_D. It is known that for rotary motion : (torque) x (angular displacement) gives work done and represents energy, Therefore, the area A1 under the T_s'-curve and the line of T_s'=0, while δ is increasing from δ_{a1} to δ_{b1} ; represents the total energy extracted from the rotating mass. As the rotor-speed starts to be less than w_o ; T_{as} will be established in its motoring form ,positive slip, in order to accelerate the retarding rotor. It has its maximum value at δ_{b1} ,minimum speed, and then decreases to zero at δ_{a2} , where $w_r = w_o$. As the position δ_{b1} is reached, both electrical torques T_s and T_{as} are assisting each other to recover the rotor kinetic-energy loss. Therefore, the rotor speed starts increasing at δ_{b1} to reach synchronous speed at δ_{a2} (Δw_r= 0), at which the lost kinetic energy is replenished. The area A2 under the T_s'-curve and the line of T_s' = 0, while δ is increasing from δ_{b1} to δ_{a2} ; represents the stored energy in the rotating mass as both T_s+T_{as} are greater than T_D. Now, it can be found out wheather the machine stability is maintained or not :

i. If area A2 greater than area A1, the compensator remains in synchronism.
ii. If area A2 equal area A1 , the synchronism is just maintained.
iii. If area A2 less than area A1 , the compensator falls out of step.



a with Infinite Bus-Bars b with Inverter

Fig.(2): Compensator Phasor Diagram.

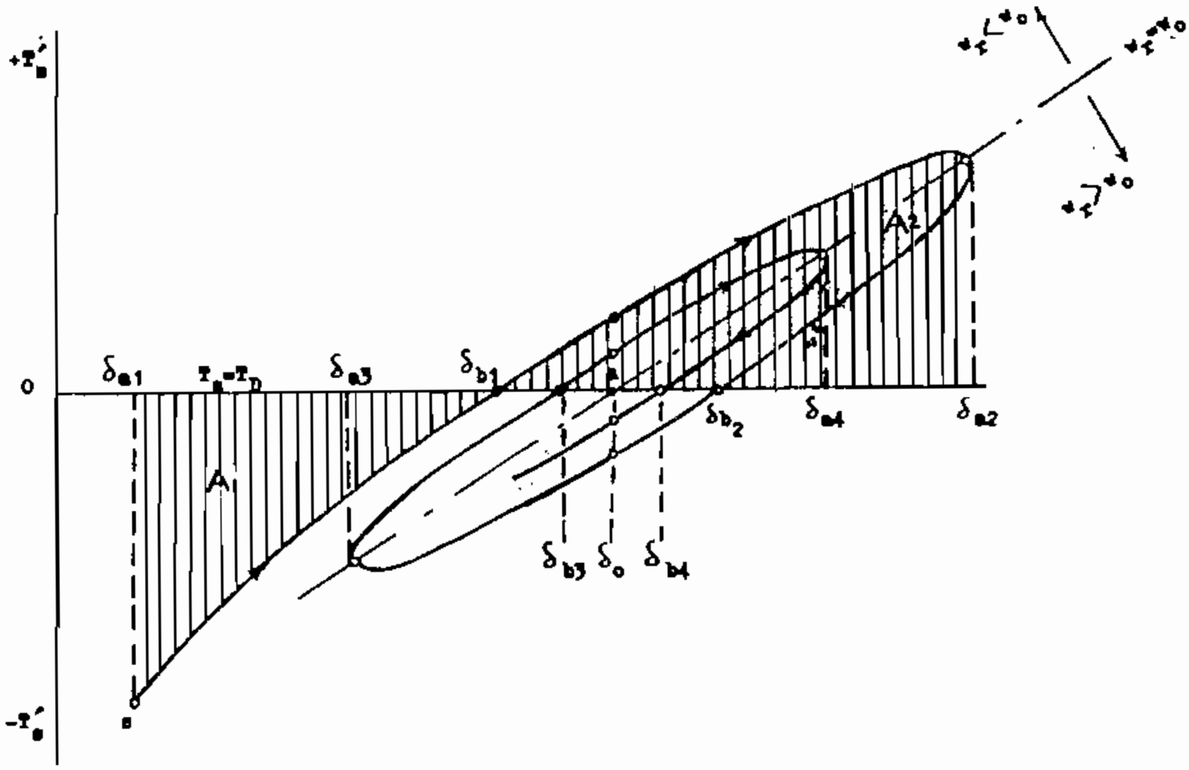


Fig.(3): Energy-Loops.

To be more sure of stability maintaining, some few successive oscillations must be studied in order to observe whether the operating point approaches the equilibruim point 'a' or not [4]. The analysis of the second oscillation beginns at δ_{a2} where the rotor is running at synchronous speed and both Δw_r and T_{as} are equal to zero, but the electrical developed torque T_s is maximum and greater than T_D. Therefore, rotor speed must increase beyond w_o. Now Δw is in the same direction of rotor speed and the load angle δ begins to decrease. On the back-ward journy from the operating point corresponding to δ_{a2}, the asynchronous torque T_{as}, beginns to be established in its generating form in order to retard the accelerating rotor. In addition, T_s will be decreasing, due to decreasing δ , to be equal to T_D at δ_{b2}. At this moment the rotor has the maximum speed beyond w_o and stops to storage energy. The energy stored in the previous period, while δ is decreasing from δ_{a2} to δ_{b2}, is given by area A3 under the corresponding portion of the T_s'-curve and the line of $T_s' = 0$. Continueing at the same moment, δ_{b2} , it will be observed that T_{as} has its maximum value while generating action beginns to be decreasing. Also T_s will be smaller than T_D and the rotor energy starts to be extracted as a trial to accelerate the decreasing rotor-speed. The load angle δ continues its decrease to reach δ_{a3} , at which Δw_r becomes zero and $w_r = w_o$. At this moment the asynchronous torque T_{as} is equal zero and stops the process of extracting the stored rotor energy which begun at δ_{a2}. The area A4 under the T_s'-curve and the line of $T_s' = 0$, while δ is decreasing from δ_{b2} to δ_{a3} , represents the extracted energy from the rotor mass through this oscillation. For stable operation the principle of equal areas still applies ; A3 = A4. Thereafter, the rotor speed will continue to decrease and a process similar to that was happen at δ_{a1} will start. Following the successive oscillations, it will be found that the curve T_s' against δ forms a spiral of approximately elliptical form ; the energy loops, as shown in Fig.(3).

Now, final decision about the machine stability can be formulated:

a- If each successive oscillation will be smaller and the T_s'-curve converges the point of equilibrium 'a' at δ_o ; the compensator machins maintains stability.

b- If each successive oscillation will be larger and the T_s'-curve diverges the point of equilibrium ; the compensator machine will go out of synchronism.

3.0 SYNCHRONOUS COMPENSATOR OPERATION WITH RIGIDLY IMPULSED INVERTER AND ISOLATED LOAD

This mode of synchronous machine operation is a different one. Neglecting higher harmonics, the complexor phasor diagram of the compensator ,Fig.(2-b), under steady-state can be constructed. The diagram shows also the load- and inverter-currents with respect to the common terminal voltage V_o. The load is assumed to be supplied at lagging power-factor and the inverter current $-I_{in}$, leads V_o by the advance angle β_o.

In such a system and under all circumstances the principle of power balance must be valied. In other wards the active power consumed by the machine and load be equal to that supplied by the inverter.

Also, the reactive power consumed by the inverter and load must be equal to that generated by the compensator.
Accordingly, and having the terminal voltage as reference :

$$(I_{in})_a = (I_s)_a + (I_L)_a \tag{9-a}$$

and

$$(I_s)_r = (I_{in})_r + (I_L)_r \tag{9-b}$$

As the system is assumed now to be steady; the load and compensator will have the same frequency as that of the inverter. Consequently, the phasor diagram will rotate with the same angular velocity $w_o = 2\pi f_o$ where f_o is the inverter frequency. Conventional analysis of synchronous compensator can be applied just in this moment and the angle δ between E_f and V_o can be taken as a measure of synchronous torque T_s ; so long $w_r = w_o$. As it is mentioned before, the system under consideration has an oscillating nature due to lack of enough damping or due to lack of proper control. Under oscillating conditions, the rotor (in turn E_f) will be oscillating around an equilibrium position with the natural frequency which depends on excitation level, synchronous impedance, and coefficients of inertia and damping ; J and D respectively. For normal excitation, $E_f = V_o$, and neglecting damping torque [3], this frequency can be determined by :

$$f = (1/2\pi) . \sqrt{V_o . I_{sc} / J . w_r} \tag{10}$$

where I_{sc} is the short-circuit current corresponding to normal excitation. This frequency is a result of solving the corresponding dynamic equation which is the differential equation of a simple harmonic oscillation :

$$J . (d^2\delta/dt^2) + K_s . \delta = 0 \tag{11}$$

where K_s is the synchronous torque-coefficient :

$$K_s = V_o . I_{sc} / w_r$$

The use of the damping winding is to damp out these oscillations and to diminish the above mentioned frequency. The corresponding damping torque plays a great role in stabilising the system. It can not be actually neglected here as the shaft torque is too small ; $T_{sh} = -T_D$. For this reason the dynamic equation, Eq.(11), does not represent the oscillating behaviour of the compensator under consideration, another reason is due to the rigid reference of frequency. Equation (11) is derived mainly in δ, the phase-difference between the rotor voltage E_f and the terminal voltage V_o. In our case, V_o is not an applied voltage. It is actually created by the compensator machine itself and does not represent a rigid reference of frequency or angular velocity as in case of the operation with infinite bus-bars. It is also oscillating in a different manner as long as E_f is oscillating and does not have a constant magnitude. A rigid frequency-reference within the machine variables can not, unfortunately, taken. It remains the rigid inverter-frequency, represented by the angular-velocity w_o of the inverter current to be taken as reference.

Because of these two reasons the load angle δ does not correspond directly to the natural oscillation of the compensator. Its variation is affected also by the tendency of both the machine and the inverter to maintain the power-balance. Therefore, the dynamic-equation becomes more complecated and does not have a linear relation with δ. Instead of this angle, the angle $\Delta(B+\delta)$ can be taken to represent the resultant variations in the phase-shift $(B+\delta)$ between E_f and I_{in}. These variations are due to the speed difference between w_r and w_o and the process of power-balance.

4.0 STABILITY CRITERION

In order to study the stability of the compensator machine, operating in such mentioned system, the principle of energy-loops can be applied. It is difficult here to have an explicit relation between T_s and the variation $\Delta(B+\delta)$ in the phase shift $(B+\delta)$ which can be taken as a proper measure of the produced oscillations. An adequate solution of this problem seems possible through the relative speed w_r/w_o . Accordingly, the following two first-order differential equations can be derived :

$$d(w_r/w_o)/dt = a.(T_s+T_{as}-T_D) \qquad (12)$$

and

$$d(B+\delta)/dt = w_o.(1-w_r/w_o) \qquad (13)$$

The first equation can be obtained from the dynamic equation, Eq.(6), and substituting for T_s, T_{as} and T_j from the equations (2-b),(3) and (4) , respectively.
Also, T_D can be obtained by :

$$T_D = T_{DO} \cdot w_r/w_o$$

where $T_{DO} = D.w_o$

The numerical solution of both equations requires the initial values of the involved torques, for each successive time interval. In order to investigate these values, an additional differential equation for I_d is almost required [1]:

$$d(I_d)/dt = V_d/L_d - I_d.R_d/L_d - 3N/L_d \cdot V \cdot \cos\beta \qquad (14)$$

This equation gives the required inter-action between the system and the DC-link, where $I_{in} = (\sqrt{6}/\pi).I_d$.
Solving the system consisting of the above mentioned differential equations, in the time domain gives the apportunity to get the relation between T_s' and $\Delta(B+\delta)$. In other wards, it will be possible to get T_s' in the angle-domain. Thereby, the energy loops can be performed as an adequate stability creterion of the system under consideration.

5.0 DIGITAL RESULTS

The method is digitally programmed in three approaches: the first approach does not consider any type of control. The second approach considers the excitation control which aims to hold V constant. The third approach considers in addition to the exci-

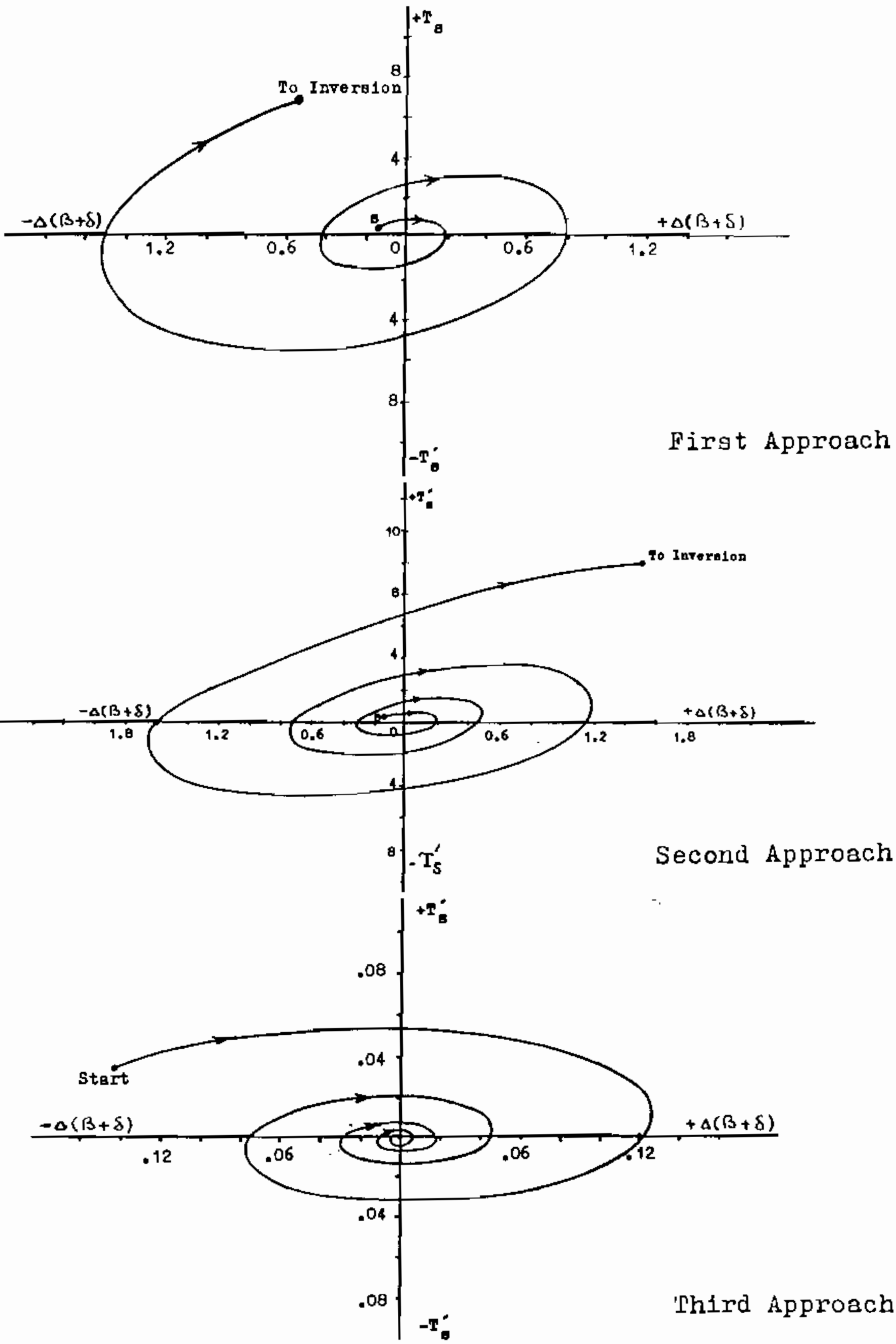


Fig.(4): Energy Loops , Unity Load P.F.

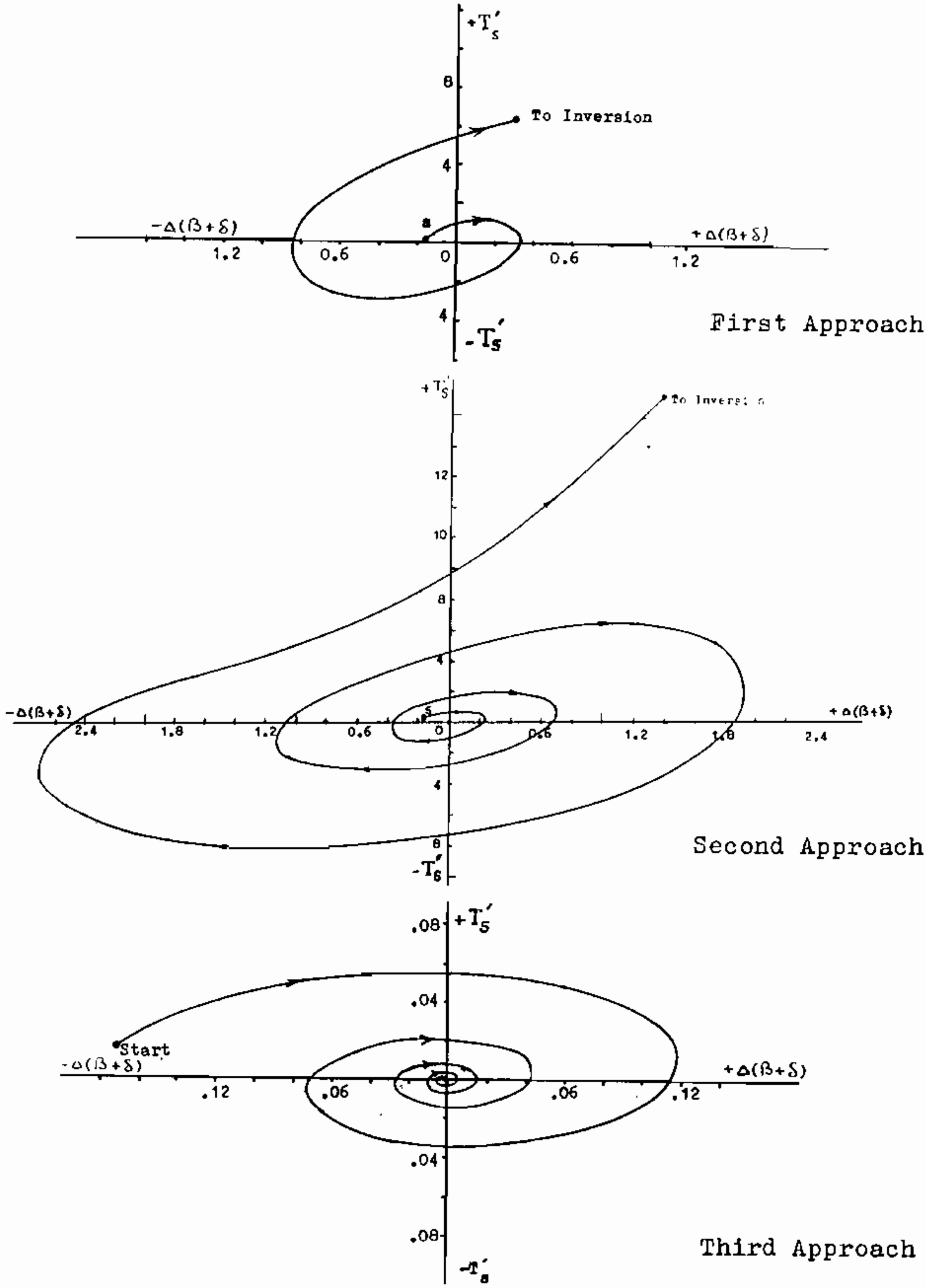


Fig.(5): Energy Loops , 0.8 Lagging P.F.

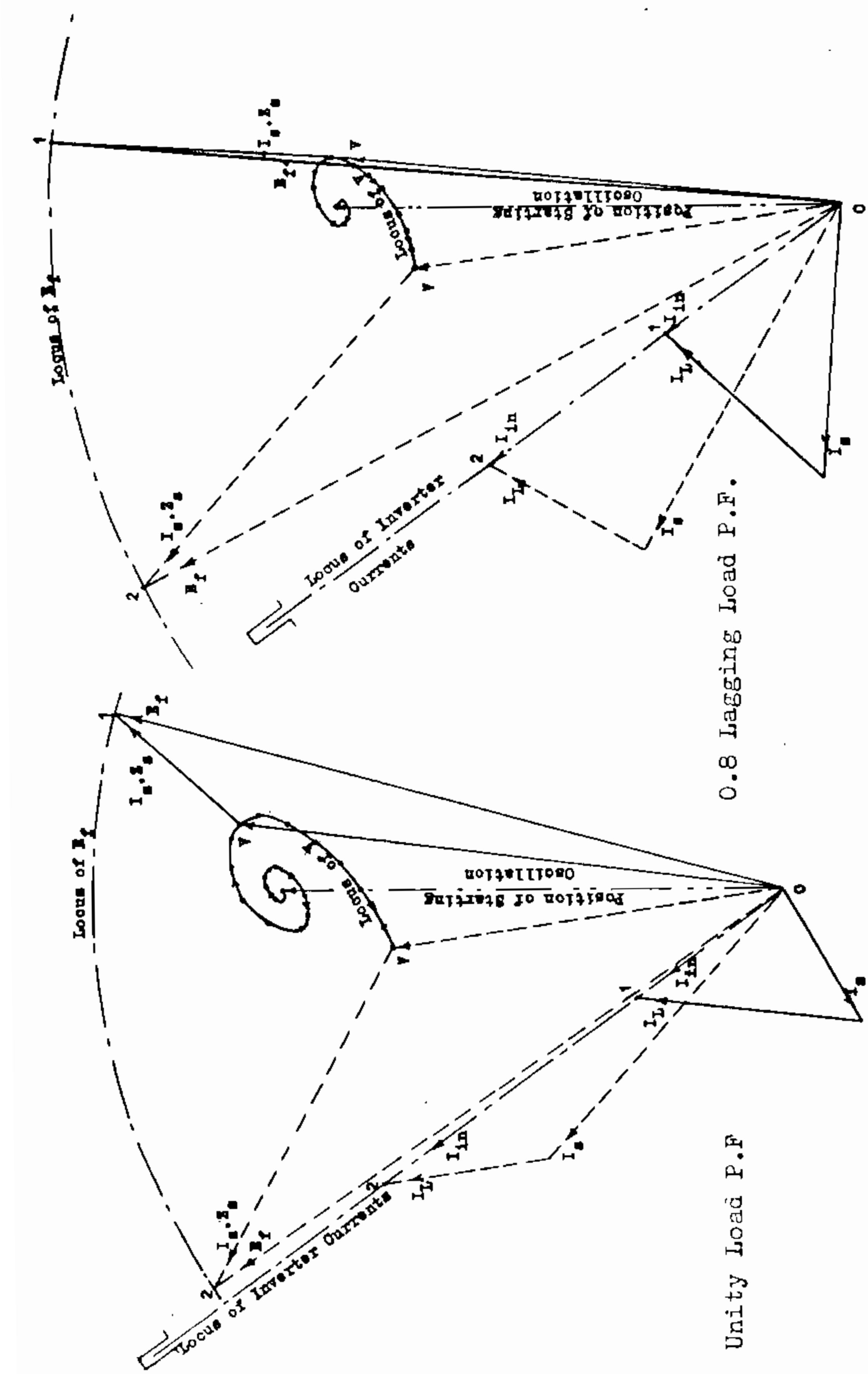
tation control ; the control of V_d in order to hold β also constant.
Test data was taken to be those of a proposed system under constructing in the electrical machine laboratory of El-Mansoura University. It may be noticed that the compensator machine has normal damping winding. Also, the terminal voltage has been reduced in order to have an over-excited machine with rated current.
In order to study the effect of load power-factor on the system stability ; two different values were taken : 0.8 lagging and unity.

5.1 Energy Loops :

The energy loops obtained by applying the three approaches for each load power-factor are given in Fig.(4 ,5).
It is seen for both 1st and 2nd approaches that the loops are going towards outside. They do not converge the equilibrium point on the line $T_s' = 0$. The variation $\Delta(B+\delta)$, in both sides, is going to be larger. It is also seen for the second approach, which applies excitation control, that the system is oscillating longer before inversion in comparison with the first approach.
The excitation control helps in maintaining stability temporary; but it fails at end due to lack of proper damping.
Applying the control of the voltage V_d behind the DC-link in addition to the excitation control for the same system, results in stable operation. As it is evident from Fig.(4,5), the energy loops converge the position of equilibrium "O", at which T_s' and $\Delta(B+\delta)$ are equal zero. Applying the two types of control results in holding both terminal voltage V_o and its shift β_o with the rigid-reference constant. In this case the compensator machine can be considered as it would be operating on infinite-bus bars. It maintains stability and stops to be oscillating.

5.2 Loci of V and E_f :

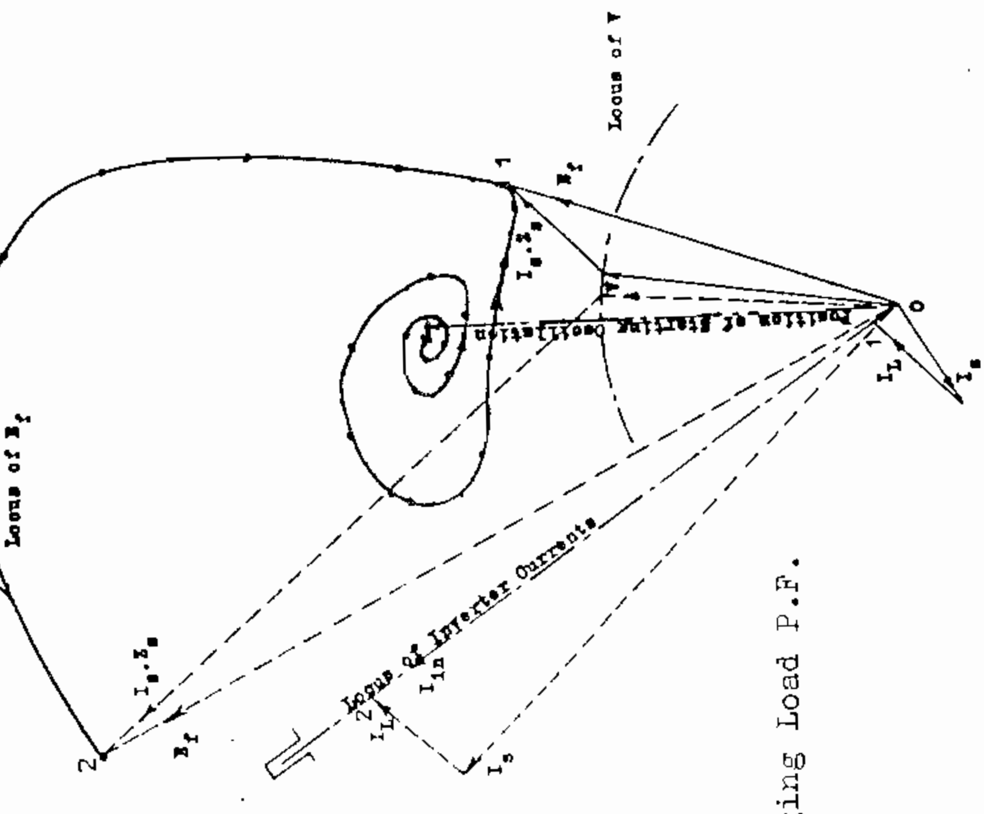
In order to get the loci of both V_o and E_f while the machine oscillating and showing instable operation, 1st and 2nd approaches; their r.m.s. values have been plotted against their polar positions β and $\beta-\delta$, respectively, w.r.t. the rigid reference I_{in} . It may be noticed here that the used convention assumes negative sign for lagging δ . It is seen from Figs.(6 and 7) that these loci has also the form of loops. As the corresponding energy loops shown unstable operation, they are also diverging. As E_f is constant in the first approach, it has a circular locus while the varying V_o has a spiral locus. In the second approach V_o is hold constant by controlling E_f. Therefore, the last voltage ,E_f, will show a spiral locus.
It is also depicted in these figures the voltage and current triangles at the two extreme values of δ in the last oscillation. They are found to be closed and show that $\Sigma V=0$, and $\Sigma I=0$.
This indicate that the principle of power-balance was valid while investigating the initial conditions for each successive interval. As the oscillations of E_f in the 3rd approach are not noticeable, the corresponding loci are not given.



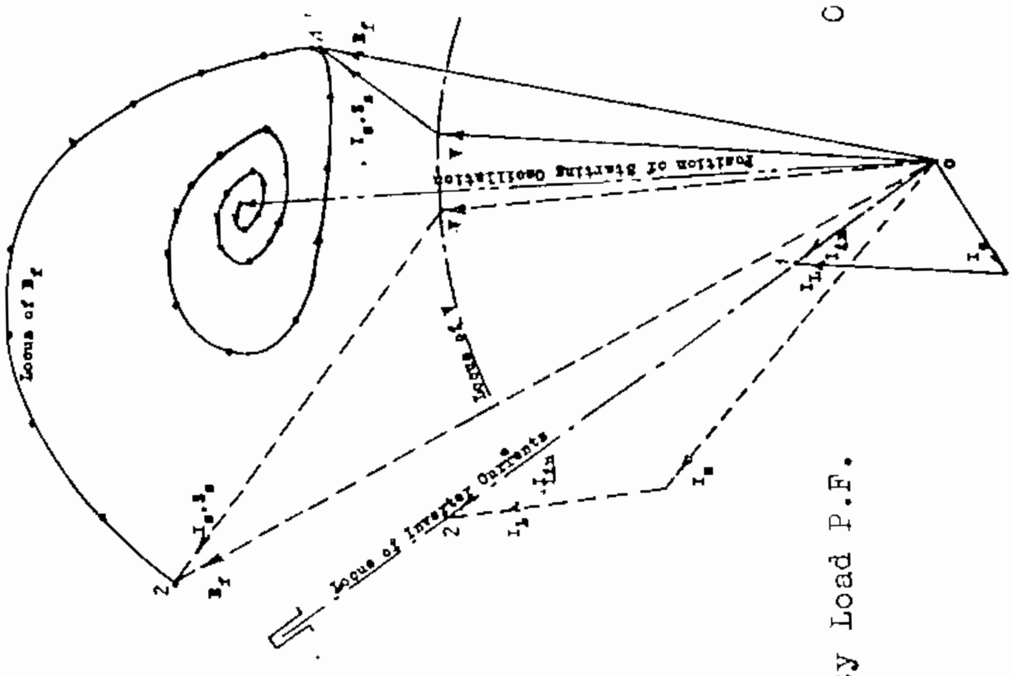
0.8 Lagging Load P.F.

Unity Load P.F

Fig.(6): Terminal Voltage Loci ,First Approach.



0.8 Lagging Load P.F.



Unity Load P.F.

Fig.(7): Excitation-Voltage Loci ,Second Approach.

6.0 CONCLUSION

An adequate stability creterion has been developed for a synchronous compensator operating with a rigidly-impulsed inverter to supply an isolated load. This creterion is based on the principle of energy loops. Conditions concerning stability are found to be :

i. If each successive loop will be smaller and converges the point of equilibrium ; the compensator will maintain stability.
ii. If each successive loop will be larger and diverges the point of equilibrium ; the compensator will go out of synchronism.
iii. If each successive loop traces the same locus ; the compensator will operate under quasi steady-state condition.

The digital results show that a synchronous compensator having a normal damper will not be able to maintain stability. Applying continuous control to the excitation voltage and the voltage behind the DC-link yields the machine analogous conditions similar to those due to operating with infinite bus-bars. Consequently, the machine will approach stability and inverter failures will be avoided.
In accordance with the system voltages and currents, it is found that their loci against the corresponding angles forms also loops. Examples for the terminal and excitation voltages are given.

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