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INVESTIGATION OF THE FLAT JOINTS SUBJECT TO A BENDING MOMENT AND NORMAL LOAD.

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ABSTRACT

The objective of this paper is to approach an approprate mathematical model to the most impotant design parameters of the flat joints of two machined rigid members.These parameters are; Bending moment, Normal load,Quality and dimensions of the two mating member surfaces.Several assumpations have been taken into consideration to simplify the deriviated mathematical model.Calculation of the flat joint stiffness and some basic optimum values would then be within the reach of designers.

INTRODUCTION

The principal design criteria of the flat joints are the stiffness properties which must be taken into consideration by the designers,since these can influence both the short-and long-term accurcy of the joint. The stiffness of the flat joint,defined by the deflection of their connected elements, is of great importance due to the following reasons : [2]+

1-In machine tools a large number of joints are present.
2-The connection area are of large sizes,therefore, the deviations from ideal shape are invitable.
3-Many joint elements of machine structure are of low rigidity as a result of which their is variable stress distribution over the loaded connection area.

+ The Number in square brackets designate Referances at end of paper.

There are many publications for both fundamental and empirecal nature which concerned with the factors affecting the characteristics of the flat joint [1-4] These investigates also the results of the study of the effect of the surface roughness,flatness deviation,initial interference pressure on the connection stiffness and the signifiance of the deflection in the overall tool-workpiece deviation of a machine.The influence of the overall-dimensions of the connecton surface and the connection recess ratios have not yet analysed.Therefore,the objective of this paper is mainly to approach an appropriate mathematical model to the most design parameters of the flat joint.

2- MATHEMATICAL MODEL

Figure 1 shows the model of the flat joint of two machined rigid elements subject to the loads(M_y and F) which transmitted across the connection interface and the main geometric parameters of the contact surface.

To determine the bending stiffness of the connected members,the following assumptions have been taken into consideration: The members to be connected are rigid, therefore,the deformation of members can be neglected and only the relative angle deviation and diplacement of the elements due to the connection deflection is considered.The normal deformation(v) due to the normal stress(σ_n),is computed by the following relation after LEVINA [2]

$$v = \propto . \sigma_n^{0,5} \qquad ....(1)$$

Therefore,it is clear that only the following design parameters can be analysed;

-The position and dimensions of the connection surface.
-The geometry and dimensions of the connection recess.
-The relation between the applied loads and the dimensions of the flat joint.

The following relative relations have been used to deive the mathematical model.

$$R_a = a_{fi}/a_f \qquad R_b = b_{fi}/b_f$$
$$\gamma = 2x_a/a_f \qquad \theta = 2x/a_f \qquad (2)$$

$$b_f(\theta) = \begin{cases} b_f & \text{for } R_a \le \theta \le 1 \\ b_f(1-R_b) & \text{for } 0 \le \theta \le R_a \end{cases} \qquad (3)$$

$$v(\theta) = a_f/2 \cdot \varphi_f(\gamma + \theta) \quad \text{, and} \qquad (4)$$

$$\sigma_n(\theta) = v^2(\theta)/\alpha^2 = a_f^2/4\alpha^2 \cdot \varphi_f^2(\gamma+\theta)^2 \qquad (5)$$

Thus, where a rectangular joint loaded by normal force and bending moment, the conditions for balanced joint are (See Fig.1):

$$\Sigma F = 0 = -\int_{-1}^{1} \sigma_n(\theta) \cdot b_f(\theta) \cdot a_f/2 \cdot d\theta + F \qquad (6)$$

and,

$$\Sigma M = 0 = -\int_{-1}^{1} \sigma_n(\theta) \cdot b_f(\theta) \cdot a_f^2/4 \cdot \theta \cdot d\theta + M_y \qquad (7)$$

To simplify the mathematical model, the contact surface between the two mating elements can be distinguished by the place of the disjoin point(γ) into four cases;

1. The point γ lies outside the joint interface.
2. The point γ lies inside the contact surface(left side).
3. The point γ lies within the joint recess.
4. The point γ lies inside the contact surface(right side)

It is clear that, case one has the highst rigidity disjoin point with respect to in cases 2,3 and 4. Therefore, these cases may be neglected because it is of less interest to the designer. The following are the mathematical models when the disjoin point lies outside the joint interface, the load is subject to a completely contact surface between the two mating elements. Thus, the relation between the external moment and the co-ordinate of the disjoin point may be derived by the integration of equations 6 and 7 with respect to the complete joint interface.

$$\frac{M_y}{F \cdot a_f} = \frac{(1 - R_a^3 \cdot R_b)}{1 - R_a^3 \cdot R_b + 3\gamma^2(1 - R_a \cdot R_b)} \qquad (8\text{-}a)$$

Before considering effecting of the bending moment(M_y) the normal stress(σ_n) may be considered uniformly distributed on the joint area (A_f). Thus;

$$\sigma_n = F / a_f \cdot b_f (1 - R_a \cdot R_b) \qquad (9)$$

Thereof,

$$\frac{M_y}{\sigma_n . a_f^2 . b_f} = \frac{\gamma(1-R_a^2 . R_b)(1-R_a . R_b)}{1-R_a^3 . R_b + 3\gamma^2 (1-R_a . R_b)} \qquad (8\text{-b})$$

The angular deflection(φ_f) may be also determined using equations 7 and 8 as follows:

$$\varphi_f^2 \frac{a_f^3 . b_f}{12\, F . \alpha^2} = \frac{1}{1-R_a^3 . R_b + 3\gamma^2(1-R_a . R_b)} \qquad (10\text{-a})$$

$$\varphi_f^2 \frac{a_f^2}{12\, \sigma_n . \alpha^2} = \frac{1-R_a . R_b}{1-R_a^3 . R_b + 3\gamma^2(1-R_a . R_b)} \qquad (10\text{-b})$$

The relationship between φ_f and the angular deflection of the joint without recess (φ_{fo}) (putting R_a=R_b=0) may be defined as:

$$\varphi_f / \varphi_{fo} = \left[\frac{o}{\gamma(1-R_a^3 . R_b)} \right]^{o,5} \qquad (11)$$

$$M_y/F.a_f = (M_y/F.a_f)_o = \gamma_o/(1+3\gamma_o^2) \qquad (12\text{-a})$$

and,

$$\frac{M_y}{\sigma_n . a_f^2 . b_f} = \left(\frac{M_y}{\sigma_n . a_f^2 . b_f} \right)_o = \frac{\gamma_o}{1+3\gamma_o^2} \qquad (12\text{-b})$$

The rotation point of the joint interface is the point which has no normal deflection under the effect of the moment(M_y).

i.e., $\partial V(\theta) / \partial M_y = 0 \qquad (13)$

Therefore,the deviation of the rotation point with respect to the joint center point (0) may be generally determined from equation(6).

$$\theta_d = -\gamma - \varphi_f . \; \partial\gamma / \partial\varphi_f \qquad (14)$$

Lastly, the rotation point may be determined by the use of equations (8 and 10).

$$\theta_d = (1-R_a^3.R_b)/\ 3\gamma(1-R_a.R_b) \qquad (15)$$

From equations (8 and 10), it is clear that the relation between φ_f and M_y is non-linear. Thus , the joint stiffness may be derived by the following partial differential.

$$C_y = \partial M_y / \partial \varphi_f \qquad (16)$$

Finally, the relation between the stiffness (C_y) and the angular deflection(φ_f) is given by the following equation:

$$C_y . \left[\frac{12\alpha^2}{a_f^5.b_f.F}\right]^{o,5} = \varphi_f . \left[\frac{a_f^3.b_f}{12F\alpha^2}\right]^{o,5} (1-R_a^3.R_b) \left[\gamma - \frac{1-R_a^3.R_b}{3\gamma(1-R_a.R_b)}\right] \qquad (17\text{-a})$$

or;

$$C_y . \left[\frac{12\alpha^2}{a_f^6.b_f^2.\sigma_n}\right]^{o,5} = \varphi_f . (a_f^2/12\alpha^2.\sigma_n)^{o,5} \left[\gamma - \frac{1-R_a^3.R_b}{3\gamma(1-R_a.R_b)}\right] (1-R_a^3.R_b) \qquad \ldots (17\text{-b})$$

ANALYSIS AND DISCUSSION

The derived mathematical models have been numerically analysed and graphically represented. The following are the discussion of the design parameters of the joint.

1- The Relationship between The External Bending Moment and The Angular Deflection:

The relationship between the bending moment and the angular deflection is represented in Fig. 2 for the centeral force (F) and in Fig.3 for a constant normal stress(σ_n). Both figures are calculated for the same recess ratios($R_a=R_b$) as dependant parameters. It shows that, the tendency of both curves is progressively increased.The non-linear relation is related to the value of the recess ratios.It shows also that the angular deflection (φ_f) increases progressively with increase of the loading magnitude($M_y/F.a_f$ or, $M_y/\sigma_n.a_f^2.b_f$).At a light load,it is clear that the relation between the external load and φ_f is approximately linear and is valid until the disjoin point $\delta=1$. After that the angular deflection increased suddenly with the gradual increase of the load.Also ,at normal load(F),the effect of joint recess ratio on the angular deflection is greater than at a constant normal stress.

This behaviour can be explained as follows; during the increasing of the joint recess, the normal stress and also the contact deflection increase,but on the other hand the inertia moment of the joint area decreases.This counter tendency decreases the effect of the joint recess so that it may be neglected.

2-The Relationship Between The Bending Moment and The Bending Stiffness.

Figs. 4 and 5 shows the relationshop between the bending moment(M_y) and the bending stiffness(C_y).It is clear for all joint recess ratios that ,the stiffness decreases when the moment increases,Also,it can be said that the stiffness value firstly decreases progressively until the disjoin point $\delta = 1$, after that it decreases degressively.

From Fig.4 it can also be seen that,at the light loading values($M_y/F.a_f \leq 0,2$),an optimum condition of the stiffness occures at $R_a=R_b \approx 0,6$, while at the heavy loading values($M_y/ F.a_f \geq 0,2$) the joint stiffness increasing of the joint recess.The highest joint stiffness in this case occures at $R_a=R_b$ =0 (i.e., without joint recess).But,the designer can better use $R_a=R_b$= 0,4 to reduce the defficulties in the joint production and minimze the production cost.

3-Effect of The Joint Recess Ratio on The Bending Stiffness:

For more comprehensive study of the effect of the recess on the stiffness value; it can be analyesd the relation between the stiffness and their angular deflection at a constant normal load(F).To simplify the analysis,it is assumed that,in the region of a light load the relation is linear(See Fig. 2).

Accordingly, substituting by the following limits $\gamma = \infty$ and $\varphi_f = 0$ in equation(8), we have;

$$C_y \left[\frac{12\ \alpha^2}{a_f^5 . b_f . F}\right]^{0,5} = \frac{1 - R_a^3 . R_b}{\left[3(1 - R_a . R_b)\right]^{0,5}} \qquad (18\text{-a})$$

$$C_y \left[\frac{12\ \alpha^2}{a_f^6 . b_f^2 . \sigma_n}\right]^{0,5} = \frac{1 - R_a^3 . R_b}{1,73} \qquad (18\text{-b})$$

When $M_y/F.a_f = M_y/\ \sigma_n . a_f^2 . b_f = 0$, the relation between the bending moment and its angular deflection is also linear. So, using

$$C_y = M_y\ /\ \varphi_f \qquad (19)$$

The linear relation between the (M_y) and the (φ_f) becomes;

$$\varphi_f \left[\frac{a_f^3 . b_f}{12\ \alpha^2 . F}\right]^{0,5} = \frac{M_y}{a_f . F} . \frac{\left[3(1 - R_a . R_b)\right]^{0,5}}{1 - R_a^3 . R_b} \qquad (20\text{-a})$$

also,

$$\varphi_f \left[\frac{a_f^2}{12\ \alpha^2 . \sigma_n}\right]^{0,5} = \frac{M_y}{a_f^2 . b_f . \sigma_n} . \frac{1,73}{1 - R_a^3 . R_b} \qquad (20\text{-b})$$

By substituting the moment of ineria of the joint

$$I_f = a_f^3 . b_f\ (1 - R_a^3 . R_b)\ /\ 12 \qquad (21)$$

surface, in equation (20), we have

$$\varphi_f = M_y . \alpha / 2\ I_f .\ \sigma_n^{0,5} \qquad (22)$$

Thus, $$C_y = 2\ I_f .\ \sigma_n^{0,5}\ / \alpha \qquad (23)$$

The effect of the joint recess ratio on the bending stiffness may be evaluated by the equation of the relative stiffness (C_R). This equation can be derived from equation(18-a). Where the joint surface without recess have the ratio $R_a = R_b = 0$.

$$(C_R)_F = \left(\frac{C_y}{C_{yo}} \right)_F = \frac{1 - R_a^3 . R_b}{(1-R_a.R_b)^{0,5}} \tag{24}$$

This equation has been presented for the parameters R_a and R_b in Figs. 6 and 7 . It is clear that, the highest relative stiffness(C_R) occurs at the ratio R_a=0,6 and R_b=1 .The excess in stiffness with respect to the joint surface without recess reaches about 24% (See Fig.6).On the other hand,at the ratio R_a=1 and R_b=0,6 the decrease in the stiffness reaches about 37%(See Fig.7) .Therefore,the position of the joint with respect to the bending moment has an effect.So, the designer must choose the convenient joint position and the joint recess ratios for his joint.Simply,for the recess ratio $R_a = R_b$ = 0,6 the stiffness has clearly an optimum condition (See Fig.7).It exceeds by approximately 9% with resepct to the closed joint surface.

4- Effect of The Bending Moment On The Position Of The Rotation Point Of The Joint Surface:

The relation between the displacement of the rotation point(θ_d) and the external load may be analysed by using equation (15).Figuire 9 represent the limits of this displacement by neglecting the effect of the joint recess ratios.It is clear that the relation may be linear in the region $0 \leq M_y/F.a_f \leq 0,2$. Therefore, for this region, the following linear relation can be derived:

$$\theta_d = M_y / F . a_f \tag{25}$$

Also, it can be seen that the displacement θ_d increases progressively after $M_y/F.a_f \approx 0,2$. The normal deflection of the joint center point(at θ= o) can be derived from Equation 4 .

$$v(o) = 0,5\ a_f .\ \varphi_f . \gamma \tag{26}$$

Figure 9 shows the relation between v(o) and the load ratio ($M_y/F.a_f$).The deflection decreases slowly until the start of the disjoining(at γ=1),after that it decreases quickly.In other words,in the region $0 \leq M_y/F.a_f \leq 0,25$ the vertical deflection of the joint center point is mainly affected by the joint recess ratio only.

CONCLUSION

The bending stiffness of the flat joint is greatly affected by the bending moment and the dimensions of the joint surface. The joint recess has a pronounced effect on the joint stiffness. Based on the results and discussion on this paper, the following optimum conditions can be recommended:

1- The angular joint deflection increases progressively with the increase of the bending moment. It can also be seen that, the relation may be considered linear until the start of the disjoining and in the load region $M_y / F.a_f \leq 0,2$.

2- For all joint recess ratios the stiffness decreases with the moment increases. An optimum condition of the stiffness occures at $R_a = R_b \approx 0,6$ for the loading value $M_y/F.a_f \leq 0,2$. Where, the stiffness increases by about 9% than the closed joint surface.

3- In the region $0 \leq M_y/F.a_f \leq 0,2$, the relation between the displacement of the joint rotation point and the moment can be considered linear.

4- The normal deflection of the joint center point is mainly affected by the joint recess ratio within the load region $0 \leq M_y / F.a_f \leq 0,25$.

REFERENCES

1- Back, N. Burdekin, M & Cowley, A. "Review of the Research on Fixed and Sliding Joints." Pro. 13th M.T.D.R Conf., (1973).

2- Levina, Z.M. "Research on the Static Stiffness of Joints in Machine Tools". Proc. 8th M.T.D.R Conf., (1967).

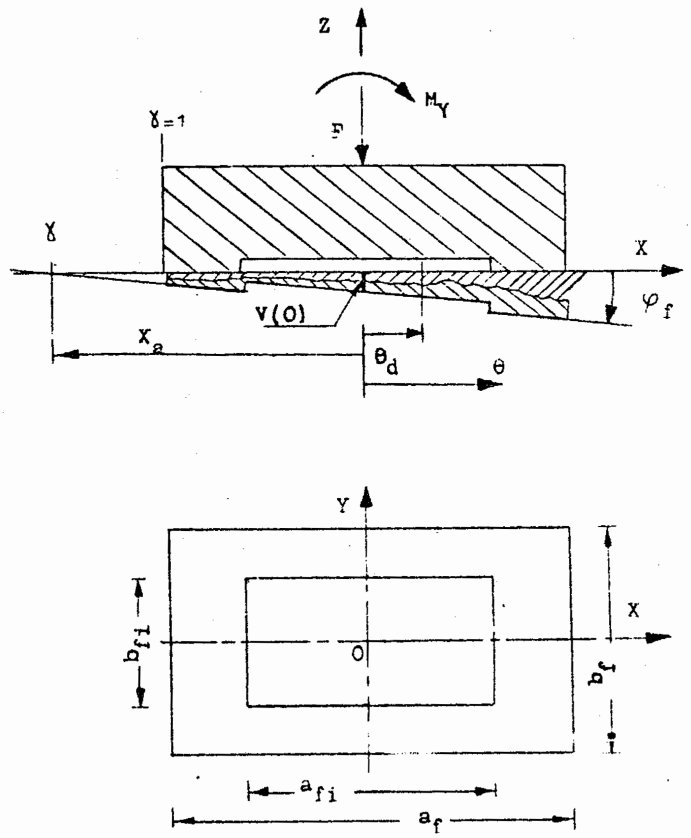
3- Pič, J. "Berechnung von Verbindungen mit Vorspannung" Strojerenstvi 14(1964)/10.

4- Wasner, O. "Berechnung von Verbindungsstellen in Werkzeugmaschinengestellen." Maschinenbautechnik 23(1974)6.

NOMENCLATURES

A_f	Joint area (mm^2).
a_f	Length of the joint(mm).
a_{fi}	Length of the joint recess(mm).
b_f	Width of the joint (mm).
b_{fi}	Width of the joint recess(mm).
R_a	Recess length ratio.
R_b	Recess width ratio.
x_a	Coordinate of the disjoin point.

γ	Relative coordinate of the disjoin point.
θ	Relative coordinate to x-direction.
α	Factor, depending on the mechanical properties of material and surface quality of the two mating parts.



$R_a = a_{fi} / a_f$, $R_b = b_{fi} / b_f$

$A_F = a_f \cdot b_f \, (1 - R_a . R_b)$

Fig.1 . Mathematical Model of The Flat Joint.

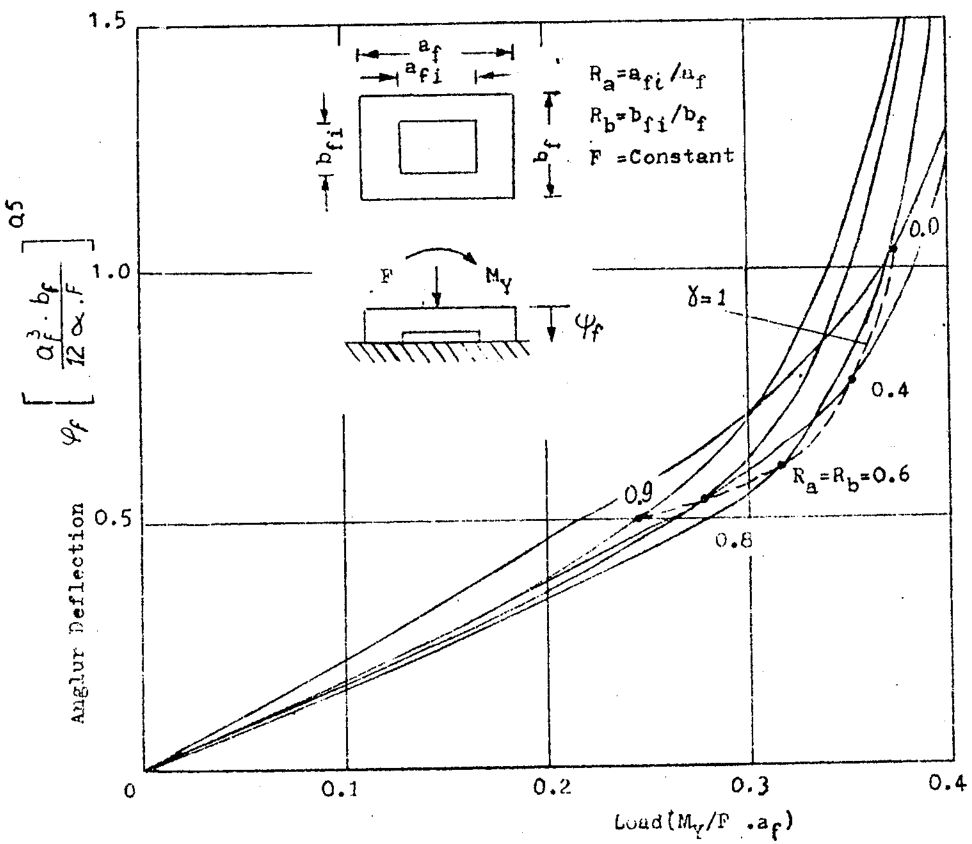


Fig.2. The Relationship between the Load Ratio and the Angular Deflection at a Constant Normal Load.

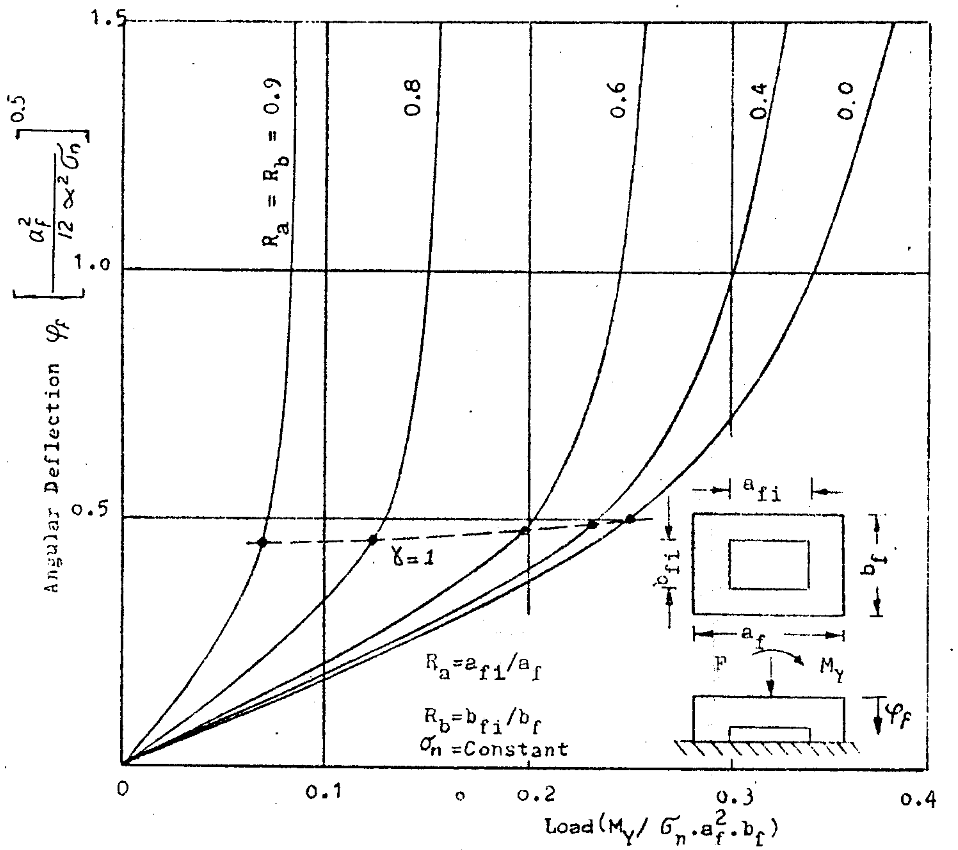


Fig.3 The Relationship between the Load Ratio and the Angular Deflection at a Constant Normal Stress.

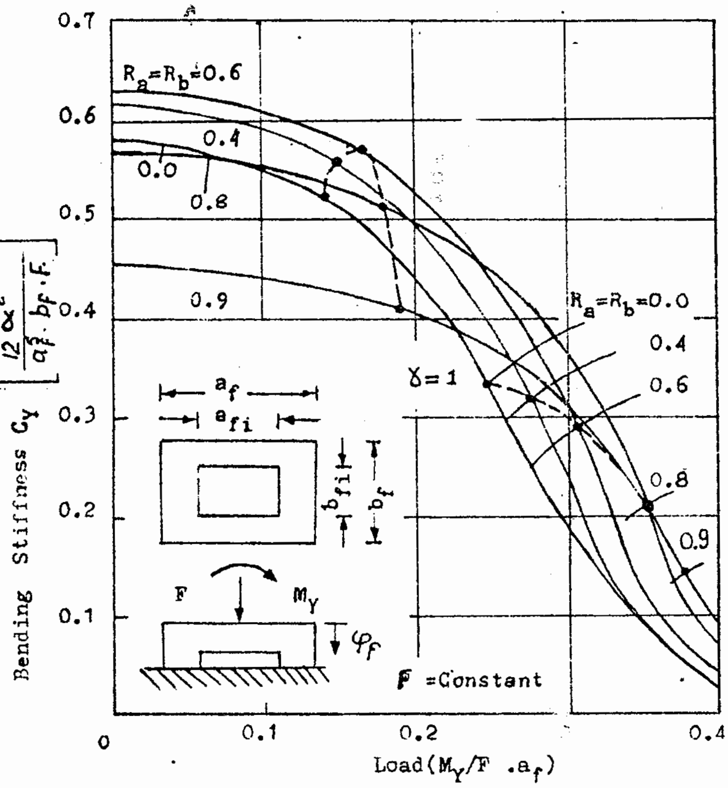


Fig.4 The Relationship between the Bending Moment and the Stiffness at a Constant Normal Load.

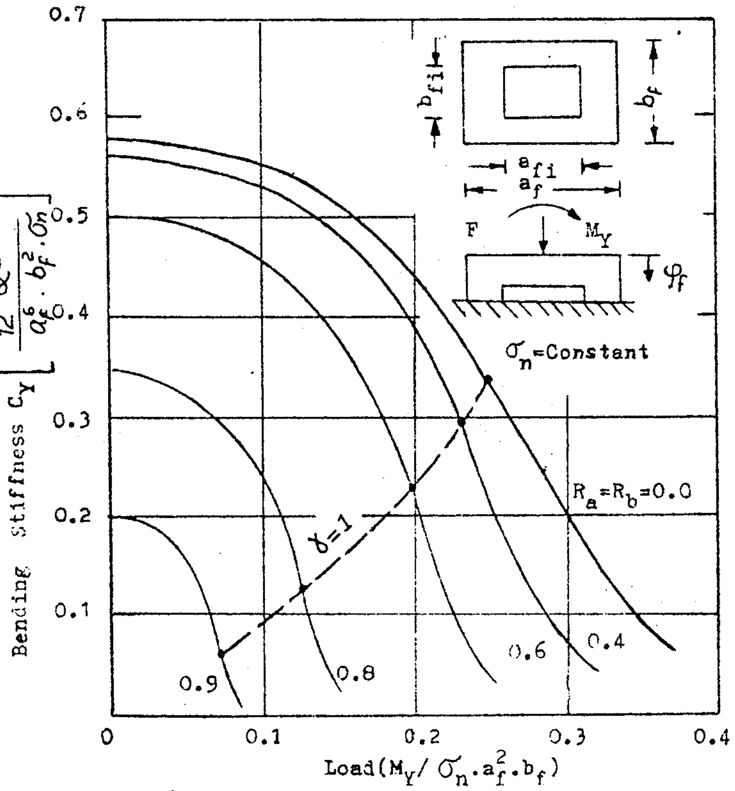


Fig. 5. The Relationship between the Bending Moment and the Stiffness at a Constant Normal Stress.

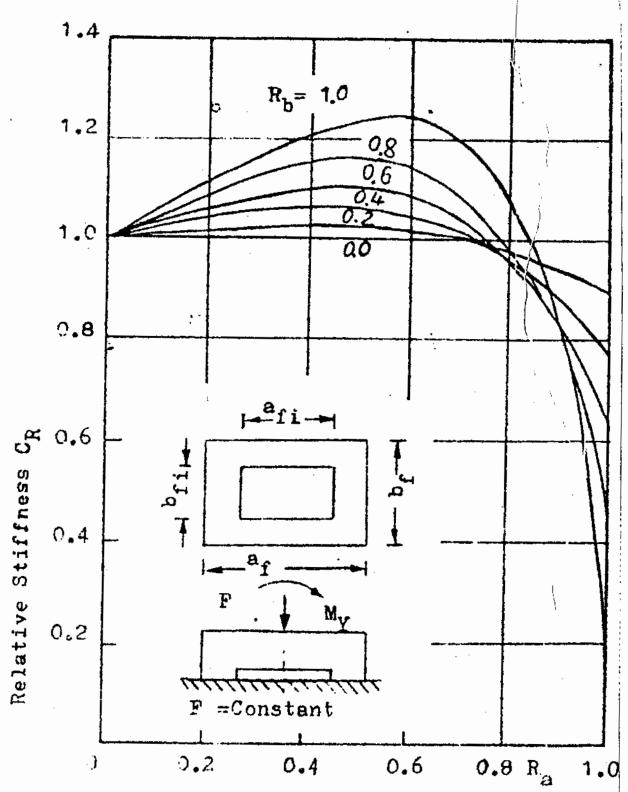


Fig.6 The Relation between the Joint Recess Ratio and the Stiffness.

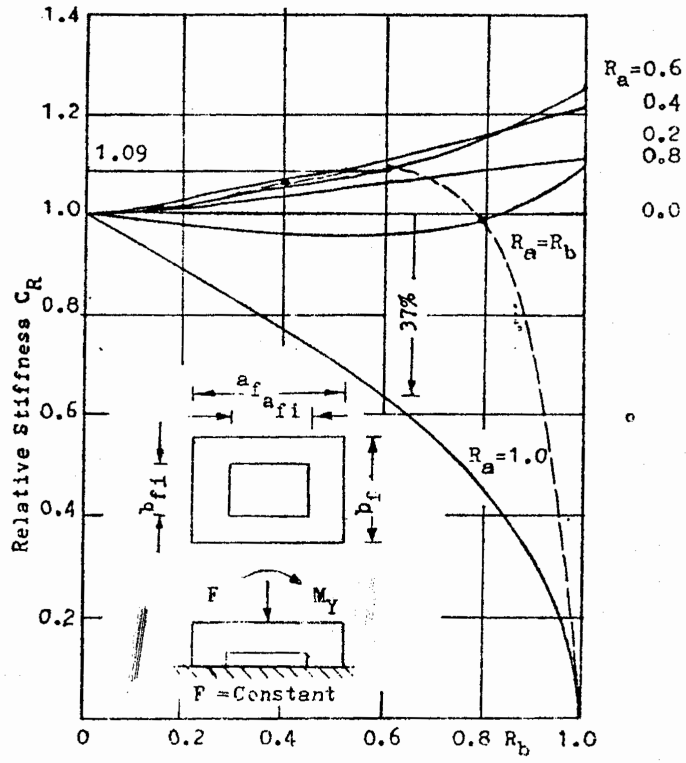


Fig.7 . The Relation between the Joint Recess Ratio and the Stiffness.

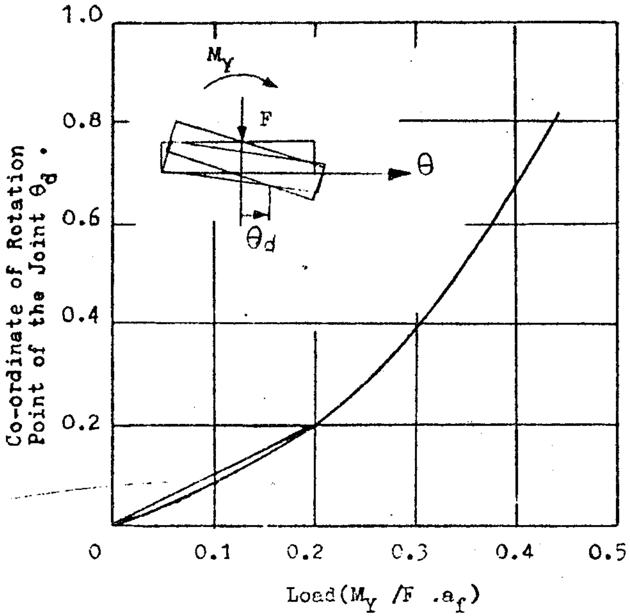


Fig. 8 The Relation between the Rotation Point and the Load.

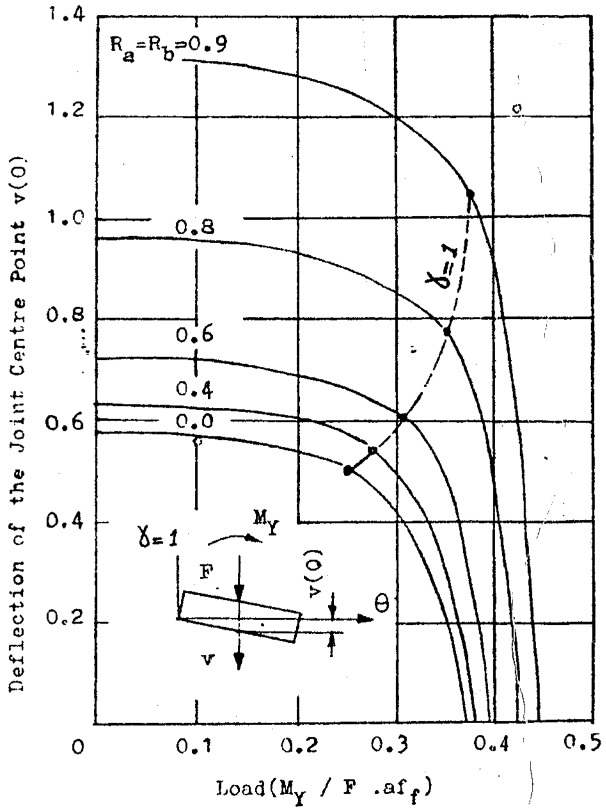


Fig. 9 The Relation between the Deflection of the Joint Centre Point and the Bending Load.