

## A TAUBERIAN-PRONY FEATURE EXTRACTION TECHNIQUE FOR ESOPHAGEAL MOTILITY PATTERNS

تقنية توبر - برونى لاستخلاص خصائص كمية لانماط حركة المريء

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الخلاصة - في هذا البحث نقدم نظاما لاستخلاص خصائص وأنماط موجات الضغط داخل المريء أثناء البلع والتي يمكن قياسها وتسجيلها باستخدام محول خاص لتحويل موجات الضغط الى اشارات كهربية ومنظومة فضاء خاصة متمثلة بحاسب لمعالجة وتحليل هذه الاشارات وتحديد هذه الخصائص منى على تحليل الموجات باستخدام طريقة تقريب توبر السببي مجموعة من نبضات داونس مخلطة من حيث السعة وزمن الحدوث . ويحدد البحث طريقة الحصول على هذه المتغيرات بدقة . وباستخدام التحليل الاحصائي لهذه المتغيرات فى حالات مخلطة من المرضى والأصحاء يمكن تصميم قاعدة بيانات لاستخدامها فى حالات التشخيص والتصنيف .

**ABSTRACT**-For the esophageal contractile activity recorded during swallowing, a feature extraction scheme has been developed. It recognizes the time, duration, and amplitudes of local peaks for each peristaltic wave. The method is based on the Tauberian approximation for modeling waveforms as a sum of identically shaped pulses with different time delays and amplitudes. Initial conditions on the pulse properties are set and an optimal solution is sought. The method is completely automated and can be utilized for characterization and classification purposes.

### I. INTRODUCTION

Recent advances in computer technology have allowed for the possibility of on-line analysis of esophageal motility tracing during swallowing using small computers. Automated analysis would help to establish normative databases and, consequently, assist making clinical evaluations of disorders and generating reports. Such an analysis would require an efficient means of characterizing the esophageal pressure profiles which is usually subject to large variabilities. This is the objective of the present paper.

Different parameters have been used to characterize the esophageal activity. These include peak amplitude, contraction duration, propagation delay and the number of peaks per peristaltic wave (Feussner et al., 1987, and Castell and Castell, 1987). However, no attempt has been made to devise a technique that extracts the essential features of the peristaltic envelope in a form that is both useful and descriptively helpful. These features are, the times, amplitudes and durations of the fundamental wavelets contributing to the peristaltic wave. The determination of these parameters helps to classify and characterize physiological and pathological motility phenomenon in the tubular esophagus objectively.

In the present work an automated technique is developed which extracts the above parameters from each peristaltic wave without a priori information or intervention. This technique is based on a Tauberian approximation and has been used successfully for automatic feature extraction of locomotor electromyographic patterns (Chen and Shiavi, 1990).

## II. MATERIAL

### II.1 Data Acquisition and Equipment

The esophageal pressure waves are recorded by an esophageal motility recorder type M19. The esophageal waves are converted into an electrical signal using a special catheter which contains three pressure transducers spaced 5 cm apart. The recorded signals are then connected to a 12-bit analog-to-digital converter with a built-in 8-channel. The pressure signal on each channel is sampled at 20 sps. To mark the transducer position and the time the swallow has occurred, another two channels are used.

The recorded data are transferred to an IEM PS2/80 computer by a program written in Basic and the transfer of data from the five channels is done in sequential order.

### II.2 Physiological Data

A 10-minute esophageal monitoring has been recorded for a group of 5 normal subjects. The catheter was inserted to each subject transnasally or through the mouth into esophagus and the subject was allowed to rest for few minutes prior to the beginning of recording. Esophageal recording was performed using a station pull-through method in which the catheter is pulled back in stepwise fashion. Baseline pressures and responses to swallows are recorded. There have been usually about 15 swallows per subject.

### II.3 Baseline Evaluation and Correction

Computer processing of the esophageal recordings requires the signal to have reasonably steady baseline since measurements of wave amplitude are made relative to the baseline. Thus, any baseline drift resulting from respiration or any other physiological factor must be reduced. For low-frequency baseline, high-pass filtering can be used; however, attempts to filter out higher frequency baseline drifts unfortunately attenuate the lower band frequency of the signal. Therefore, a polynomial of

order three or a cubic spline has been found appropriate since fewer constants need to be evaluated. In addition, the power spectral density of the cubic spline interpolators closely approximates the band-limited white noise spectrum (Horowitz, 1974).

The baseline removal process consists of estimating the baseline drift by fitting a third-order polynomial through a set of predetermined points (before and after each peristaltic wave), and then subtracting the estimate from the original signal. The results of baseline removal are illustrated in Fig.1.

#### II.4 Selection of Peristaltic waveforms

Automatic selection of peristaltic waveforms are performed to detect their location. The peak of each swallow is identified as the maximum point which is above the mean level by three times the standard deviation. An automatic search was done to locate the position of the beginning and end of the swallowing act. The samples of the detected swallows have been saved with 1s of data prior to the beginning and 1s after the end. This approach captures a complete peristaltic wave and has been adopted by Pfister et al.(1989).

### III. METHODOLOGY

The feature extraction scheme consists of three steps. Each will be discussed separately. The first step is amplitude normalization which is necessary because of the variabilities involved in the measurements. The second step is the spectral analysis of the peristaltic waves because the approximation technique is a function of the harmonic contents of these waves. Finally, the theory of the Tauberian approximation and the practical adaptations necessary to make it implementable will be described in detail.

#### III.1 Amplitude Normalization

A major consideration when comparing peristaltic waves across individuals is to recognize that absolute magnitudes are meaningless because of variation in measuring environment: tissue resistances, transducer placements and other factors. Amplitude normalization has been introduced to remove this artifact and allow concentration on the shape of the peristaltic waves. It is implemented simply by dividing the peristaltic wave values by a designated factor. This factor has been chosen to be the peak of the within-subject ensemble peristaltic peaks. This reduces the variability between the different subjects and allows inter-subject comparison as well.

#### III.2 Spectral Analysis

As mentioned the Tauberian approximation relies on the spectral composition of the peristaltic waves. To ensure uniformity, the average power spectrum of a number of peristaltic waves of each section of the esophagus is calculated and is shown in Fig.2 ( usually the esophageal tube is divided into three sections: proximal, middle and distal). Fig.3 shows the

corresponding percentage cumulative power density and the ensemble standard deviation. It should be noted that 95 percent of the power resides within 7 Hz with only a deviation of 3 percent.

### III.3 Tauberian Approximation

Since the peristaltic waves created by swallowing is a composite of successive waves of muscular contraction, an appropriate mathematical model of these waves is one whose components are pulse-like. Such a technique is known as the Tauberian approximation (Defigueiredo and Hu, 1982). It uses a pulse basis function,  $x(t)$ , whose shape models adequately the peaks of the peristaltic wave. The peristaltic envelope,  $y(t)$ , is then modeled as a weighted sum of pulses with different lag times and is represented mathematically by

$$y(t) = \sum_{i=1}^M a_i x(t-\tau_i) \quad (1)$$

where  $a_i$  and  $\tau_i$  are parameters designating the amplitudes and times of occurrence of the pulses, and  $M$  is the number of pulses.

Examination of the peristaltic waves shows that the function that most resembles the local peaks of each wave is the unnormalized and truncated Gaussian function

$$x(t) = \exp \left( -t^2 / 2 \sigma^2 \right) \quad |t| \leq 2.4 \sigma \quad (2)$$

Fig.4 shows  $x(t)$  for  $\sigma = 1.6$ . Since the model approximation is a sum of weighted and lagged functions, the features can be more efficiently determined in the frequency domain and calculated using Prony's method (Hilderbrand,1965).

### III.4 Prony's Algorithm

The desired waveform can be expressed as a sum of weighted and time shifted basis functions as in Eqn.(1). The Fourier transform of this equation is:

$$Y(\omega) = \sum_{i=1}^M a_i \exp(-j\omega\tau_i) X(\omega) \quad (3)$$

It follows that

$$H(\omega) = Y(\omega)/X(\omega) = \sum_{i=1}^M a_i \exp(-j\omega\tau_i) \quad (4)$$

$H(\omega)$  is computable since  $y(\omega)$  and  $X(\omega)$  can both be determined by Fourier transforming the given function  $y(t)$  and the basis function  $x(t)$ . Using the discrete Fourier transform, we calculate the samples of  $H(\omega)$  at equidistant frequency points;

$$\begin{aligned} \omega_k &= \omega_0 + \Delta\omega, & k=0, \dots, K-1 & \quad \text{with } 2M < K \\ H_k &= H(\omega_k) \end{aligned} \quad (5)$$

$$H_k = \sum_{i=1}^M a_i \exp(-j\tau_i \omega_k), \quad k=0, \dots, K-1 \quad (6)$$

The general problem of determining the values  $a_i$  and  $\tau_i$  is difficult because of the nonlinear dependence of  $H_k$  on  $\tau_i$ . The timings  $\{\tau_i\}$  can be optimized by using the principle which lies behind Prony's algorithm. It transforms this nonlinear problem into a problem of finding the zeros of one single function.

The first step in the method is to note that satisfaction of (6) together with (4) and (5) implies that there holds the following difference equation in the complex coefficients  $c_0, \dots, c_M$  with  $c_0=1$  and  $c_1, \dots, c_M$  to be determined:

$$\sum_{l=0}^M H_{k-l} c_l = 0, \quad k=M, M+1, \dots, K-1 \quad (7)$$

whose characteristic equation is

$$\sum_{l=0}^M c_l z^l = 0 \quad (8)$$

with roots

$$z_i = \exp(-j\tau_i \Delta\omega), \quad i=1, 2, \dots, M \quad (9)$$

Letting

$$H = \begin{bmatrix} H_{M-1} & \dots & H_0 \\ \vdots & & \vdots \\ H_{K-2} & \dots & H_{K-M-1} \end{bmatrix} \quad (10a)$$

$$c = \begin{bmatrix} c_1 \\ \vdots \\ c_M \end{bmatrix}, \quad h = \begin{bmatrix} H_M \\ \vdots \\ H_{K-1} \end{bmatrix} \quad (10b)$$

The set of (7) can be rewritten as

$$H c = -h \quad (11)$$

The general solution of this equation is given by

$$c = -[H^* H^{-1}] H^* h \quad (12)$$

where  $*$  denotes the conjugate complex transpose of a matrix (Hermitian). However, for the sake of simplicity and computational efficiency, the problem can be solved in the real domain following the variant of the above procedure explained in detail by DeFligueiredo and Hu, (1982).

This leads to the characteristic equation:

$$\sum_{l=0}^{2M} \tilde{c}_l z^l = 0 \quad (13)$$

which has  $M$  pair of conjugate roots.  $\tilde{c}_l$  is related to the real part ( $H_R$ ) of  $H_k$  in the following way:

$$\sum_{l=0}^{M-1} (H_{R,k+l} + H_{R,k+2M-l}) \tilde{c}_l = -H_{R,k+M} \quad (14)$$

The timings can be computed from the roots  $z_i$  of (12) using:

$$\tau_i = 1/\Delta\omega \left\{ \arctg( -\text{Im } z_i / \text{Re } z_i ) \right\} \quad (15)$$

The inverse of the tangent function is not unique, therefore  $\pi$  or  $2\pi$  has to be added whenever convenient.

Substituting  $\tau_i$ 's into the following equation:

$$H_{Rk} = \sum_{i=1}^M a_i \cos \tau_i \omega_k \quad (16)$$

leads to a system of linear equations in the  $a_i$ 's the amplitudes of the basis functions.

#### IV. COMPUTER IMPLEMENTATION

In order to practically implement the approximation model, several important conditions must be considered.

1- Fig.5 shows the amplitude spectrum of the pulse function  $X(\omega)$ . The frequency band is limited to 1.5 Hz (9 harmonics), while that of the average amplitude spectrum of the peristaltic waves  $Y(\omega)$  is limited to about 8 Hz (50 harmonics). Since  $X(\omega)$  and  $Y(\omega)$  are both low frequency processes, the frequency range of  $H(\omega)$  must be limited to that of  $X(\omega)$  to avoid dividing by small values and thereby creating artifactual amplitudes of  $H(\omega)$ . However, limiting the spectral content of  $H(\omega)$  may drastically degrade the quality of the approximation. To circumvent this problem, we adopt an alternative approach (Abou-Chadi et al., 1991) Instead of performing the division  $Y(\omega)/X(\omega)$  in the frequency domain, we perform it in the  $z$ -domain using a conventional long division. The  $z$ -transform of (1) is:

$$Y(z) = X(z) \sum_{i=1}^M a_i z^{-\tau_i} \quad (17)$$

or

$$H(z) = Y(z)/X(z) = \sum_{i=1}^M a_i z^{-\tau_i}$$

$$\begin{aligned}
 &= \frac{b_0 + b_1 z^{-1} + \dots + b_k z^{-k}}{1 + a_1 z^{-1} + \dots + a_k z^{-k}} \quad (18) \\
 &= d_0 + d_1 z^{-1} + \dots + d_k z^{-k}
 \end{aligned}$$

where  $a_k$  and  $b_k$  are not necessarily non-zero valued.

Having determined  $H(z)$  in the form of a power series of  $z^{-1}$ ,  $h(t)$  can be obtained and subsequently  $H(\omega)$  and one can proceed with Prony algorithm in the frequency domain as explained previously. It should be noted that the use of the  $z$ -transform is not elaborate since the  $a_k$ 's and  $b_k$ 's are the sample values of  $x(t)$  and  $y(t)$ , respectively. Similarly,  $d_k$ 's are the sample values of  $h(t)$ .

2- Since the peristaltic time duration is mapped onto the unit circle, the fact that the roots are complex pairs poses a specific problem. Because any positive angle has its corresponding negative equivalent on the unit circle, one does not know which of the pair is correct. The solution is to place one root of a pair outside of the feasibility range, i.e., its lag time is greater than the peristaltic duration. This was accomplished by extending the time duration to double its value by simply padding with zeros.

3- Because of the noise imposed on  $H_k$ 's, a substantial property of Prony's algorithm is lost; the absolute values of the roots  $z_i$  in Eqn.(8) are different from unity. This effect has been reduced by smoothing the recorded esophageal waveform using a Hanning filter whose coefficients are 0.25, 0.5, 0.25 right after digitization. This has removed most artifacts interpreted as unwanted noise.

4- Particular attention should be given to the various types of roots that are obtained. Because the coefficients of the polynomial are real and symmetric, for every root  $z$ , both its inverse  $1/z$  and its complex conjugate  $\bar{z}$  are also roots. Three main cases may arise: 1) A real root. This corresponds to zero delay or  $\tau=128$  and is rejected as is its inverse. 2) A complex root of unit modulus. This, together with its complex conjugate, poses a special problem. Because any positive angle has its corresponding negative equivalent on the unit circle, one does not know which of the pair is correct. The solution is to place one root of a pair outside of the feasibility range, i.e. to make its lag time greater than the waveform length. This is accomplished by extending the waveform length to double its duration by simply padding with zeros. 3) A complex root with modulus not equal to unity. If  $z$  is a root of this type, then  $1/z$ ,  $\bar{z}$ , and  $1/\bar{z}$  are all distinct roots. If the modulus is within the range from 0.5 to 2.0 and therefore not too different from unity (Meyer et al., 1989), a single delay within the range is retained.

5- Because not every root is retained due to the above cases the final number of roots is often less than the initial order  $M$ ; the actual number is usually unpredictable. The optimum number of the pulses,  $M$ , required to approximate all peristaltic waves was found to be 10 and has been used here.

6- The delays which are computed using (15) have yet to be rounded because the timings must have discrete values. This process is found to have negligible effect on the  $a_i$ 's which are obtained from (16).

7- The dimension of the matrices in the iterative Prony algorithm is  $K-2*M$ , where  $K$  is the number of nonzero points in the discrete representation of  $H(\omega)$ . It has been found that this dimension should be greater than five for accurate solutions. In order to ensure this, the negative frequency region was also utilized as suggested by Chen and Shiavi, (1990). This greatly increased the accuracy of the results.

#### V. RESULTS AND DISCUSSION

Fig.6 shows two peristaltic waves and their Tauberian approximation models with  $\sigma = 1.6$ . The amplitudes and lags are also listed. It is seen that the waves are modeled quite well. However, there are also several small amplitudes occurring. These represent extraneous information and may be eliminated.

Most peristaltic waves have been satisfactorily modelled with  $\sigma = 1.6$ . However, for some cases larger values of  $\sigma$  have been found more appropriate to get a better model with fewer pulses.

Errors in solution are reflected by the roots of (8) being off the unit circle. Observation of the corresponding envelopes showed that they contain either brief periods or long duration peaks; that is, the model either contained too many pulses or  $\sigma$  was not large enough. Automated determination of the cause of errors is based on the fact that local peaks with long durations produced a set of  $a_i$  that are nearly equal. Thus, on an error condition, if the variance of  $a_i$  is small then  $\sigma$  is increased, otherwise  $M$  is decreased and the lags are recalculated.

The feature extraction technique has been applied to the peristaltic waves for each subject. The final features of each peristaltic wave is a set of parameters comprising the value of  $\sigma$  and pairs of values of time-lag and amplitude of the corresponding basis functions (pulses). The number of sets depends on the number of local peaks in each peristaltic.

As with any automated evaluative procedure, a normative database of feature sets needs to be established. The present method would be helpful in this context as it can be used to establish classes of parameters for different normal and pathological esophageal motility patterns. The statistical distribution of the parameters in each class would become the basis for assessing any peristaltic waveform via its derived feature set. This is the subject of a current investigation which aims to design a classification scheme for esophageal motility patterns that is completely automated and can operate without any user intervention.



## VI. CONCLUSION

A feature extraction scheme has been developed which recognizes the time, amplitude and duration of local peaks of the esophageal pressure patterns generated during human swallowing. The scheme utilizes a Tauberian-Prony approximation to analyze the envelope of the peristaltic waves. The waves are modeled as a summation of pulses which differ in times of occurrence, duration and amplitude. These wave parameters can be used for characterization and classification purposes.

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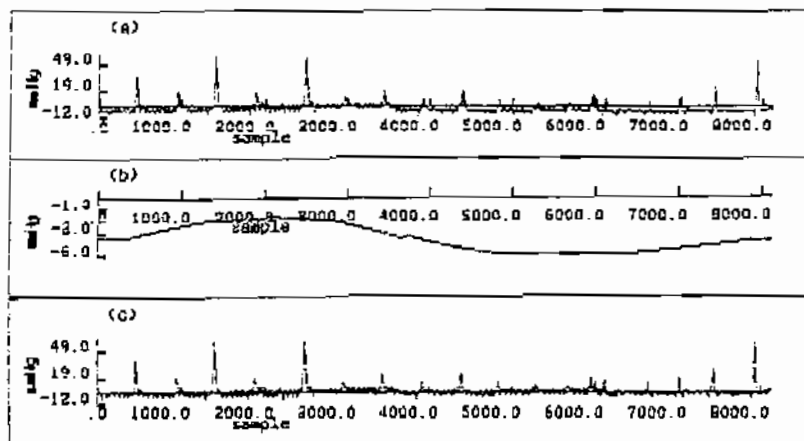


Fig.1 Typical tubular esophageal recording  
 (a) Original signal  
 (b) Estimated baseline drift  
 (c) Resultant signal after baseline removal

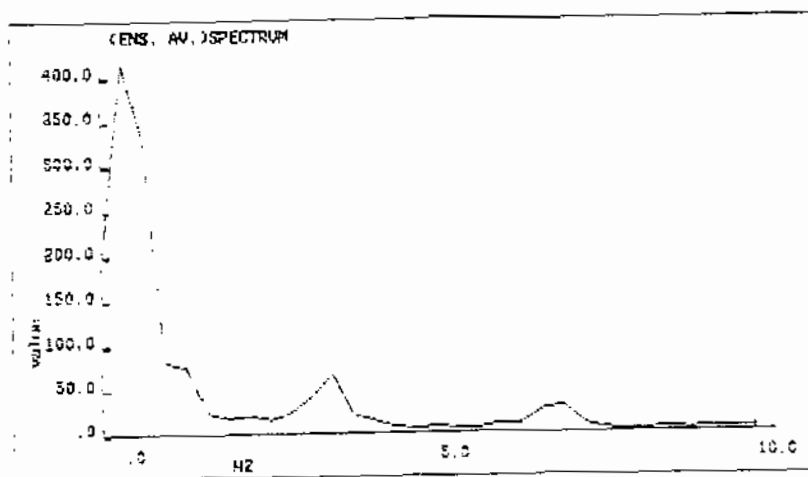


Fig.2 The ensemble average power spectrum  
 of peristaltic waves

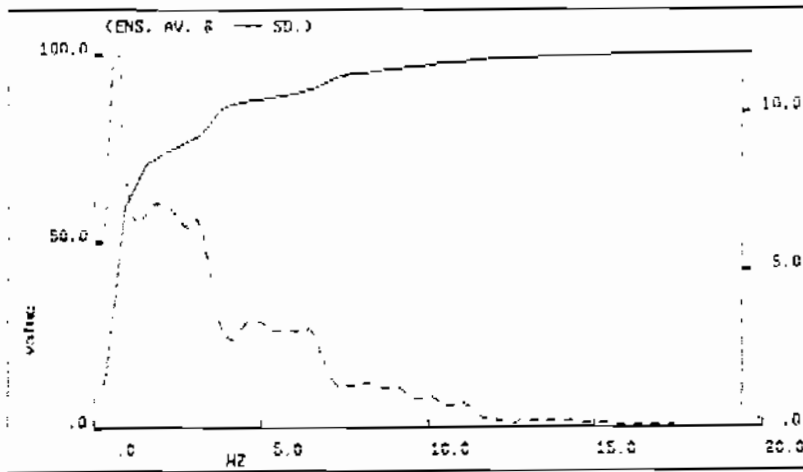


Fig.3 The percentage cumulative power density (—) and the ensemble standard deviation (---)

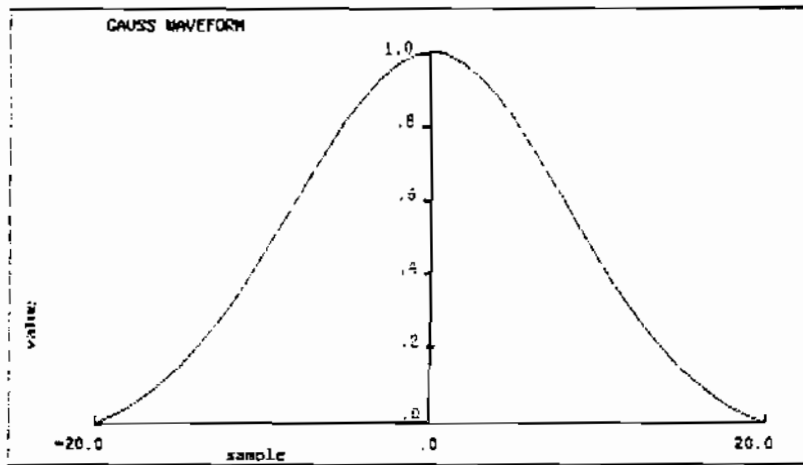
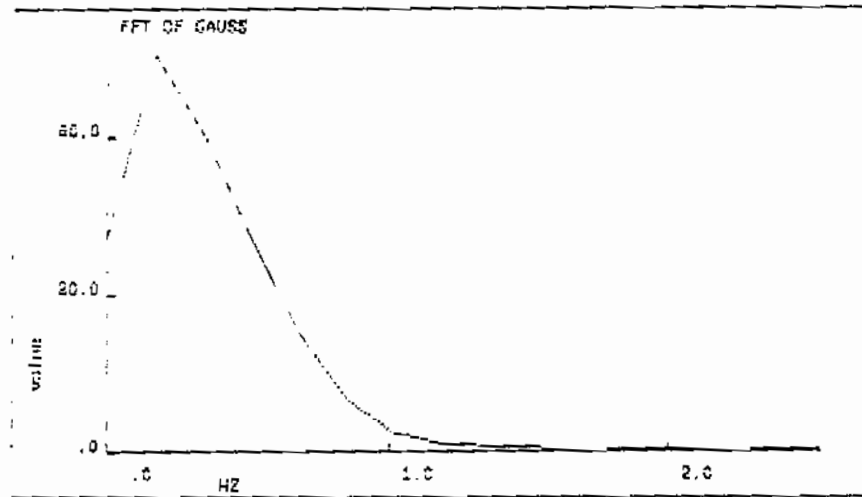
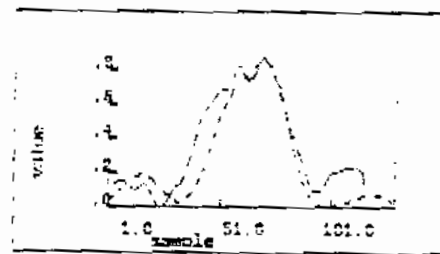
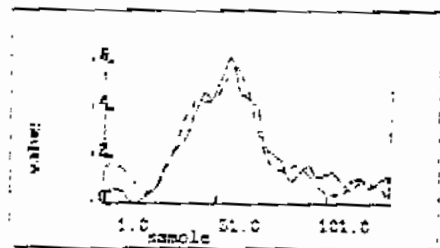


Fig.4 The Gaussian basis function  $x(t)$  for  $\sigma = 1.6$

Fig.5 Amplitude spectrum of the pulse function  $X(\omega)$ 

$\tau_i$ (samples)	$a_i$
15	0.194
53	0.708
69	0.837
74	0.175
117	0.073



$\tau_i$ (samples)	$a_i$
4	0.165
39	0.438
59	0.545
81	0.206
84	0.100
116	0.091

Fig.6 Examples of peristaltic waves (—) with their Tauberian modeling (- -). The accompanying table lists the lags and amplitudes of the pulses