DIGITAL COMPUTER SIMULATION OF INDUCTION MACHINES

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ABSTRACT

A digital simulation of an induction machine is described. The simulation is suitable for analyzing the operation for either a motor or generator with different speeds. The induction generator load is represented in the simulation by symmetrical or unsymmetrical R, L circuits. The computer program described in this paper calculates both the transient and steady-state performance of the machine from its equivalent circuit parameters. The effect of the magnitude and direction of the generator shaft speed on its stability is discussed. The method of simulation is relatively easy to program and can be solved with a personal digital computer. The example given was applied to study and illustrates the performance of the induction generator under different operating conditions.

1. INTRODUCTION:

The dynamic and steady-state behavior of the induction machines has an important effect on the overall performance of the system of which it is a part. Although induction machines are generally operated as a motor under balanced conditions, various unbalanced or unsymmetrical conditions can occur, especially if the machine operates as an induction generator.

Numerous simulations (2, 4, 5, 6, 7) resolve the three phase stator current into two axes, namely the direct and quadrature; they then resolve the rotating-rotor currents along these two stationary axes. The zero-sequence components are added[2] to obtain a complete transformation of the machine. Another transformation[1] that transforms the stator currents into zero-sequence and non-zero-sequence currents uses two stator currents as stationary axes and the rotor currents along these axes. In the three-wire system the third stator and rotor currents can be directly calculated by taking the negative summation of the two computed currents.

To use the digital computer to study the performance of three-phase induction generators under different operating conditions, a scheme of simulation is used. The method, first described by Robertson & Heblar[1], uses only two stator currents, which reduces the number of differential equations representing the simulation. This transformation eliminates the time-varying elements and reduces the computer time needed to solve the simulation problem. The machine considered here is a three-wire wound rotor, which can be supplied through its rotor circuit while the load is connected to the stator terminals. The shaft will be driven at a constant speed; therefore the total input power is the sum of shaft torque and the input power to the rotor. This power will vary according to the load connected to the generator. Balanced and unbalanced loads are considered, and the performance of the generator at each operating condition may be obtained by using this computer program. The load is assumed to be three-wire system, which is usually the condition in any ship distribution system where these generators are used.

The machine considered is symmetrical, which is an idealized assumption. There are some important factors that affect the performance of the actual machine such as, (a) a nonlinear magnetic circuit. (b) a harmonic component of the mmf wave due to the distributed winding, and (c) the effect of slotting on the magnetic flux distribution. These factors are not considered in this work and it will be a primary targets for future research.

This work is part of a complete investigation concerning a new type of generator. This new machine is somewhere between an induction machine and a synchronous machine. It operates over a wide range of speeds, while suppling power at a constant frequency and is called a Variable-Speed Constant Frequency (VSCF) machine.

In its modelling, a VSCF machine is much like an induction machine, especially those with wound rotors. Part of the input power needed to overcome the changes in speed and to keep the output frequency constant is supplied to the rotor by a special type of converter with variable output frequency in a closed-loop control circuit. To minimize the power fed to the rotor, due to transformer action, a new construction is to be found taking into account the significiant effect of the air gap length. As a first step in this direction, the induction machine considered in this paper is used as a VSCF machine to predict its behavior under these conditions.

2. BASIC EQUATIONS:

2.1. Voltage and Current Equations:

To obtain equations that completely describe a three-phase induction machine the following assumptions have been applied:

a) The stator and rotor windings are assumed to be symmetrical and perfectly distributed to produce a sinusoidal mmf wave in space.

b) The stator and rotor are assumed to be smooth cylindrical surfaces. The effect of slot and teeth on the permeance of the air gap is ignored.

c) The magnetic circuit is assumed to be linear, i.e., neglecting magnetic saturation.
d) The rotor coils are arranged so that, for any fixed time, the rotor mmf has the same number of poles as the stator mmf wave.

The voltage equation of an induction machine can be written in the matrix form as:

$$\begin{bmatrix} V^{s}_{123} \\ V^{r}_{123} \end{bmatrix} = \begin{bmatrix} R^{s}_{123} + pL^{s}_{123} & pL^{sr}_{123} \\ pL^{rs}_{123} & R^{r}_{123} + pL^{r}_{123} \end{bmatrix} \begin{bmatrix} i^{s}_{123} \\ i^{r}_{123} \end{bmatrix} \quad (1)$$

where

V,i	are the voltage and current for a particular winding on either stator or rotor.
p	the operator d/dt.
M	maximum mutual inductance between stator and roter coil.
n	number of pole pairs.
K^s, K^r	coefficients of mutual coupling between two stator or rotor coils respectively. The values of these factors are independent of a and slightly less than 0.5 because of leakage flux.
α	mechanical radians.

Solving these equations is difficult because of the non-linearity of the differential equations.

The equations can be simplified by a transformation of axes to obtain equations which are linear for different speeds. The proper change of variable eliminates the variation of mutual inductances with the displacement angle α . The power invariant transformation(3), which uses Tensor analysis, is employed.

If the voltage equation is

$$[V] = [R][i] + \frac{d}{dt}(Li) \tag{2}$$

and if the circuit configuration is changed so that the same equation is described by

$$[V`] = [R`][i`] + \frac{d}{dt}[L`i`] \tag{3}$$

where the currents $[i]$ and $[i`]$ are related by the connection matrix $[C]$, then $[i] = [C].[i`]$ and $[V`] = [C]_T [V]$.

The relation between $[R]$ and $[R`]$ and $[L]$ and $[L`]$ are given by

$$[V] = [R][i] + \frac{d}{dt}[Li]$$

$$[C]_T[V] = [C]_T \cdot \{[R]\cdot[C]\cdot[i`] + [L]\cdot[C]\frac{di`}{dt} + \frac{d}{dt}[L]\cdot[C]\cdot[i`]\}$$

$$[V`] = [C]_T \{ [R][C] + \frac{d}{dt}[L]\cdot[C] \} i` + [C]_T[L]\cdot[C]\frac{di`}{dt} \tag{4}$$

From equation (3) and (4)

$$[R`] = [C]_T \{ [R]\cdot[C] + \frac{d}{dt}[L]\cdot[C] \} \tag{5}$$

and

$$[L`] = [C]_T \cdot[L]\cdot[C] \tag{6}$$

Therefore, if the connection matrix $[C]$ is known, then variables in any two reference frames can be related in a straightforward manner.

To obtain the connection matrix the current for both stator and rotor are first separated into zero and nonzero components. For a three-wire system the zero sequence com-

$$i^{s}_{1,2,3} = i^{s}_{a,b,c} + i^{s}_{o} = i^{s}_{a,b,c} \tag{7}$$

Therefore

$$i^{s}_{a} + i^{s}_{b} + i^{s}_{c} = 0 \quad \text{or} \quad i^{s}_{b} = -(i^{s}_{a} + i^{s}_{c}) \tag{8}$$

and for the rotor circuit

$$i^{r}_{1} + i^{r}_{2} + i^{r}_{3} = 0 \quad \text{or} \quad i^{r}_{2} = -(i^{r}_{1} + i^{v}_{3}) \tag{9}$$

The second step is to transform the rotor current quantities to three stationary reference axes, as shown in Fig.2. The new transformed values of rotor currents should produce the same mmf as the actual currents. Transforming the current into and perpendicular to R_a axes,

$$i^{r}_{1}\text{Cos}\,(n\alpha) + i^{r}_{2}\text{Cos}\,(n\alpha+2\pi/3) + i^{r}_{3}\text{Cos}\,(n\alpha-2\pi/3) = i^{r}_{a} + i^{r}_{b}\text{Cos}\,(2\pi/3) + i^{r}_{c}\text{Cos}\,(2\pi/3) \tag{10}$$

and

$$i^{r}_{1}\,\text{Cos}\,(n\alpha+\pi/2) + i^{r}_{2}\,\text{Cos}\,(n\alpha+7\pi/6) + i^{r}_{3}\,\text{Cos}\,(n\alpha-\pi/6) = i^{r}_{a}\,\text{Cos}\,(\pi/2) + i^{r}_{b}\,\text{Cos}\,(7\pi/6) + i^{r}_{c}\,\text{Cos}\,(\pi/6) \tag{11}$$

From symmetry $i^{r}_{a} + i^{r}_{b} + i^{r}_{c} = 0$ (12)

From equations (9), (10), (11) and (12) the relation between the transformed and actual rotor current is given by

$$2/\sqrt{3}\begin{bmatrix} \text{Cos}(n\alpha+\pi/6) & -\text{Sin}(n\alpha) \\ \text{Sin}(n\alpha) & \text{Cos}(n\alpha-\pi/6) \end{bmatrix} \cdot \begin{bmatrix} i^{r}_{a} \\ i^{r}_{c} \end{bmatrix} = \begin{bmatrix} i^{r}_{1} \\ i^{r}_{3} \end{bmatrix} \tag{13}$$

The rotor voltage can be transformed from $[V^r]$ into $[V^{r`}]$ by the same method. Finally, the connection matrix for both primary and secondary currents referred to as a stationary axis is given by

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2/\sqrt{3}\,\mathrm{Cos}\,(n\alpha+\pi/6) & -2/\sqrt{3}\,\mathrm{Sin}(n\alpha) \\ 0 & 0 & -2/\sqrt{3}[\mathrm{Cos}(n\alpha+\pi/6)+\mathrm{Sin}(n\alpha)] & 2/\sqrt{3}[\mathrm{Sin}(n\alpha)-\mathrm{Cos}(n\alpha-\pi/6)] \\ 0 & 0 & 2/\sqrt{3}\,\mathrm{Sin}(n\alpha) & 2/\sqrt{3}\,\mathrm{Cos}\,(n\alpha-\pi/6) \end{bmatrix} \cdot \begin{bmatrix} i_a^s \\ i_b^s \\ i_a^r \\ i_c^r \end{bmatrix} = \begin{bmatrix} i_1^s \\ i_2^s \\ i_3^s \\ i_1^r \\ i_2^r \\ i_3^r \end{bmatrix} \quad (14)$$

With equations (5) and (6), the connection matrix can be used to transform the voltage, resistance, and inductance.

$$[V^`] = \begin{bmatrix} V_1^s - V_2^s \\ V_3^s - V_2^s \\ V_1^{r`} \\ V_2^{r`} \end{bmatrix} \quad (15)$$

where

$$V_1^{r`} = 2/\sqrt{3}\,\mathrm{Cos}\,(n\alpha+\pi/6)\,V_1^r - (2/\sqrt{3})\,\{\mathrm{Cos}\,(n\alpha+\pi/6) + \mathrm{Sin}\,n\alpha\}\,V_2^r + 2/\sqrt{3}\,\mathrm{Sin}\,(n\alpha)\,V_3^r$$

$$V_2^{r`} = -2/\sqrt{3}\,\mathrm{Sin}(n\alpha)\,V_1^r + (2/\sqrt{3})\,\{\mathrm{Sin}(n\alpha)-\mathrm{Cos}(n\alpha-\pi/6)\}\,V_2^r + 2/\sqrt{3}\,\mathrm{Cos}(n\alpha-\pi/6)V_3^r$$

The transformed time-varying resistance matrix and the constant inductance matrix are given by

$$[R`] = \begin{bmatrix} 2R^s & R^s & 0 & 0 \\ R^s & 2R^s & 0 & 0 \\ 0 & -3\sqrt{3}/2\ Mn\dot{\alpha} & 2R^r & R^r-3\sqrt{3}/2L^r n\dot{\alpha} \\ 3\sqrt{3}/2Mn\dot{\alpha} & 0 & R^r+3\sqrt{3}/2L^r n\dot{\alpha} & 2R^r \end{bmatrix} \tag{16}$$

$$[L`] = \begin{bmatrix} 3L^s_{pn} & 3/2L^s_{pn} & 3M & 3/2M \\ 3/2L^s_{pn} & 3L^s_{pn} & 3/2M & 3M \\ 3M & 3/2M & 3L^r_{pn} & 3/2L^r_{pn} \\ 3/2M & 3M & 3/2L^r_{pn} & 3L^r_{pn} \end{bmatrix} \tag{17}$$

where

$L^s_{pn} = 2/3\ (1+K^s)L^s$ and $L^r_{pn} = 2/3\ (1+K^r)L^r$

Because $[L`]$ has become time independant, the voltage equation (2) in terms of the transformed variable is given by

$$[V`] = [R`]\cdot[i`] + [L`]\ p[i`] \tag{18}$$

From equation (18), for constant speed or steady state-analysis, the transformation used changes the voltage equations into ordinary first-order differential equations, which are easy to solve with digital computer.

2.2. Torque Equation:

The power supplied to the machine is given by

$$P_{input} = [i']_T \cdot [V'] = [i']_T \cdot [[R'] \cdot [i'] + [L'] \; p \; [i']] \tag{19}$$

The $[\dot{L}]$ matrix does not contribute anything to the torque after the transformation to a time-inarying matrix. Therefore only the first term of equation (19) contributes to the machine torque. Terms involving the product of any two currents and a resistance represent the copper losses of the machine which is transformed into dissipated heat and does not produce any torque. Thus from (19) substituting from equation (16), the mechanical output power is given by

$$P_m = \frac{3\sqrt{3}\, M\alpha\dot{n}}{2} \left(i_a^s \, i_c^r - i_c^s \, i_a^r \right) \tag{20}$$

Therefore

$$\text{Torque} = \frac{P_m}{\dot{\alpha}} = \frac{3\sqrt{3}}{2} Mn \left(i_a^s \, i_c^r - i_c^s \, i_a^r \right) \tag{21}$$

3. Induction Machine As a Generator Under Different Load Conditions:

This part presents and illustrates the performance characteristics of an induction machine when it operates as a generator. The generator has a three phase voltage applied to the wound rotor terminals. The stator terminals are connected to a three phase load as shown in Fig. 3. For a wide variety of symmetrical and unsymmetrical load the equations that describe the behavior of the induction generator are illustrated as follows.

In this case the stator can be presented as short circuited with an additional load impedence in series with the stator circuit.

Thus

$$z_1 = Z_1^s + Z_1^L = R_1^s + R_1^L + p(L_1^s + L_1^L) \tag{22}$$

Howevers, the stator windings have equal internal impedances for the three phases.

Therefore

$$(R^s + R^L_{1,2,3}) + p(L^s + L^L_{1,2,3}) = R_{1,2,3} + p\ L_{1,2,3} \tag{23}$$

while the stator voltages become

$$V_1^s = V_2^s = V_3^s = 0 \tag{24}$$

The mutual coefficients K^s and K^r are assumed 0.5.

Following the same steps as in (2-2) by transforming the rotor supplied voltages and currents into a stationary axis and using the same assumption as before, the time varying resistance matrix and time-invarying inductance matrix may be obtained as

$$[R`] = \begin{bmatrix} R_1 + R_2 & R_2 & 0 & 0 \\ R_2 & R_2 + R_3 & 0 & 0 \\ 0 & -3\sqrt{3}/2\ Mn\alpha^{\bullet} & 2R^r & R^r - 3\sqrt{3}/2\ L^r_{pn}n\alpha^{\bullet} \\ 3\sqrt{3}/2\ Mn\alpha^{\bullet} & 0 & R^r + 3\sqrt{3}/2\ L^r_{pn}n\alpha^{\bullet} & 2R^r \end{bmatrix} \tag{25}$$

and

$$[L] = \begin{bmatrix} 3/2(L^{s}_{pn_1} + L^{s}_{pn2}) & 3/2\ L^{s}_{pn2} & 3\ M & 3/2\ M \\ 3/2\ L^{s}_{pn2} & 3/2(L^{s}_{pn2} + L^{s}_{pn3}) & 3/2\ M & 3\ M \\ 3M & 3/2\ M & 3L^{r}_{pn} & 3/2\ L^{r}_{pn} \\ 3/2\ M & 3M & 3/2\ L^{r}_{pn} & 3L^{r}_{pn} \end{bmatrix} \quad (26)$$

where

$$(L_{1\ 2,3} + K^{s}L^{s}) = 3/2\ L^{s}_{pn\ 1,2,3} \quad \text{and} \quad L^{r}(1+K^{r}) = 3/2\ L^{r}_{pn}$$

From equation (4)

$$[V] = [R]\,[I] + [L]\ p\,[I]$$

or

$$[I] = \int_{0}^{t} [L]^{-1} \{[V] - [R][I]\} \quad (27)$$

where $[L]^{-1}$ is the inverse of the matrix $[L]$.

Solving equation (27) with the digital computer, the currents in both stator and rotor windings are obtained. The stator voltage across the load terminals may be given by

$$[V^{L}] = [R^{L}]\cdot[i^{s}] + [L^{L}]\cdot p[i^{s}] \quad (28)$$

Also the torque, which should be supplied to the generator shaft from the prime mover, can be calculated from equation (21) as input of useful torque.

$$T = \frac{\sqrt{3}}{2} \; Mn \, (i_a^s \, i_c^r - i_c^s \, i_a^r) \tag{29}$$

The input power to the generator consists of the electric power supplied to the rotor terminal and the mechanical power that drives the rotor shaft. Each component as percentage of the total input power, depends on the construction of the machine, especially the air gap length (due to the transformer action).

4. Computer Simulation:

In a computer simulation it is essential to observe the important system variables and desirable to obtain this requirement in a minimum amount of computer time. A complete simulation of the induction machines with balanced or unbalanced load can be implemented from equations (27), (28), and (29), which include the machine and the load parameters. The parameters presented in the $[R^{`}]$ and $[L^{`}]$ matrix vary with the type of loads and the rotor speeds.

To solve equation (27), an integration method is used with the digital computer. This method should be chosen carefully, compromising between accuracy of results, computer time, and stability of the method. Runge-Kutta, and Milne's predictor-corrector give more accurate results, but they require a large computing time for the execution of each integration step. A computer program with a fixed integration step has been developed that gaves good accuracy with a short computer time when compared with the more accurate methods.

The results were plotted for dynamic and steady state conditions. For example, the stator output current and the torque, which should supply the shaft to obtain that current, are presented.

The machine considered in this work has a wound rotor and three-wire symmetrical systems with the following parameters: R^s = 0.2 ohm, R^r = 0.253 ohm, L = 0.0825 H, M = 0.081 H, the number of pole pairs (n) = 2, and the output voltages of the stator windings are assumed to have constant frequency ($W_s = W_r + \alpha°$).

Figure 4 shows the stator current and the shaft torque when the stator is shorted and the rotor speed ($\alpha°$)= -185; Figure 5 shows the same current and torque at a different rotor speed $\alpha°$ = 125. Comparing the two curves shows that the effect of the shaft speed on the output current increases by more than three times and that the machine takes more time to become stable than in the case of Figure 4.

Figure 6 shows the condition of the generator at an unbalanced resistive load with rotor shaft speed $\alpha°$= -185. Figure 7 shows the balanced resistance and unbalanced inductive loads with rotor shaft speed $\alpha°$ = 125 and the rotor frequency W_r = 127. This figure shows the unstability in both output stator current and input torque which is mainly due to the harmonic generated in this case. Similar results presented in Figure 8, is obtained for an unbalanced resistive load, whereas Figure 9 shows the effect of unbalanced resistive load with balanced inductive load.

5. CONCLUSIONS:

A technique for the simulation of an induction machine has been described when it operates as an induction generator. The technique obtains the characteristic performance of the generator under different operating conditions. For balanced load the stability and the transient period depends on the mechanical power supplied to the rotor in the form of speed and mechanical torque. Unbalanced load distorts the behavior and performance of the machine rapidly, depending on the degree of imbalance. One reason for this distortion is the appearance of harmonics. Under this condition the stability is affected, which is one of the important unsolved problems

The simulation model can be used to study the induction motor performance with an unbalanced power supply. By modifying the model it can account for the space harmonics of the mmf in the study of its effect on the performance of induction machines.

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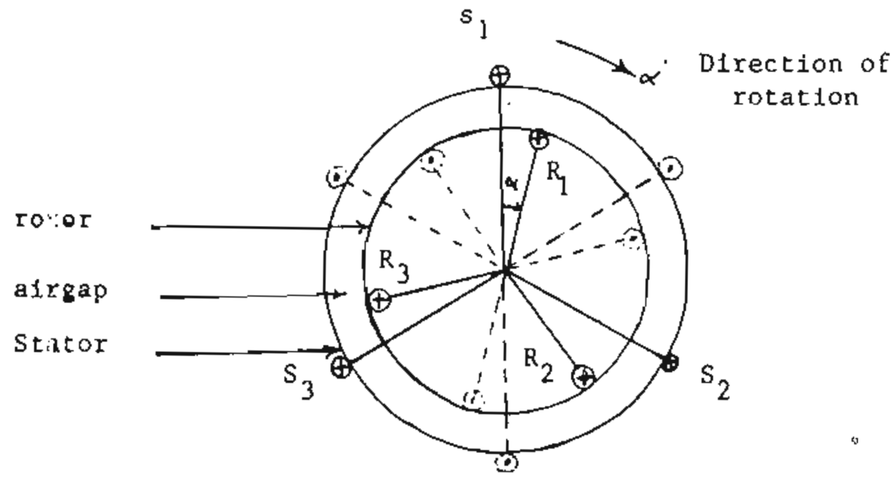
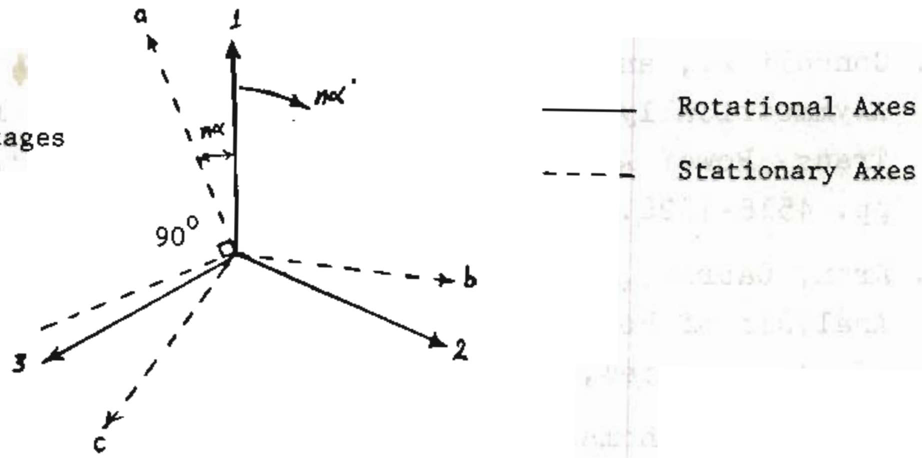
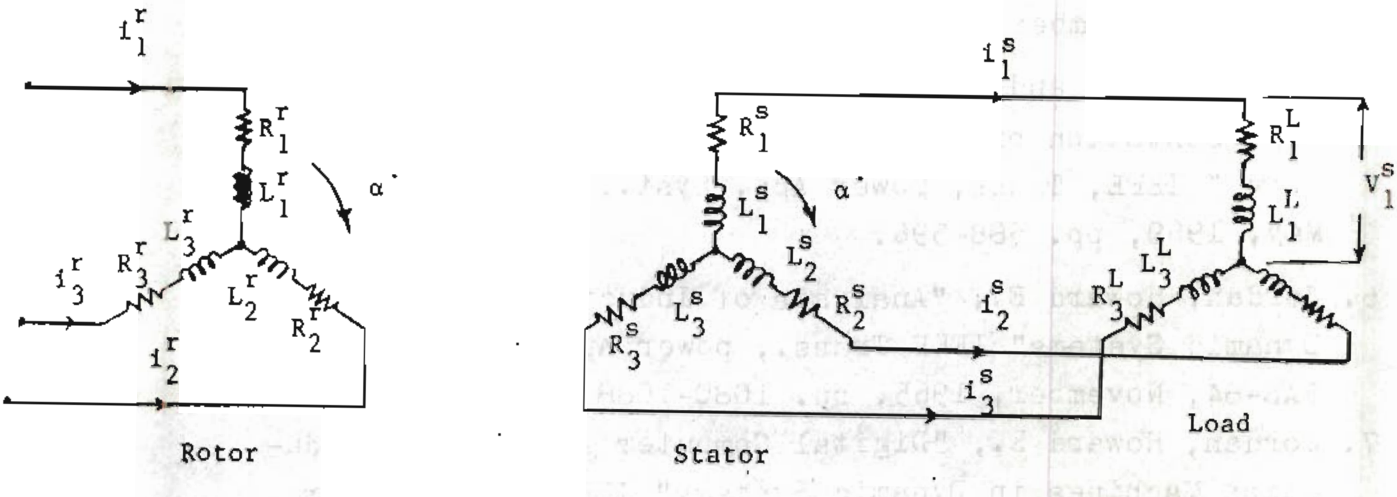


Fig.(1) A three phase symmetrical Induction Machine

Fig.(2) Transformation of rotor Currents and Voltages into a stationary axes.



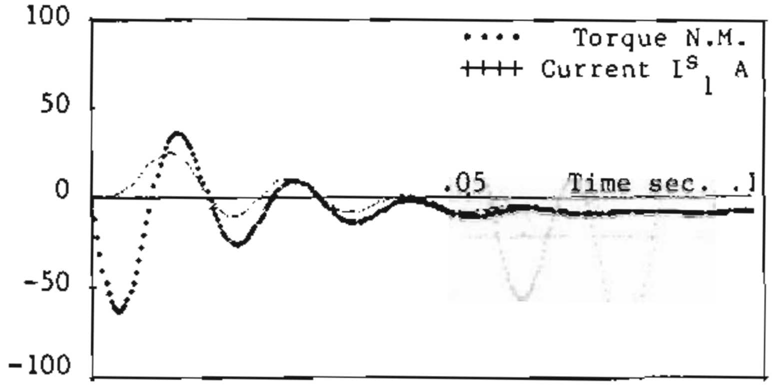


ig.(4). The generator output
urrent and input torque at:

rotor frequency $w_r = 377$

rotor speed $\dot{\alpha} = -185$

$R^L = L^r = 0.0$

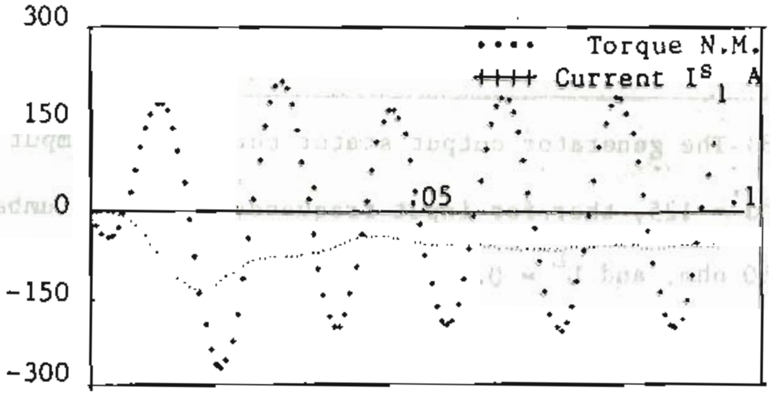


'ig.(5) The generator output
urrent and input torque at :

rotor frequency wr =377

rotor speed $\dot{\alpha} = 125$

$R^L = L^L = 0$



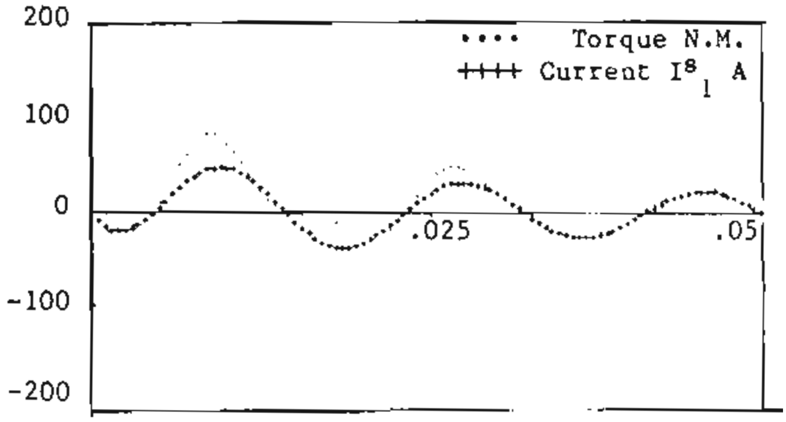
'ig.(6) The output current and
.nput torque at:

rotor freq. Wr = 377

rotor speed $\dot{\alpha} = -185$

R^L = 0, 50, 0 ohm

$L^L = 0.0$



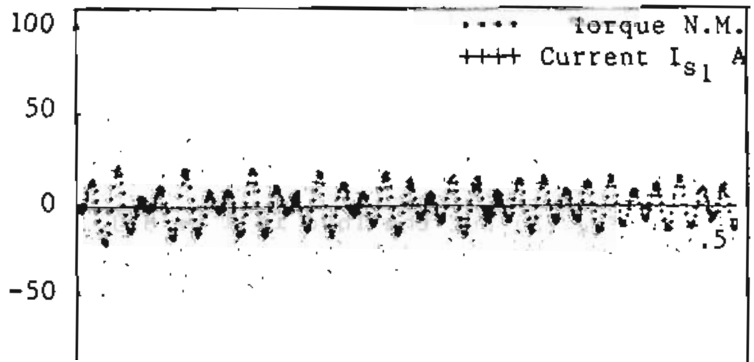
'ig.(7) the output current and
input
'orque at :

rotor freq. wr = 127

rotor speed $\dot{\alpha} = 125$

R^L = 2,2,2 ohm

L^L = 0.0, 0.5, 0.2 Hen.



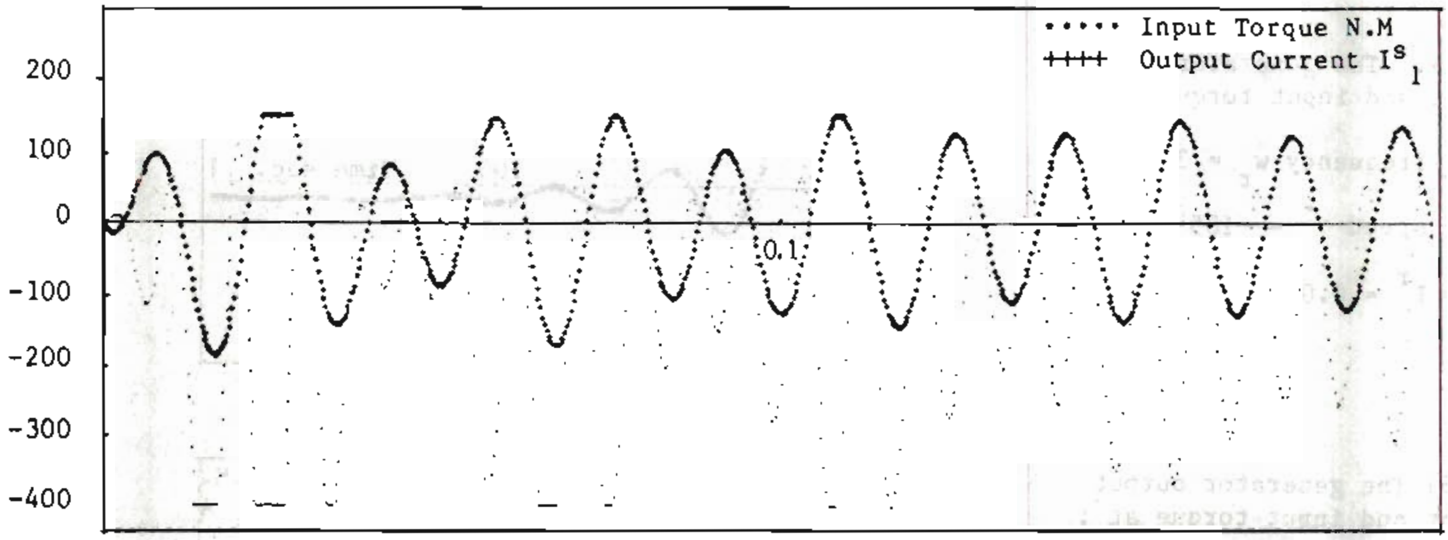


Fig.(8) The generator output stator current and imput shaft torque at: the rotor speed $\dot{\alpha}$ = 125, therefor input frequency w_r = 127, unbalanced resistance load R^L =0.0, 2.0, 50 ohm, and L^L = 0.0

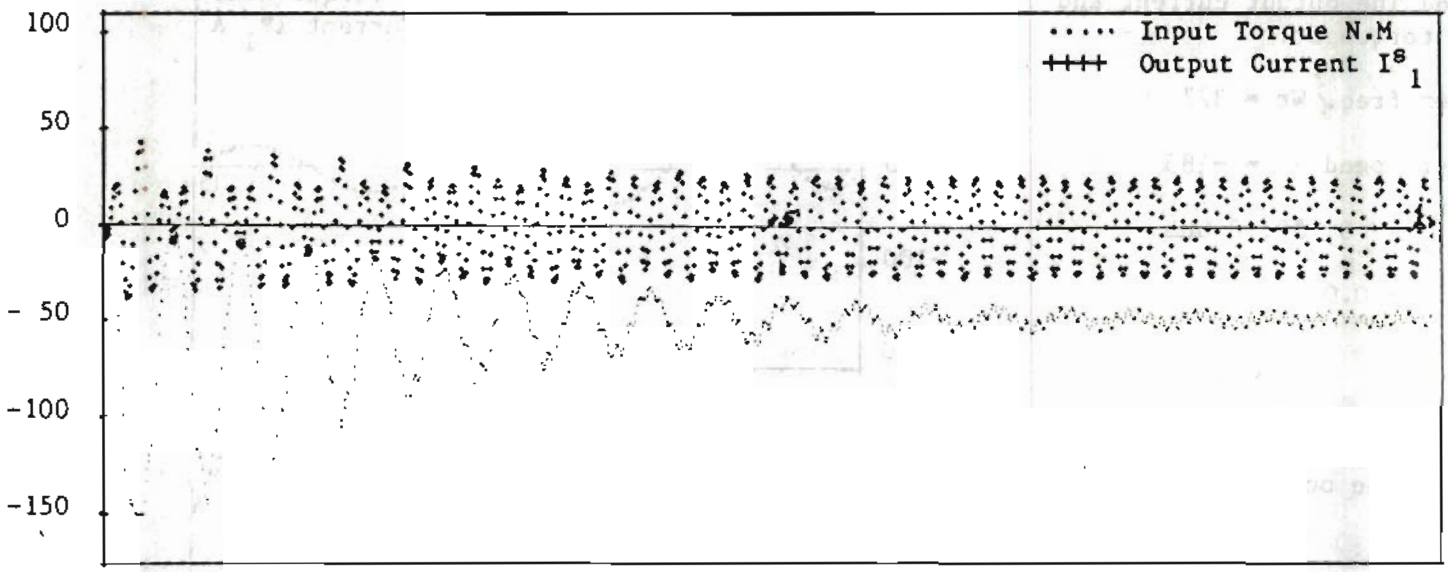


Fig.(9). The stator current and input torque at the same rotor speed and input frequency as in Fig.(8) but the load is R^L = 8.0, 10, 10 ohm and L^L = 0.03, 0.03, 0.03 Hen.