Mansoura University
Faculty of Engineering
Math. & Eng. Physics. Dept.

Mathematics 4

1st year Civil Engineering Dept.
Final Exam, June 2012
Time Allowed: 3 Hours.

يتألف الإختبار من 4 أسئلة في ورقة واحدة من وجهين. برجاء بدء إجابة كل فرع من إحدى نهايتي ورقة الإجابة.

<u>يسمح بالإجابة باللغة العربية في فرع التحليل العددي عند الحاجة للشرح</u>

Part (A)

Question (1): [25 points]

(a) Use the successive approximation method to find one root (accurate to three decimal places) of the equation $x^3 - 3 = 2x$. Take $x_0 = 1.32235$. **[5 points]**

(b) Use Taylor series method of third order to approximate y at $x = 0.1$ for the differential equation $y' = 3x + y^2, y(0) = 1$ **[10 points]**

(c) Use the finite difference method to approximate the solution of

$y'' + 4y = \cos x$, $0 \le x \le \frac{\pi}{4}$, subject to the conditions $y(0) = 0$, $y(\frac{\pi}{4}) = 0$.

Divide the interval into three segments. **[10 points]**

Question (2): [30 points]

(a) Solve the following PDE numerically

$$u_t - u_{xx} = 0,\ 0 < x < 2,\ 0 < t$$

where the solution equals zero at both boundaries and equals $x(2-x)$ at the initial step. Take $\Delta x = 0.5$ then choose an appropriate Δt to find the approximate solution at $t = 0.3$

[15 points]

(b) Deduce the stability condition when solving the equation $u' = a\, u$, where a is a real constant, using implicit (backward) Euler method for: (i) $a > 0$ (ii) $a < 0$. **[10 points]**

(c) A function is required to pass through 20 readings. What is the best method to do it: Newton divided difference, cubic splines, least squares fitting? Why? **[5 points]**

Part (B)

3. (a) Evaluate the following integrals

   i. $\int_0^\infty \sqrt{x}\ e^{-\sqrt{x}}dx$

   ii. $\int_0^{\pi/2} \sqrt{\tan\theta}\ d\theta$

   iii. $\int x^{n+3}J_n(x)\ dx$

   (b) Find the the general solution of the differential equation $x^2y'' - xy' + (4x^2-3)y = 0$.

   (c) For the second order ordinary differential equation

$$(x^2-1)y'' + 4xy' + 2y = 0,$$

   i. Locate and classify its singular points, if exist.

   ii. Use the power series method to find the series solution about the ordinary point $x = 0$.

   (d) Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\ \cos x.$ [**Hint:** $\Gamma(n+\frac{1}{2}) = \frac{\sqrt{\pi}\ 2n!}{2^{2n}\ n!}$]

4. (a) Find the first two nonzero terms in the Legendre expansion of

$$g(x) = \begin{cases} 0 & -1 < x < 0, \\ 4 & 0 < x < 1. \end{cases}$$

   (b) Determine the regions in the xy-plane for which the equation $\frac{\partial^2 u}{\partial y^2} + y\frac{\partial^2 u}{\partial x^2} = 0$ is hyperbolic, parabolic, or elliptic.

   (c) Given the function

$$f(x) = \begin{cases} -1-x & -1 < x < -\frac{1}{2} \\ x & -\frac{1}{2} < x < \frac{1}{2}, \\ 1-x & \frac{1}{2} < x < 1 \end{cases}$$

   which has the property $f(x) = f(x+2)$. Answer the following questions :

   i. Sketch the function within three complete successive periods

   ii. Check whether the function is odd or even or neither

   iii. Evaluate the Fourier coefficients

   iv. Write the corresponding Fourier series

   (d) By using the method of separation of variables find the solution $u(x,t)$ to the diffusion equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 1, \quad t > 0,$$

   where $u(x,t)$ satisfies the initial and boundary conditions

$$u(x,0) = \begin{cases} x & \text{for} \quad 0 < x < \frac{1}{2} \\ 1-x & \text{for} \quad \frac{1}{2} < x < 1 \end{cases},$$

$$u(0,t) = 0, \qquad u(1,t) = 0,$$

   [Hint: you may find the answer to part (c) useful.]

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Math (4)
(من الخارج)

1^{st} year.
Final Exam, May 2012
Time Allowed: 3 hour

Exam Guidelines:
This Exam contains **4** questions in **2** pages. Please start the answer of each branch on the opposite ends of your answer sheet.

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Question (1) [25 points]

(a) Evaluate the integrals: $\int_0^\infty \sqrt[3]{y}\, e^{-y}\, dy$ **[5 points]**

(b) Find the series solution of the differential equation about $x_0 = 0$

$$y'' + x^2 y = 0$$ **[10 points]**

(c) Prove that for any positive integer *n*, $J_{-n}(x) = (-1)^n J_n(x)$. **[10 points]**

Question (2) [30 points]

(a) Obtain the Fourier series of the function $f(x)$ defined by:

$$f(x) = \begin{cases} 0 & -\pi \le x < 0 \\ \pi - x & 0 \le x \le \pi. \end{cases}$$

with $f(x) = f(x + 2\pi)$. Hence, prove that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$ **[15 points]**

(b) Use the separation of variables technique to solve the heat equation

$$u_{xx} = \frac{1}{k} u_t, \quad 0 < x < L, \quad t > 0$$

subjected to the conditions

$$u(0,t) = 0$$
$$u(L,t) = 0$$
$$u(x,0) = f(x)$$ **[15 points]**

Part II – Numerical Methods

Question (3): [29 Marks]

(a) Deduce the normal equations for the least square line $y = A + Bx$ [7 Marks]

(b) Consider the function $g(x) = 2L_0(x) + L_1(x) - (L_2(x) + 3)^2$, where $L_i(x)$ [8 Marks]
Lagrange's coefficient polynomial based on nodes x_0, x_1, x_2.

(i) Find the values of $g(x_i)$, $i = 0,1,2$

(ii) What is the degree of the polynomial $g(x)$.

(c) Use Newton-Raphson method to find the root of the equation $5x - e^x = 0$, correct to 4 decimal places. [7 Marks]

(d) Deduce the convergence condition for the simple iteration method. [7 Marks]

Question (4): [24 Marks]

(a) Use the iterative form $x_i^{(k)} = \frac{1}{a_{ii}}[b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k-1)}]$ to solve the system

$$2x - 4y + 10z = 6$$
$$5x - y - 3z = -2$$
$$x + 5y - 2z = 10$$

[8 Marks]

Four iterations are required.

(b) Given the initial-value problem $2y' = 2xy + 3$, $y(0) = 1$, find:

(i) $y(0.2)$ by Euler's method. [8 Marks]

(ii) $y(0.3)$ by the fourth order Runge-Kutta method.

(c) Use finite difference method to solve the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < 2, \; 0 < y < 2, \text{ where}$$

[8 Marks]

$$u(x,0) = u(x,2) = x(2-x), \qquad 0 < x < 2$$
$$u(0,y) = u(2,y) = 0, \qquad 0 < y < 2, \qquad h = k = 0.5$$