Mansoura University
Faculty of Engineering
Math. & Eng. Physics Dept.

Nonlinear programming

September 18, 2013

2^{nd} year Postgraduate Students

Time allowed: 3 Hours

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Answer the following problems (Full mark 100 pts)

1. (a) **[5 pts]** Define the following items: Slack variables, Unrestricted variables, Feasible region, Objective function, Optimal solution.

   (b) **[15 pts]** Leather Limited manufactures two types of belts the deluxe model and regular model. Each type requires 1 sq yd of leather. A deluxe belt requires 2 hour of skilled labor, and a regular belt requires 1 hours of skilled labor. Each week; 40 sq yd of leather and 60 hours of skilled labor are available. Each deluxe belt contributes 4 LE to profit, and each regular belt contributes 3 LE to profit.
   (i) Use the graphical method to solve the linear programming problem
   (ii) Verify the graphical solution using the Simplex algorithm.

2. (a) **[15 pts]** A furniture company manufactures desks, tables, and chairs. The manufacture of each type of furniture requires lumber and two types of skilled labor: finishing labor and carpentry labor. The amount of each resource needed to make each type of furniture is given in the following table.

Resource	Desk	Table	Chair
Lumber(board ft)	8	6	1
Finishing hours(hours)	4	2	1.5
Carpentry hours(hours)	2	1.5	0.5

At present, 48 board ft of lumber, 20 finishing hours, and 8 carpentry hours are available. A desk sells for 60 LE, a table for 30 LE, and a chair for 20 LE.
(i) Formulate a mathematical model that can be used to maximize the company's total revenue.
(ii) Use the Simplex algorithm to solve this problem
(iii) Write the dual problem of this problem and then solve it

   (b) **[15 pts]** Use the penalty method to solve the problem: Minimize $z = x_1 + 4x_2$, subject to $x_1 + 3x_2 = 3$, $3x_1 + 4x_2 \geq 6$, $2x_1 + x_2 \leq 4$, $x_1 \geq 0$, $x_2 \geq 0$.

3. (a) [5 pts] State what is meant by: Degeneracy, Unbounded solution, Alternative optima; Balanced transportation problem, Dummy demand point.

   (b) [15 pts] Suppose that a medical company intends to install air conditioning in three of its building during a two weeks vacation. The company invited three contractors to submit separate offers for the work involved in each of the three buildings. The received offers are arranged in the following table in 1000 LE.

	Building 1	Building 2	Building 3
Contractor 1	53	96	37
Contractor 2	47	87	41
Contractor 3	60	92	36

Each contractor can install the air conditioning for only one building during the two week vacation, so the company must assign a different contractor for each building. Use the Hungarian method to determine to which building should each contractor be assigned in order to minimize the sum of the corresponding offers. What is the minimum offer?.

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4. (a) [15 pts] Talkha company has three electric power plants that supply the power needs for four cities. Each power plant can supply the following numbers of kilowatt-hours(kwh) of electricity: plant 1, 35 million; plant 2, 50 million; plant 3, 40 million. The peak power demands in these cities are as follows (in kwh): city 1, 45 million; city 2, 20 million; city 3, 30 million, city 4, 30 million. The cost of sending 1 million kwh of electricity from plant to city depend on the distance the electricity must travel (See the following table). Use Vogel's method to minimize the cost of meeting each city's peak power demand.

From	city 1	city 2	City 3	city 4	Supply
plant 1	8	6	10	9	35
plant 2	9	12	13	7	50
plant 3	14	9	16	5	40
Demand	45	20	30	30	

   (b) [15 pts] Use the branch-and-bound algorithm to solve the problem:

$$\text{maximize} \quad z = 5x_1 + 4x_2$$

subject to

$$\begin{aligned} x_1 &+ x_2 &\le 5 \\ 10x_1 &+ 6x_2 &\le 45 \\ x_1, x_2 &\ge 0 \text{ and integer} \end{aligned}$$