

علو في الامتحان

Menofiya University
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Department of Basic Eng. Sciences.
2nd Semester Exam., 2016-2017
Date of Exam. : 10 / 6 / 2017



Subject: Eng. Mathematics
Code: BES 712
Year : Grade 700
Time allowed : 3 hrs.
Total marks: 100 marks

Answer all the following questions

Question(1) [25 marks]

a) If $f(z)=e^z$, use the change of variables to show that $\int_c f(z)dz = \int_{c'} dw$, and if c is the straight line $x=0$ in the z -plane, show that c is mapped into the unit circle c' in the w -plane. Deduce the integral and check your result.

b) Using Chayley Hamilton theorem solve and check the system of equations

$$3x^2+2y^2+z^2=20$$

$$2x^2+4y^2-z^2=9$$

$$x^2-5y^2+3z^2=8$$

c) Using the complex variables solve and check the system of equations

$$x^2 - y^2 - 3x - 2y + 5 = 0$$

$$2xy + 2x - 3y - 1 = 0$$

Determine the geometric interpretation of the system of equations with its solution.

Question (2) [25 marks]

a) Use Fourier expansion for the function $f(x) = \begin{cases} -a, & -\pi < x < 0 \\ a, & 0 < x < \pi \end{cases}$ to show that

$$\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots = \frac{\pi}{4}, \quad 0 < x < \pi$$

b) Classify the equation $y[n+2] - 4y[n]=0$. Using z -transform method solve and check this equation with the initial conditions $y_0=1, y_1=2$.

c) Test the validity of the relations: $\widetilde{ab} = \widetilde{ba}$, $\widetilde{(ab)} = ba^T - ab^T$, $\widetilde{(a+b)} = \widetilde{a} + \widetilde{b}$

with the two vectors $a = [-2, 1, -3]^T$ and $b = [1, -2, 4]^T$.

Question (3) [25 marks]

a) Use Fourier transform to solve the wave equation in an infinite domain

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty, t > 0$$

$$u(x, t) \rightarrow 0 \quad \text{as} \quad x \rightarrow \begin{cases} \infty \\ -\infty \end{cases}$$

$$u(x, 0) = f(x) \quad \text{and} \quad \frac{\partial u(x, 0)}{\partial t} = 0 \quad \text{for} \quad -\infty < x < \infty$$

b) Construct the trigonometric interpolating polynomial of degree four in the interval $[-\pi, \pi]$ for the function $f(x) = \pi(x - \pi)$ using

i) Direct Calculations

ii) Fast Fourier transform

Question (4) [25 marks]

a) i) Define the tensor, rank of tensor and give an example.

ii) Compare between covariant tensor and contra-variant tensor.

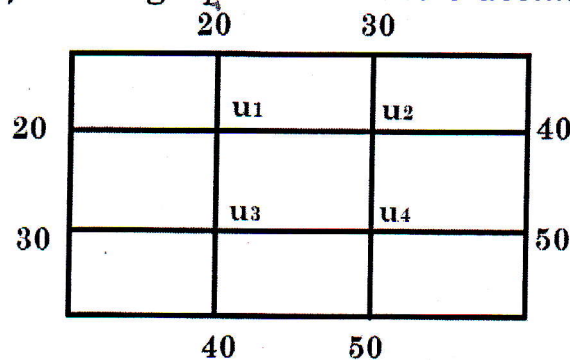
iii) If $u = (T^i)$ is a contra-variant 1st order tensor, show that the partial

derivatives $T_j^i = \frac{\partial T^i}{\partial x^j}$ defined in each coordinate system according to the

$$\text{rule} \quad \bar{T}_j^i = T_s^r \frac{\partial \bar{x}^i}{\partial x^r} \frac{\partial x^s}{\partial \bar{x}^j} + T_r \frac{\partial^2 \bar{x}^i}{\partial x^r \partial x^s} \frac{\partial x^s}{\partial \bar{x}^j}$$

c) Solve the elliptic partial differential equations equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

at pivotal points of the following square mesh to 3 decimal places.



Question Number	Q1-a	Q1-b	Q3-b	Q2-b	Q4-a	Q1-c	Q2-c	Q4-b	Q3-a	Q2-a	
Skills											
	Knowledge & understanding skills				Professional Skills						

With best wishes

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