Mansoura University Faculty of Engineering Eng. Math. and Phys. Dept.	*Final First Term Exam.* *2010-2011*	Mathematics (1). تخلفات Time allowed: 3 Hours.
من فضلك إبدأ حل الجبر من إحدى جهتى ورقة الإجابة و التفاضل من الجهة الأخرى		

Algebra

Question.1 [35 marks]

(a) Use De Moivre's theorem to solve the equation $z^3 + 8 = 0$.

(b) Use mathematical induction to prove that $5^n - 2^n$ is divisible by 3 for any positive integer n.

(c) Decompose the fraction $\dfrac{2x+3}{(x^2-1)(x^2+1)}$ into its partial fractions.

(d) Find the roots of the equation $x^4 - x^3 + 3x^2 - 9x - 54 = 0$.

Question.2 [35 marks]

(a) Prove that the product of orthogonal matrices is orthogonal.

(b) If A is $3*3$ matrix and $\det A = 5$, find:

(i) $\det(2A)$ (ii) $\det A^{-1}$

(c) If $A = \begin{bmatrix} -1 & 3 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 \\ 1 & 0 \\ -3 & -1 \end{bmatrix}$ evaluate:

(i) AB (ii) $2A^T$ (iii) A^{-1}.

(d) Using (c), solve the system $AX + 2M = N$, where $M = \begin{bmatrix} -2 \\ 3 \\ 3 \end{bmatrix}$ and $N = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$.

باقي الأسئلة بالخلف

اداری

Answer the following questions [Full mark 130 points]

1. (a) Solve for x

(i) $\ln(x+2)=\ln(x-7)+\ln 4$, (ii) $\pi=2\sin^{-1}(x+4)$ [4+4 points]

(iii) $\dfrac{x^2-4x+3}{x+2}\geq 0$ [4 points]

(b) If $f(x)=\dfrac{1}{x}$ and $g(x)=\sin x$, compute $(g\circ f)(\dfrac{2}{\pi})$. [4 points]

(c) Evaluate each of the following limits

(i) $\lim\limits_{x\to 0}(3\csc x-\cot x)$, [4 points]

(ii) $\lim\limits_{x\to\infty}\dfrac{\sinh x}{3+\tan^{-1}x}$ [4 points]

(iii) $\lim\limits_{x\to 0^+}(1+\sin 3x)^{\csc x}$ [4 points]

(d) Find the domain, range and discuss the symmetry of $y=\tanh x$. Sketch the graph of this function and then prove that $\tanh(\ln x)=\dfrac{x^2-1}{x^2+1}$. [7 points]

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2. (a) Find $\dfrac{dy}{dx}$ for each of the following functions

(i) $y=\sec^{-1}(\cosh 5x)+\operatorname{csch}^{-1}(3x)$ [6 points]

(ii) $(\cos x)^y+\operatorname{sech} y=\cos^{-1}x+\log_3(x^4+1)$ [8 points]

(iii) $y=a(1-\cos t)$, $x=a(t-\sin t)$ [4 points]

(b) If $y=\sinh(m\sinh^{-1}x)$, prove that

$$(1+x^2)y''+xy'-m^2y=0,$$ [5 points]

hence, or otherwise, deduce that

$$(1+x^2)y^{(n+2)}+(2n+1)xy^{(n+1)}+(n^2-m^2)y^{(n)}=0.$$ [5 points]

(c) Find the Taylor's expansion of order 4 of the function $f(x)=\sin x$ about the point $x=\pi/3$ and hence evaluate $\cos 61^\circ$ from this expansion. [7 points]