Post graduate Exam (Basic Engineering Sciences)
Branch: Engineering Mathematics (Master 600)

Menofia University Faculty of Engineering Academic Year: 2014-2015 Department: Basic Eng. Sci.	Subject: Mathematical Physics (1) Time Allowed: 3 hours Date: 8 / 6 / 2015 Max Marks: 100

Answer all the following questions:

Q.1 (A) The vibrations of an elastic string is governed by the partial differential equation

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$$

The length of the string is π and the ends are fixed. The initial velocity is zero and the initial deflection is $u(x,0)=2(\sin x+\sin 3x)$. Find the deflection $u(x,t)$ of the vibrating string for $t>0$.

(B) The ends A and B of a rod 20 cm long have the temperatures at 30°C and at 80°C until steady state prevails. The temperatures of the ends are changed to 40°C and 60°C respectively. Find the temperature distribution in the at time t.

(C) A periodic square wave function $f(t)$, in terms of unit step functions is written as : $f(t)=k\left[u_0(t)-2u_a(t)+2u_{2a}(t)-2u_{3a}(t)+....\right]$

Show that the Laplace transform of $f(t)$ is given by

$$L\left[f(t)\right]=\frac{k}{s}\tanh\left[\frac{as}{2}\right].$$

[Q.1 (50 mark)]

Q.2 (A) By using Fourier sin transforms, Solve the heat equation $\frac{\partial u}{\partial t}=k\,\frac{\partial^2 u}{\partial x^2}$ for $x\geq 0,\ t\geq 0$ under the given conditions $u=u_0$ at $x=0,\ t>0$ with initial condition $u(x,0)=0,\ x\geq 0$.

(B) Use the method of separation of variables to solve the equation :

$$\frac{\partial^2 v}{\partial x^2}=\frac{\partial v}{\partial t}$$

given that $v=0$ when $t\longrightarrow\infty$, as well as $v=0$ at $x=0$ and $x=l$.

(C) Find the solution of the following wave equation using D'Almbert's method:

$$\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$$

with initial condition $y(x,0)=f(x)$, $\frac{\partial y}{\partial t}=0$ when $t=0$.

Q.2 (50 mark)]

With my best wishes *Dr. M.A. El-Shorbagy*