MANSOURA JOURNAL OF Mathematics
Official Journal of Faculty of Science, Mansoura University, Egypt
E-mail: scimag@mans.edu.eg ISSN: 2974-4946

ThePancharatnam phase and the geometric phase behaviors for a system of a three-level atom in the presence of Kerr-like medium

A. Makled [a], A. H. Hamid [b] and M. Abdel-Aty [c]

[a] *Faculty of Science, Mansoura University, Mansoura, Egypt*

[b] *Faculty of Science, Al-Azhar University, Assiut, Egypt*

[c] *Zewail City For Science and Technology, 6 October City, Cairo, Egypt*

Received:19/8/2019
Accepted: 2/10/2019

Abstract: In this publication, for a model descibed by the quantum theory of electromagnetic field interaction with a three-level atom, we study the Pancharatnam phase and the geometric phase that arises in the state vector, with an extra Kerr-like medium for one mode. The results demonstrate that adding Kerr-like medium has a significant impact on the characteristics of the Pancharatnam and the geometric phases. In the Pancharatnam phase, it has been shown that the information on the field statistics and atomic coherence are expressly provided. Numerical results show The general conclusions attained.

keywords: Pancharatnam phase; geometric phase; Kerr-like medium

1.Introduction

In quantum physics, the geometric phase (berry phase) [1] and the Pancharatnam phase [2] have a significant effect within a variety of quantum phenomena. The geometric phase relies on the quantum theory's state space structure [3] and it is very essential for and it is very important in all physics branches. For example, the Jahn-Teller effect [4], the spin-orbit interaction [5], the quantum Hall effect [6] and in many topics of quantum optics [[7] - [11]]. Also, decoherent effect can be included in quantum computation by applying geometric phases [[12] - [25]]. Studies show the importance of the Pancharatnam phase in the spread of a light beam in which its state polarization changes regularly [26]. The total phase (or the Pancharatnam phase) includes both the dynamical and the geometric components and it obtained through a qauntum system's wave function.

The dynamical phase was previously defined for the state as system was Hamiltonian dependent. Also, the geometric phase was dependant on the path selected in the space covering all probable quantum system condtions. The Pancharatnam phase for a two-level atom one mode model was studied [27]. Furthermore, Some researchers in the recent years has studied The geometric phase for a three-level atom crosses a double cavity [28].

The model consists in the current work of a single three-level atom with a single field of the mode surrounded in an ideal cavity by a nonlinear kerr-like medium. The Kerr-like medium can be specifically developed as anharmonic oscillator with frequency ω. This paper will be sorted out as follows. In Section 2, we present at any time $t > 0$ the model and the expressions for the final state vector. In Section 3, when Kerr-like medium impacts are included we display the Pancharatnam and the geometric phases calculations. Finally in section 4, we present our conclusion.

2 Basic equation

We consider the Hamiltonian for a single three-levl atom with a single mode in an ideal cavity ($Q = \infty$) containing a Kerr-like medium in this cavity. The effective Hamiltonian of the system in the rotating-wave approximation (RWA)can be written as:

$$H_{eff} = \chi \hat{a}^{\dagger 2}\hat{a}^2 + \lambda_1\left(e^{-i\Delta_1 t}\hat{a}\sigma_+^{12} + e^{i\Delta_1 t}\hat{a}^\dagger\sigma_-^{12}\right) + \lambda_2\left(e^{-i\Delta_2 t}\hat{a}\sigma_+^{13} + e^{i\Delta_2 t}\hat{a}^\dagger\sigma_-^{13}\right), \quad (1)$$

where $\Delta_1 = \omega - (\omega_1 - \omega_2)$, $\Delta_2 = \omega - (\omega_1 - \omega_3)$ and χ denotes the coupling to the third-order nonlinearity Kerr-like medium. We have an atom whose levels have energies ω_1, ω_2 and ω_3. The operators σ_-^{ij}, σ_+^{ij} $(i, j = 1,2,3)$ are the lowering and raising operators between the two levels i and j. Also, λ_1 and λ_2 are the coupling constants between the field and the atom. The initial state of the total atom-field system is

$|\Psi(0)\rangle = |\Psi\_A\ (0)\rangle \otimes \top |\Psi\_F\ (0)\rangle$, (2)

where $|\Psi_A(0)\rangle$ is the initial state of atom and $|\Psi_F(0)\rangle$ is is the initial state of field. Anywise, we take $|\Psi(0)\rangle$ in the form

$|\Psi(0)\rangle = \sum▒(\gamma\_1\ |n, 1\rangle + \gamma\_2\ |n + 1,2\rangle + \gamma\_3\ |n + 2,3\rangle)$, (3)

Where

$\gamma\_i = q\_n\ \kappa\_(i + \eta\_i\ )$ $(\eta\_1 = 0, \eta\_2 = \eta\_3 = 1)$, $q\_n = \langle n|\Psi\_F\ (0)\rangle$ is the number-state expansion coefficients and κ_i is the amplitude of finding the atom in state i and $\kappa\_i = \langle n|\Psi\_A\ (0)\rangle$. At any time $t > 0$, the wave function becomes

$|\Psi(t)\rangle = \sum\_n▒(A\_n\ (t)|n, 1\rangle + B\_n\ (t)|n + 1,2\rangle + C\_n\ (t)|n + 2,3\rangle)$. (4)

For simplicity, we set $\Delta_1 = \Delta_2 = \Delta$, and hence using the following schrodinger equation when $\hbar = 1$.

$i\ (d|\Psi(t)\rangle)/dt = H\_eff\ |\Psi(t)\rangle$. (5)

Now, we solve Equation.(5), so the amplitudes of Eq. (4) are given by:

$$A_n(t) = (\gamma_1 \cos(\nu_n t) - i a^1 \sin(\nu_n t))e^{-i\Omega_n t},$$

$$B_n(t) = b^1 e^{-i\alpha_n t} + (b^{11}\cos(\nu_{n+2} t) + i\lambda_1 b^{111}\sin(\nu_{n+2} t))e^{-i\beta_n t},$$

$$C_n(t) = c^1 e^{-i\alpha_{n+1} t} + (c^{11}\cos(\nu_{n+3} t) + i\lambda_2 c^{111}\sin(\nu_{n+3} t))e^{-i\beta_{n+1} t}.$$

The abbreviations in that variables are given by the following :

$$\alpha\_n = \chi n(n + 1),$$

$$\beta\_n = \chi(n + 1)^2 + \Delta/2,$$

$$\alpha\_(n + 1) = \chi(n + 1)(n + 2),$$

$$\beta\_(n + 1) = \chi(n + 2)^2 + \Delta/2,$$

$$\theta\_n = \chi\ (n - 1) + \Delta/2,$$

$\theta\_(n + 2) = \chi(n + 1) + \Delta/2,$

$\theta\_(n + 3) = \chi(n + 2) + \Delta/2,$

$\nu\_n = \sqrt{}(\theta\_n^2 + n(\lambda\_1^2 + \lambda\_2^2)),$

$\nu_{n+2} = \sqrt{\theta_{n+2}^2 + (n + 2)(\lambda_1^2 + \lambda_2^2)},$

$\nu_{n+3} = \sqrt{\theta_{n+3}^2 + (n + 3)(\lambda_1^2 + \lambda_2^2)},$

$\Omega\_n = (\chi(n - 1)^2 - \Delta/2),$

$a^1 = (\gamma\_1\ \theta\_n + \epsilon\sqrt{}n)/\nu\_n$,

$a^11 = (\gamma\_1\ \theta\_(n + 2) + \epsilon\sqrt{}(n + 2))/\nu\_(n + 2)$,

$a^111 = (\gamma\_1\ \theta\_n + \epsilon\sqrt{}(n + 3))/\nu\_(n + 3)$,

$\epsilon = \lambda_1\gamma_2 + \lambda_2\gamma_3,$

$b^1 = (\lambda\_2^2\ \gamma\_2 - \lambda\_1\ \lambda\_2\ \gamma\_3)/(\lambda\_1^2 + \lambda\_2^2\ ),$

$b^11 = (\epsilon\lambda\_1)/(\lambda\_1^2 + \lambda\_2^2\ ),$

$b^111 = (\epsilon\theta\_(n + 2))/(\lambda\_1^2 + \lambda\_2^2\ ) - (\gamma\_1\ \sqrt{}(n + 2)),$

$c^1 = (\lambda\_1^2\ \gamma\_3 - \lambda\_1\ \lambda\_2\ \gamma\_2)/(\lambda\_1^2 + \lambda\_2^2\ ),$

$c^11 = (\epsilon\lambda\_2)/(\lambda\_1^2 + \lambda\_2^2\ )$

$c^111 = (\epsilon\theta\_(n + 3))/(\lambda\_1^2 + \lambda\_2^2\ ) - (\gamma\_1\ \sqrt{}(n + 3)).$

3 Pancharatnam and geometric phases

The Pancharatnam phase (or the total phase ϕ_t) between the two state vectors $|\Psi(0)\rangle$, $|\Psi(t)\rangle$ is given by [2]:

$\phi_t = \arg\langle\Psi(0)|\Psi(t)\rangle = Tan^{-1}(Y(t)/X(t))$, (6)

where

$X(t) = \sum_n\ (\gamma_1(\gamma_1 \cos\Omega_n t \cos\nu_n t - a^1 \sin\Omega_n t \sin\nu_n t)$

$+\gamma\_2\ (b^1\ \cos\alpha\_n\ t + b^11\ \cos\beta\_n\ t\cos\nu\_(n + 2)\ t) + \gamma\_3\ (c^1\ \cos\alpha\_(n + 1)\ t$

$+c^{11}\cos\beta_{n+1} t \cos\nu_{n+3} t) + \lambda_1\gamma_2 b^{111}\sin\beta_n t \sin\nu_{n+2} t$

$+\lambda\_2\ \gamma\_3\ c^111\ \sin\beta\_(n + 1)\ t\sin\nu\_(n + 3)\ t),$

$Y(t) = -\sum\_n▒(\gamma\_1\ (\gamma\_1\ \sin\Omega\_n\ t\cos\nu\_n\ t + a^1\ \cos\Omega\_n\ t\sin\nu\_n\ t)$

$+\gamma\_2\ (b^1\ \sin\alpha\_n\ t + b^11\ \sin y\_n\ t\cos\nu\_(n + 2)\ t) + \gamma\_3\ (c^1\ \sin\alpha\_(n + 1)\ t$

$+c^11\ \sin\beta\_(n + 1)\ t\cos\nu\_(n + 3)\ t) - \lambda\_1\ \gamma\_2\ b^111\ \cos\beta\_n\ t\sin\nu\_(n + 2)\ t$

$-\lambda\_2\ \gamma\_3\ c^111\ \cos\beta\_(n + 1)\ t\sin\nu\_(n + 3)\ t).$

In addition, the dynamical phase ϕ_d is achieved with the time integral of the expectation value of the Hamiltonian during the time $t = 0$ to t for any quantum evolution from time $t = 0$ to t,

$$\phi_d = -\int_0^t \langle\Psi(t)|H_{eff}|\Psi(t)\rangle dt, \quad \hbar = 1. \qquad (7)$$

After compensation for $|\Psi(t)\rangle$, we get

$$\begin{aligned}\phi_d = \sum \Big(&\zeta_1 t + \frac{\zeta_2}{4\nu_n}\sin 2\nu_n t + \frac{\zeta_3}{4\nu_{n+2}}\sin 2\nu_{n+2} t \\ &+ \frac{\zeta_4}{\Gamma_{n+2}}\sin\Gamma_{n+2} t - \frac{\zeta_5}{\Lambda_{n+2}}\sin\Lambda_{n+2} t \\ &+ \frac{\zeta_6}{4\nu_{n+3}}\sin 2\nu_{n+3} t + \frac{\zeta_7}{\Gamma_{n+3}}\sin\Gamma_{n+3} t \\ &- \frac{\zeta_8}{\Lambda_{n+3}}\sin\Lambda_{n+3} t + i\Big(\xi_1 + \xi_2\Big(\frac{a^1+\gamma_1}{\Gamma_n}\exp^{i\Gamma_n t} \\ &- \frac{a^1-\gamma_1}{\Lambda_n}\exp^{i\Lambda_n t}\Big) - \xi_3\Big(\frac{a^{11}+\gamma_1}{\Gamma_{n+2}}\exp^{-i\Gamma_{n+2} t} \\ &- \frac{a^{11}-\gamma_1}{\Lambda_{n+2}}\exp^{-i\Lambda_{n+2} t}\Big) \\ &- \xi_4\Big(\frac{a^{111}+\gamma_1}{\Gamma_{n+3}}\exp^{-i\Gamma_{n+3} t} - \frac{a^{111}-\gamma_1}{\Lambda_{n+3}}\exp^{-i\Lambda_{n+3} t}\Big) \\ &+ \frac{\xi_5}{4\nu_n}(\cos 2\nu_n t - 1) - \frac{\xi_6}{4\nu_{n+2}}(\cos 2\nu_{n+2} t - 1) \\ &- \frac{\xi_7}{4\nu_{n+3}}(\cos 2\nu_{n+3} t - 1)\Big)\Big),\end{aligned}$$

where

$$\zeta_1 = \zeta_{11} + \zeta_{12} + \zeta_{13} + \zeta_{14} + \zeta_{15} + \frac{1}{2}\mu,$$

$$\zeta_{11} = -\frac{1}{2}\chi n(n-1)({a^1}^2 + \gamma_1^2),$$

$$\zeta_{12} = -\frac{1}{2}\alpha_n(2{b^1}^2 + {b^{11}}^2 + \lambda_1^2 {b^{111}}^2),$$

$$\zeta_{13} = -\frac{1}{2}\alpha_{n+1}(2{c^1}^2 + {c^{11}}^2 + \lambda_2^2 {c^{111}}^2),$$

$$\zeta_{14} = -\frac{1}{2}(\lambda_1\gamma_1 b^{11}(\sqrt{n} + \sqrt{n+2}) + \lambda_2\gamma_1 c^{11}(\sqrt{n} + \sqrt{n+3})),$$

$$\zeta_{15} = \frac{1}{2}(\lambda_1^2 a^{11} b^{11}\sqrt{n+2} + \lambda_2^2 a^{111} c^{111}\sqrt{n+3}),$$

$$\mu = a^1\sqrt{n}(\epsilon\theta_n - \gamma_1\sqrt{n}(\lambda_1^2 + \lambda_2^2)),$$

$$\zeta_2 = \chi n(n-1)({a^1}^2 - \gamma_1^2) - \sqrt{n}\gamma_1(\lambda_1 b^{11} - \lambda_2 c^{11}) - \mu,$$

$$\zeta_3 = \alpha_n(\lambda_1^2 {b^{111}}^2 - {b^{11}}^2) + \sqrt{n+2}(\lambda_1^2 a^{11} b^{111} - \lambda_1\gamma_1 b^{11}),$$

$$\zeta_4 = \alpha_n b^1(\lambda_1 b^{111} - b^{11}),$$

$$\zeta_5 = \alpha_n b^1(\lambda_1 b^{111} + b^{11}),$$

$$\zeta_6 = \alpha_{n+1}(\lambda_2^2 {c^{111}}^2 - {c^{11}}^2) + \sqrt{n+3}(\lambda_2^2 a^{111} c^{111} - \lambda_2\gamma_1 c^{11}),$$

$$\zeta_7 = \alpha_{n+1} c^1(\lambda_2 c^{111} - c^{11}),$$

$$\zeta_8 = \alpha_{n+1} c^1(\lambda_2 c^{111} + c^{11}),$$

$$\begin{aligned}\xi_1 = &\frac{\sqrt{n}(\lambda_1 b^1 + \lambda_2 c^1)}{\Gamma_n\Lambda_n}(a^1\nu_n - \gamma_1\theta_n) - \frac{\lambda_1 b^1\sqrt{n+2}}{\Gamma_{n+2}\Lambda_{n+2}} \\ &\times (a^{11}\nu_{n+2} - \gamma_1\theta_{n+2}) - \frac{\lambda_2 c^1\sqrt{n+3}}{\Gamma_{n+3}\Lambda_{n+3}}(a^{111}\nu_{n+3} - \gamma_1\theta_{n+3}),\end{aligned}$$

$$\xi_2 = \frac{1}{2}\sqrt{n}(\lambda_1 b^1 + \lambda_2 c^1),$$

$$\xi_3 = \frac{1}{2}\lambda_1 b^1\sqrt{n+2},$$

$$\xi_4 = \frac{1}{2}\lambda_2 c^1\sqrt{n+3},$$

$$\xi_5 = \frac{\gamma_1\mu}{a^1} + \sqrt{n}\lambda_1 a^1 b^{11},$$

$$\xi_6 = \sqrt{n+2}(\lambda_1^2\gamma_1 b^{111} + \lambda_1 a^{11} b^{11}),$$

$$\xi_7 = \sqrt{n+3}(\lambda_2^2\gamma_1 c^{111} + \lambda_2 a^{111} c^{11}),$$

$$\Lambda_n = \theta_n - \nu_n,$$

$$\Lambda_{n+2} = \theta_{n+2} - \nu_{n+2},$$

$$\Lambda_{n+3} = \theta_{n+3} - \nu_{n+3},$$

$$\Gamma_n = \theta_n + \nu_n,$$

$$\Gamma_{n+2} = \theta_{n+2} + \nu_{n+2},$$

$$\Gamma_{n+3} = \theta_{n+3} + \nu_{n+3}.$$

The geometric phase under Schrodinger evolution is obtained by subtracting the dynamical phase from the Pancharatnam phase as follows [29].

$$\phi_g = \phi_t - \phi_d \qquad (8)$$

$$\phi_g = i\int_0^\tau \langle\psi(t)|d|\psi(t)\rangle dt \qquad (9)$$

$$\phi_g = 2\pi/n\langle\psi(t)|\hat{a}^\dagger\hat{a}|\psi(t)\rangle \qquad (10)$$

With $\tau = 2\pi/n$ being the period, which is proportional to the expectation value of the photon number operator, and inversely proportional to the transition photon-number n. After restitution for $|\Psi(t)\rangle$, we get

$$\phi_g = 2\pi \sum_n [\{\gamma_1^2(\cos\nu_n t)^2 + {a^1}^2(\sin\nu_n t)^2\}$$
$$+\{(n+1)/n\}\{{b^1}^2 + 2b^1 b^{11}\cos\nu_{n+2}t\cos\theta_{n+2}t$$
$$+2\lambda_1 b^1 b^{111}\sin\nu_{n+2}t\sin\theta_{n+2}t + {b^{11}}^2(\cos\nu_{n+2}t)^2$$
$$+\lambda_1^2 {b^{111}}^2(\sin\nu_{n+2}t)^2\} + \{(n+2)/n\}\{{c^1}^2$$
$$+2c^1c^{11}\cos\nu_{n+3}t\cos\theta_{n+3}t + 2\lambda_2 c^1 c^{111}$$
$$\times \sin\nu_{n+3}t\sin\theta_{n+3}t + {c^{11}}^2(\cos\nu_{n+3}t)^2$$
$$+\lambda_2^2 {c^{111}}^2(\sin\nu_{n+3}t)^2\}].$$

4 Conclusion and results

The pancharatnam phase and the geometric phase for a three-level atom in the presenence of Kerr-Like medium was studied in the previous section. For our plots we have assumed that $\lambda_1 = \lambda_2 = \lambda$ and $\Delta_1 = \Delta_2 = \Delta$. The time t has been scaled; one unit is obtained by the inverse of the coupling Parameter λ. We introduce the Pancharatnam phase and the geometric phase for the initial coherent field with different values of Kerr-Like medium parameter $\frac{\chi}{\lambda}$. In our calculations, we have taken $\frac{\Delta}{\lambda} \in [0,0.01]$

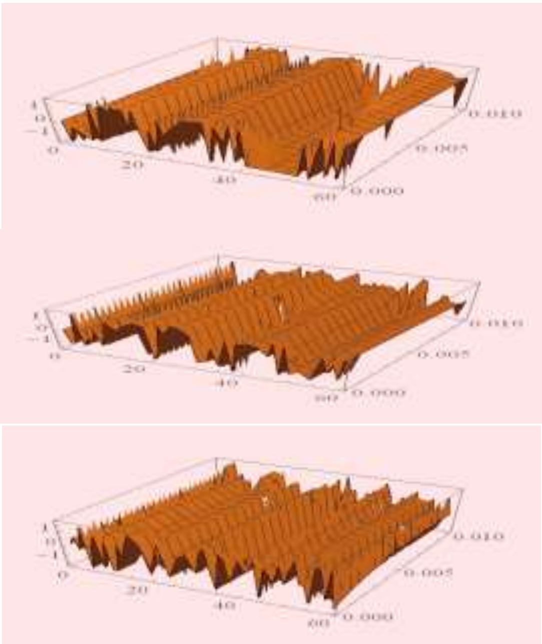

Figure 1: Plots of The pancharatnam phase as a function of the scaled time λt of the initial coherent field equal to $\bar{n} = 25$ and $\frac{\chi}{\lambda} = 0,0.05,0.5$

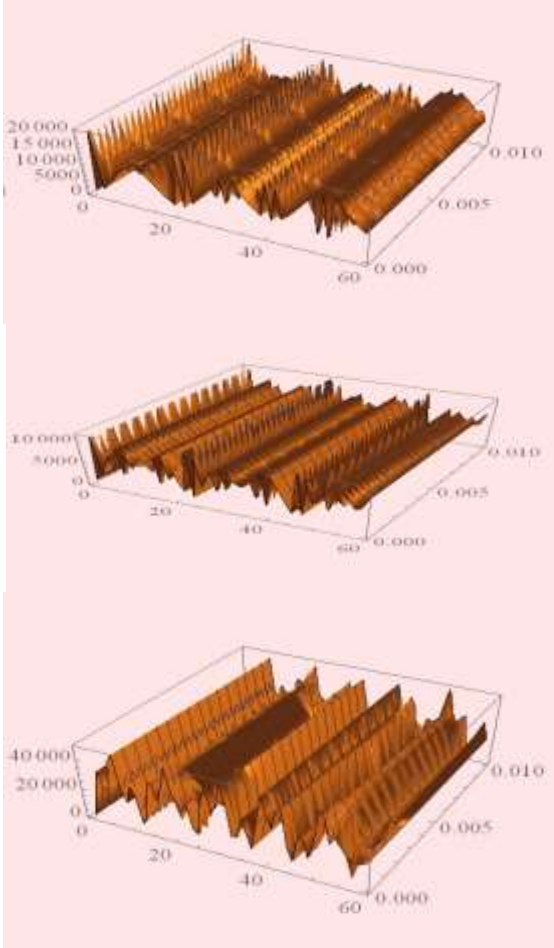

Figure 2: Plots of The Geometric phase as a function of the scaled time λt of the initial coherent field equal to $\bar{n} = 25$ and $\frac{\chi}{\lambda} = 0,0.05,0.5$

4. References

1 M.V. Berry, Proc. R. Soc(1984). A 392, 45.

2 Pancharatnam, (1956) Proc. Ind. Acad. Sci. A 44, 217.

3 H. Lyre, Studies in History and Philosophy of Modern Physics **48**, (2014) 45.

4 ES. Ham, (1987) Phys. Rev. Lett. 58, 725.

5 H. Mnthur, (1991) Phys. Rev. Lett. 67, 3325.

6 D. Thouless et al., (1983) Phys. Rev. Lett. **49**, 405.

7 C.C. Gerry, (1989) Phys. Rev. A **39**, 3204.

8 S. Chaturvedi, M.S. Sriram, V. Srinivasan, (1987) *J. Phys*. A **20**, L1071 .

9 D. Ellinas, S.M. Barnett, M.A. Reupertuis, (1989) Phys. Rev. A 39, 3228.

10 A. Joshi, A.K. Pati, A. Banerjee, (1994) Phys. Rev. **A 49**, 5131.

11 T. Sen, J.L. Milovich, (1992) Phys. Rev. A 45, 45.
12 G. Dattoli, R. Mignani, A. Torre, (1990) *J. Phys*. **A 23**, 5795.
13 Y.Y. Jiang, et al., (2010) Phys. Rev. A 82, 062108.
14 E. et al., , (2000) Phys. Rev. Lett. 852845.
15 A. Carollo, I. Fuentes-Guridi, M.F. Santos, V. Vedral, (2003) Phys. Rev. Lett. 90, 160402.
16 K.M. Fonseca Romero, A.C. Aguiar, M.T.Thomaz, (2002) Physica A 307, 142.
17 Z.S. Wang, L.C. Kwek, L.C. Lai, C.H. Oh, (2006) Europhys. Lett. 74, 958.
18 Z.S. Wang, et al., (2007) Phys. Rev. A 75, 024102.
19 Z.S. Wang, (2009) *Int. J. Theor. Phys*. 48, 2353.
20 A. Nazir, T.P. Spiller, W.J. Munro, (2002)Phys. Rev. A 65, 042303.
21 R.S. Whitney, Y. Gefen, (2003) Phys. Rev. Lett. 90, 190402.
22 G. De Chiara, G. MPalma, (2003) Phys. Rev. Lett. 91, 090404.
23 Z.S. Wang, (2012) *Int. J. Theor. Phys*. **51**, 3647.
24 H.-Y. Rao, et al., (2012) Acta Phys. Sin. **61,** 020302.
25 H.-l. Xu, et al., (2012) *J. Magn. Reson*. 223, 25.
26 R. Bhandari, J. Samuel, (1988) Phys. Rev. Lett. **60**, 1210.
27 H.Y. Rao, H.L. Xu, G.L. Fu, Y.B. Liu, B. Lv, Y.Y. Jiang, Y.X. Yu, (2014) *Int. J. Theor. Phys.* **53**, 1033.
28 Q.V. Lawande, S.V. Lawande, A. Joshi, (1999) Phys. Lett. A 251, 164.
29 Liu Ni,Wang Yue-Ming, Liang Jiu-Qing, Commun. (2012) Theor. Phys. **58**, 271.