TRANSIENT STABILITY ANALYSIS OF MULTIMACHINE POWER SYSTEM CONSIDERING GENERATOR FLUX DECAY

H. Shaaban
Faculty of Engineering, Shebin El-Kom, Egypt

Abstract:

In the paper, transient stability analysis of an N-machine power system is carried out using the decomposition-aggregation via vector Lyapunov function method. It is considered in the analysis, transfer conductances, non-uniform mechanical damping, and generators flux decay effect. Each of the system generators is represented by a more sophisticated model, that is, the one-axis model in which the generator internal voltage component E'_q is assumed to be changed with time. Note that, using the stability direct methods the voltage E'_q is usually assumed, for simplicity, constant. The mathematical model of the whole system is derived and is decomposed into [(N–1) / 3] eleventh-order interconnected subsystems, each of them includes three machines in addition to the reference machine. The system aggregation is carried out using a constructed vector Lyapunov function whose elements are scalar Lyapunov functions, each in the form of " quadratic form + sum of the integrals of six nonlinear functions". It is obtained a square aggregation matrix of the order [(N–1) / 3], and stability of this matrix implies asymptotic stability of the system equilibrium.

In a numerical example, the developed stability approach is used to carry out transient stability studies of a 10-machine,11-bus power system. The stability computations are carried out assuming occurrence of a 3 - phase short circuit fault near a bus, and also for connection of a pulsating load to one of the system buses. In addition it is assumed two composite faults defined as, disconnection of two tie-lines (due to false operation of circuit breakers near fault location), or addition of a pulsating load, just after clearing a 3-phase short circuit fault (the faulted line is switched off) at two different locations. It is found that the developed stability approach is suitable and can be easily used for practical, and on-line stability studies of large- scale power systems (number of machines may be more than 10)

1 . Introduction

The numerical integration methods used for power system stability analysis, although very effective in handling different models, are very expensive in terms of computation requirement. For this reason the research for a direct method has continued.

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The scalar Lyapunov function method appeared one of the most powerful methods for stability studies of power systems [1]. However, this method did not seem suitable, owing to the continuous increase in size and complexity of power systems, and in particular when the problem of the stability domain estimate of the system is attacked [2]. Attempts to overcome the drawbacks of the scalar Lyapunov approach have led to the decomposition-aggregation via vector Lyapunov function method. The expected advantages of the decomposition-aggregation method are, however, manifold [3]. On the one hand, the Lyapunov function of a disconnected (free) low-order subsystem can handle more sophisticated generator and transmission models. Further, an analytic expression of transient stability index may be derived, which can be a good basis for further investigations such as sensitivity analysis.

In the last two decades, the decomposition–aggregation method has been used for stability analysis of large-scale power systems [4–13]. It is to be noted that, the power system stability analysis was carried out in the papers [4–12] considering the generator classical model (the internal voltage E' is assumed constant). However, this is equivalent to neglecting the effect of generators flux decays.

In the papers[14–17], the transient stability analysis of multimachine power systems was carried out considering the flux decay effect. However, the authors introduced different forms for the used scalar Lyapunov functions, which were constructed under the assumption that transfer conductances G_{ij}, are all negligible.

In the work [13], each generator was represented by the two-axis model and transfer conductances were considered. The system decomposition was carried out using the "two- machine" decomposition. The develo-ped approach was applied to a 3-machine, 4-bus power system.

Now, in the present paper an N-machine power system is considered, and the flux decay effect is taken into consideration (the generator voltage component E'_q is assumed to be changed with time). The system loads are represented by constant impedances to ground, and then the system network is simplified by eliminating all the nodes, except generators internal nodes. The system mathematical model (non-uniform mechanical damping case is assumed and the transfer conductances are included) is obtained, and is decomposed so that each free subsystem contains six (the largest number) nonlinearities. Finally, asymptotic stability of the system equilibrium is implied by stability of an obtained (square) aggregation matrix of the order [(N − 1) / 3].

2. Power system model

Consider an N-machine power system (the generator stator resistances are neglected) with mechanical damping. Representing each machine by the one-axis model [18], in which the voltage component E'_q is assumed to be changed with the time, the absolute motion of the i-th machine is described by the following equations (see Notation)

$$M_i \ddot{\delta}_i + D_i \dot{\delta}_i = P_{mi} - P_{ei}$$
$$T'_{doi} E'_{qi} = E_{fdi} - E'_{qi} + (X_{di} - X'_{di}) I_{di} \qquad (1)$$

where, M_i and P_{mi} are assumed constant, and P_{ei} is given in the form,

$$P_{ei} = E'_{di} I_{di} + E'_{qi} I_{qi} - (X'_{qi} - X'_{di}) I_{di} I_{qi}, \quad i = 1,2,\ldots,N \qquad (2)$$

It is to be noted that, the voltage E_{fdi}, is equal to its pre-transient value E^o_{fdi}, since the effect of the automatic voltage regulator (AVR) has been neglected in the paper.

Under the assumption $X'_{di} = X'_{qi}$, (generators with solid cylindrical rotors are considered) we get[18].

$$P_{ei} = \sum_{j=1}^{N} Y_{ij} \{E'_{qi} [E'_{qj} \cos(\theta_{ij} - \delta_{ij}) - E'_{dj} \sin(\theta_{ij} - \delta_{ij})] + E'_{di} [E'_{dj} \cos(\theta_{ij} - \delta_{ij}) + E'_{qj} \sin(\theta_{ij} - \delta_{ij})] \} \quad , i = 1,2,\dots,N \quad (3)$$

Now, selecting the Nth machine as a comparison machine, and introducing the following (3N−1) state variables

$$\sigma_{iN} = \delta_{iN} - \delta^{o}_{iN} \qquad , i \neq N$$

$$\omega_i = \dot{\delta}_i \quad ; \qquad E_{Qi} = E'_{qi} - \hat{E}_{qi} \qquad , i = 1,2,\dots,N \quad (4)$$

the overall system motion is governed by the state equations,

$$\dot{\sigma}_{iN} = \omega_i - \omega_N = \omega_{iN}$$

$$\dot{\omega}_i = -\lambda_i \omega_i - (1/M_i) \Big[G_{ii} (E^2_{Qi} + 2 E_{Qi} \hat{E}_{qi}) + \sum_{j \neq i}^{N} Y_{ij} \{ A_{ij} f_{ij}(\sigma_{ij}) + \hat{A}_{ij}\, g_{ij}(\sigma_{ij}) + [\hat{E}_{qi} E_{Qj} + E_{Qi} (E_{Qj} + \hat{E}_{qj})] \cos(\theta_{ij} - \delta_{ij}) + [\hat{E}_{di} E_{Qj} - \hat{E}_{dj} E_{Qi}] \sin(\theta_{ij} - \delta_{ij}) \} \Big]$$

$$\dot{E}_{Qi} = -\Gamma_i E_{Qi} + K_i \sum_{j \neq i}^{N} Y_{ij} [\hat{E}_{dj} f_{ij}(\sigma_{ij}) - \hat{E}_{qj}\, g_{ij}(\sigma_{ij}) + E_{Qj} \sin(\theta_{ij} - \delta_{ij})] \qquad , i = 1,2,\dots\dots, N \qquad (5)$$

where

$$f_{ij}(\sigma_{ij}) = \cos(\sigma_{ij} + \delta^{o}_{ij} - \theta_{ij}) - \cos(\delta^{o}_{ij} - \theta_{ij})$$

$$g_{ij}(\sigma_{ij}) = \sin(\sigma_{ij} + \delta^{o}_{ij} - \theta_{ij}) - \sin(\delta^{o}_{ij} - \theta_{ij}) \qquad (6)$$

3. *Power system decomposition*

The considered N-machine system is decomposed, in the paper, as follows:

1– All the system loads are represented by constant impedances to ground (those impedances are obtained from the pre-transient conditions in the system).

2– Eliminating all the system nodes, except the generators internal nodes, it is obtained the system Nth-order reduced admittance matrix Y.

3– Referring to the obtained Y-matrix, the system is decomposed into [(N −1) / 3] interconnected subsystems, each consisting of four machines one of them is the comparison machine [11] .

Now, defining the state vector X_I in the form

$$X_I = [\sigma_{iI,N}, \sigma_{iI+1,N}, \sigma_{iI+2,N}, \omega_{iI}, \omega_{iI+1}, \omega_{iI+2}, \omega_N, E_{QiI}, E_{QiI+1}, E_{QiI+2}, E_{QN}]^T = [X_{I1}, X_{I2}, X_{I3}, \dots\dots, X_{I11}]^T \qquad (7)$$

we can decompose the mathematical model of the whole system (eqn. 5) into S =[(N −1)/3], eleventh-order interconnected subsystems, each can be written in the general form

$$\dot{X}_I = P_I X_I + B_I F_I(\sigma_I) + h_I(X) \quad , \sigma_I = C^T_I X_I \qquad , I = 1,2,\dots, S \quad (8)$$

where P_I, B_I and C_I are constant matrices with appropriate dimensions, and $F_I(\sigma_I)$ is a nonlinear vector function, whose elements are arbitrary chosen. It is to be noted that each subsystem of Eq. (8), can be decomposed into the free subsystem

$$\dot{X}_I = P_I X_I + B_I F_I(\sigma_I) \quad , \quad \sigma_I = C^T_I X_I \qquad , I = 1,2,\ldots\ldots, S \quad (9)$$

and the interconnectons $h_I(X)$.

Referring to Eqs. 5 and 7, the matrix P_I is derived in the form

$$P_I = \left[\begin{array}{c|cc|c} & I_3 & -\mathbf{b}_I & \mathbf{O}_{3\times4} \\ \hline \mathbf{O}_{11\times3} & -\mathbf{P}_{I1} & & -\mathbf{P}_{I2} \\ \hline & \mathbf{O}_{4\times4} & & -\mathbf{P}_{I3} \end{array}\right] \qquad (10)$$

where, $\mathbf{O}$ and $\mathbf{I}$ are zero and identity (square) matrices, respectively, of the indicated dimensions, and where

$$\mathbf{b}_I = [1.0 \ , 1.0 \ , 1.0]^T \quad ; \quad \mathbf{P}_{I1} = \mathrm{diag}\,[\lambda_{iI} \ , \lambda_{iI+1} \ , \lambda_{iI+2} \ , \lambda_N]$$
$$\mathbf{P}_{I2} = \mathrm{diag}\,[\mu_{iI} \ , \mu_{iI+1} \ , \mu_{iI+2} \ , \mu_N]$$
$$\mathbf{P}_{I3} = \mathrm{diag}\,[\Gamma_{iI} \ , \Gamma_{iI+1} \ , \Gamma_{iI+2} \ , \Gamma_N] \qquad (11)$$

Now, after expanding the free subsystem twenty-four functions, it is found that there are at most six nonlinearities which satisfy the Lurie's sector condition, and these functions are given as,

$$f_{I1}(\sigma_{I1}) = \sin(\sigma_{iI,N} + \delta^o_{iI,N}) - \sin\delta^o_{iI,N}$$
$$f_{I2}(\sigma_{I2}) = \sin(\sigma_{iI+1,N} + \delta^o_{iI+1,N}) - \sin\delta^o_{iI+1,N}$$
$$f_{I3}(\sigma_{I3}) = \sin(\sigma_{iI+2,N} + \delta^o_{iI+2,N}) - \sin\delta^o_{iI+2,N}$$
$$f_{I4}(\sigma_{I4}) = \sin(\sigma_{iI,iI+1} + \delta^o_{iI,iI+1}) - \sin\delta^o_{iI,iI+1}$$
$$f_{I5}(\sigma_{I5}) = \sin(\sigma_{iI,iI+2} + \delta^o_{iI,iI+2}) - \sin\delta^o_{iI,iI+2}$$
$$f_{I6}(\sigma_{I6}) = \sin(\sigma_{iI+1,iI+2} + \delta^o_{iI+1,iI+2}) - \sin\delta^o_{iI+1,iI+2} \qquad (12)$$

Note carefully that the six functions given by Eq.(12), satisfy the following conditions

$$f_{Ik}(0) = 0 \quad ; \quad 0 \le \sigma_{Ik}\, f_{Ik}(\sigma_{Ik}) \le \xi_{Ik}\, \sigma^2_{Ik} \quad , k = 1,2,\ldots\ldots, 6 \qquad (13)$$

on bounded intervals, where the positive constants ξ_{Ik} may be determined as

$$\xi_{Ik} = \left| \partial f_{Ik}(\sigma_{Ik}) / \partial \sigma_{Ik} \right|_{\sigma_{Ik}=0} \qquad , k = 1,2,\ldots., 6 \qquad (14)$$

Now, assuming the six nonlinear functions of Eq.(12) to be the elements of F_I we define the following matrices,

$$F_I(\sigma_I) = [f_{I1}(\sigma_{I1}) , f_{I2}(\sigma_{I2}) , \ldots\ldots\ldots , f_{I6}(\sigma_{I6})]^T \qquad (15)$$

$$C^T_I = \left[\begin{array}{ccc|c} & I_3 & & \\ \hline 1 & -1 & 0 & O_{6\times8} \\ 1 & 0 & -1 & \\ 0 & 1 & -1 & \end{array}\right] \qquad (16)$$

$$B_I = \left[\begin{array}{cccccc} \multicolumn{6}{c}{O'_{3\times 6}} \\ \hline -d_{iI,N} & 0 & 0 & -d_{iI,iI+1} & -d_{iI,iI+2} & 0 \\ 0 & -d_{iI+1,N} & 0 & d_{iI+1,iI} & 0 & -d_{iI+1,iI+2} \\ 0 & 0 & -d_{iI+2,N} & 0 & d_{iI+2,iI} & d_{iI+2,iI+1} \\ d_{iI} & d_{iI+1} & d_{iI+2} & 0 & 0 & 0 \\ q_{iI,N} & 0 & 0 & q_{iI,iI+1} & q_{iI,iI+2} & 0 \\ 0 & q_{iI+1,N} & 0 & -q_{iI+1,iI} & 0 & q_{iI+1,iI+2} \\ 0 & 0 & q_{iI+2,N} & 0 & -q_{iI+2,iI} & -q_{iI+2,iI+1} \\ -q_{N,iI} & -q_{N,iI+1} & -q_{N,iI+2} & 0 & 0 & 0 \end{array}\right] \quad (17)$$

where, O and O' are zero matrices of the indicated dimensions and the following constants are defined,

$$d_k = (A_{kN}\, B_{kN} - \hat{A}_{kN}\, G_{kN}) / M_N \;, k \varepsilon J_I$$

$$d_{kj} = (A_{kj} B_{kj} + \hat{A}_{kj}\, G_{kj}) / M_k \;,\; k \neq j \;, k \varepsilon J_I \;,\; j \varepsilon J_{IN}$$

$$q_{jk} = K_j\, (\hat{E}_{dk}\, B_{jk} - \hat{E}_{qk}\, G_{jk}) \;,\; k \neq j \;,\; k, j \varepsilon J_{IN}$$

Using Eqs. (10,15 –17), the free subsystem of eqn. 9 is completely defined.

Now, the interconnection (vector) matrix $h_I(X)$ is obtained in the form

$$\mathbf{h}_I(X) = [\, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{h}_{I4}(X), \mathbf{h}_{I5}(X), \ldots\ldots\ldots\ldots, \mathbf{h}_{I11}(X) \,]^T \quad (18)$$

where

$$\mathbf{h}_{I4}(X) = -(1/M_{iI})\Big[G_{iI,iI} X^2_{I8} + C_{iI,N}\, \hat{f}_{I1}(\sigma_{I1}) + C_{iI,iI+1} \hat{f}_{I4}(\sigma_{I4}) + + C_{iI,iI+2}\, \hat{f}_{I5}(\sigma_{I5}) + \sum S_{iI,j} + \sum^{N}_{j\neq iI} \{\hat{L}_{iI,j} + X_{I8} L_{iI,j}\}\Big]$$

$$\mathbf{h}_{I5}(X) = -(1/M_{iI+1})\Big[G_{iI+1,iI+1} X^2_{I9} + C_{iI+1,N}\, \hat{f}_{I2}(\sigma_{I2}) + C_{iI+1,iI}\, \hat{f}_{I4}(\sigma_{I4}) + C_{iI+1,iI+2}\, \hat{f}_{I6}(\sigma_{I6}) + \sum S_{iI+1,j} + \sum^{N}_{j\neq iI+1} \{\hat{L}_{iI+1,j} + X_{I9} L_{iI+1,j}\}\Big]$$

$$\mathbf{h}_{I6}(X) = -(1/M_{iI+2})\Big[G_{iI+2,iI+2} X^2_{I10} + C_{iI+2,N}\, \hat{f}_{I3}(\sigma_{I3}) + C_{iI+2,iI}\, \hat{f}_{I5}(\sigma_{I5}) + C_{iI+2,iI+1}\, \hat{f}_{I6}(\sigma_{I6}) + \sum S_{iI+2,j} + \sum^{N}_{j\neq iI+2} \{\hat{L}_{iI+2,j} + X_{I10}\, L_{iI+2,j}\}\Big]$$

$$\mathbf{h}_{I7}(X) = -(1/M_N)\Big[G_{N,N} X^2_{I11} + C_{N,iI}\, \hat{f}_{I1}(\sigma_{I1}) + C_{N,iI+1}\, \hat{f}_{I2}(\sigma_{I2}) + + C_{N,iI+2}\, \hat{f}_{I3}(\sigma_{I3}) + \sum S_{N,j} + \sum^{N-1}_{j=1} \{\hat{L}_{N,j} + X_{I11} L_{N,j}\}\Big]$$

$$\mathbf{h}_{I8}(X) = K_{iI}\Big[\tilde{C}_{iI,N}\, \hat{f}_{I1}(\sigma_{I1}) + \tilde{C}_{iI,iI+1}\, \hat{f}_{I4}(\sigma_{I4}) + \tilde{C}_{iI,iI+2}\, \hat{f}_{I5}(\sigma_{I5}) + \sum \tilde{S}_{iI,j} - \sum^{N}_{j\neq iI} \tilde{L}_{iI,j}\Big]$$

$$\mathbf{h}_{I9}(X) = K_{iI+1}\Big[\tilde{C}_{iI+1,N}\, \hat{f}_{I2}(\sigma_{I2}) + \tilde{C}_{iI+1,iI}\, \hat{f}_{I4}(\sigma_{I4}) + \tilde{C}_{iI+1,iI+2}\, \hat{f}_{I6}(\sigma_{I6}) + \sum \tilde{S}_{iI+1,j} - \sum^{N}_{j\neq iI+1} \tilde{L}_{iI+1,j}\Big]$$

$$h_{I10}(X) = K_{iI+2}\left[\tilde{C}_{iI+2,N}\,\hat{f}_{I3}(\sigma_{I3}) + \tilde{C}_{iI+2,iI}\,\hat{f}_{I5}(\sigma_{I5}) + \tilde{C}_{iI+2,iI+1}\,\hat{f}_{I6}(\sigma_{I6}) + \sum \tilde{S}_{iI+2,j} - \sum_{j \neq iI+2}^{N} \tilde{L}_{iI+2,j}\right]$$

$$h_{I11}(X) = K_N\left[C_{N,iI}\,\hat{f}_{I1}(\sigma_{I1}) + C_{N,iI+1}\,\hat{f}_{I2}(\sigma_{I2}) + C_{N,iI+2}\,\hat{f}_{I3}(\sigma_{I3}) + \sum \tilde{S}_{N,j} - \sum_{j=1}^{N-1} \tilde{L}_{N,j}\right] \quad (19)$$

Note that Σ is given as $\sum_{j \notin JI}^{N-1}$ and the following constants are defined ,

$$\begin{aligned}
L_{kj} &= \{(E_{Qj}+\hat{E}_{qj})\cos(\sigma_{kj}+\delta^{\circ}_{kj}-\theta_{kj}) + \hat{E}_{dj}\sin(\sigma_{kj}+\delta^{\circ}_{kj}-\theta_{kj})\}Y_{kj} \\
\hat{L}_{kj} &= \{\hat{E}_{qk}\cos(\sigma_{kj}+\delta^{\circ}_{kj}-\theta_{kj}) - \hat{E}_{dk}\sin(\sigma_{kj}+\delta^{\circ}_{kj}-\theta_{kj})\}\,Y_{kj}E_{Qj} \\
\tilde{L}_{kj} &= E_{Qj}\,Y_{kj}\,\sin(\sigma_{kj}+\delta^{\circ}_{kj}-\theta_{kj}) \\
S_{kj} &= Y_{kj}\{A_{kj}f_{kj}(\sigma_{kj}) + \hat{A}_{kj}g_{kj}(\sigma_{kj})\} \\
\tilde{S}_{kj} &= Y_{kj}\{\hat{E}_{dj}f_{kj}(\sigma_{kj}) - \hat{E}_{qj}g_{kj}(\sigma_{kj})\} \qquad , \; k \in J_{IN} \\
C_{kj} &= A_{kj}G_{kj} - \hat{A}_{kj}B_{kj} \\
\tilde{C}_{kj} &= \hat{E}_{dj}G_{kj} + \hat{E}_{qj}B_{kj} \qquad , k \neq j \;,\; k, j \in J_{IN} \qquad (20)
\end{aligned}$$

In Eq. (19), the following nonlinear function are defined ,

$$\begin{aligned}
\hat{f}_{I1}(\sigma_{I1}) &= \cos(\sigma_{iI,N}+\delta^{\circ}_{iI,N}) - \cos\delta^{\circ}_{iI,N} \\
\hat{f}_{I2}(\sigma_{I2}) &= \cos(\sigma_{iI+1,N}+\delta^{\circ}_{iI+1,N}) - \cos\delta^{\circ}_{iI+1,N} \\
\hat{f}_{I3}(\sigma_{I3}) &= \cos(\sigma_{iI+2,N}+\delta^{\circ}_{iI+2,N}) - \cos\delta^{\circ}_{iI+2,N} \\
\hat{f}_{I4}(\sigma_{I4}) &= \cos(\sigma_{iI,iI+1}+\delta^{\circ}_{iI,iI+1}) - \cos\delta^{\circ}_{iI,iI+1} \\
\hat{f}_{I5}(\sigma_{I5}) &= \cos(\sigma_{iI,iI+2}+\delta^{\circ}_{iI,iI+2}) - \cos\delta^{\circ}_{iI,iI+2} \\
\hat{f}_{I6}(\sigma_{I6}) &= \cos(\sigma_{iI+1,iI+2}+\delta^{\circ}_{iI+1,iI+2}) - \cos\delta^{\circ}_{iI+1,iI+2} \qquad (21)
\end{aligned}$$

and the nonlinear functions $f_{ij}(\sigma_{ij})$ and $g_{ij}(\sigma_{ij})$ are given by Eq.(6) .

4 . *Power system aggregation*

As the first step, we accept for each free subsystem of eqn.9 a Lyapunov function in the form [4-7, 9 -13],

$$V_I(X_I) = X^T_I H_I X_I + \sum_{m=1}^{6} \gamma_{Im} \int_0^{\sigma I} f_{Im}(\sigma_{Im})\, d\sigma_{Im} \quad , I = 1,2,....,S \quad (22)$$

where H_I is an eleventh-order symmetric positive definite matrix , γ_{Im} are arbitrary positive numbers , and the nonlinear functions f_{Im} are given by Eq. (12) .
Following the aggregation procedure in [19], it is constructed an aggregation matrix, A =[α_{IJ}], the elements (real numbers) of this matrix obey the inequality

$$\dot{V}_I(X_I) \leq \sum_{J=1}^{S} \alpha_{IJ} \|X_I\| \, \|X_J\| , I = 1,2,.........,S \quad (23)$$

where $\dot{V}_I(X_I)$, is the total time derivative of the function $V_I(X_I)$, along the motion of the ith interconnected subsystem of eqn. 8 . It is to be noted that V_I, can be written as

$$\dot{V}_I(X_I) = V_I(X_I)_f + [\text{grad}\, V_I\ (X_I)]^T\ h_I(X) \qquad (24)$$

where $V_I(X_I)_f$ is the total time derivative of the function V_I, along the motion of the ith free subsystem .

4.1 Stability criterion

According to theorem 1 of Ref. 19, stability of the aggregation matrix, $A=[\alpha_{ik}]$, or equivalently, if it is satisfied the Hick's conditions

$$(-1)^k \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1k} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2k} \\ \vdots & & & \vdots \\ \alpha_{k1} & \alpha_{k2} & \cdots & \alpha_{kk} \end{bmatrix} > 0 \qquad k=1,2,....,S \qquad (25)$$

implies asymptotic stability of the system equilibrium .

4.2 Aggregation matrix

As a first step, the two terms in the right-hand side of Eq. (24) are computed, then a number of majorizations are introduced and used to majorize the left-hand side of eqn. 24. Finally, elements of the (square) aggregation matrix , $A = [\alpha_{IK}]$, of order [(N-1) / 3] are obtained and defined as

$$\alpha_{IK} = \begin{cases} -\lambda^*_I & ,K=I \\ 2Z_2(\hat{Z}_I;\tilde{Z}_I) & ,K\neq I \end{cases} \qquad K,I=1,2,...,S=N-1 \qquad (26)$$

where λ^* is the minimal (positive) eigenvalue of the 14th-order symmetric matrix R_I whose elements are given by eqn. (A-1), and $\hat{Z}_I$ and $\tilde{Z}_I$ are defined by eqn. (A-2).

5. Numerical example

The developed approach is used, in this example, to carry out transient stability studies of the 10-machine, 11-bus system shown in Fig. 1. The (pre-transient) steady state values of the system angle δ and voltages E'_q, E'_d and E'_{fd} are computed and given in Table I .

Table I. Post-fault equilibrium state results.

Bus No.	δ^o(deg)	$\hat{E}_q$	$\hat{E}_d$	E^o_{fdi}
1	13.07	1.05158	– 0.01496	1.05484
2	10.64	1.14617	– 0.00936	1.15335
3	7.58	1.13267	–0.00554	1.13668
4	4.21	1.06205	–0.00984	1.06710
5	2.29	1.07639	–0.00348	1.07920

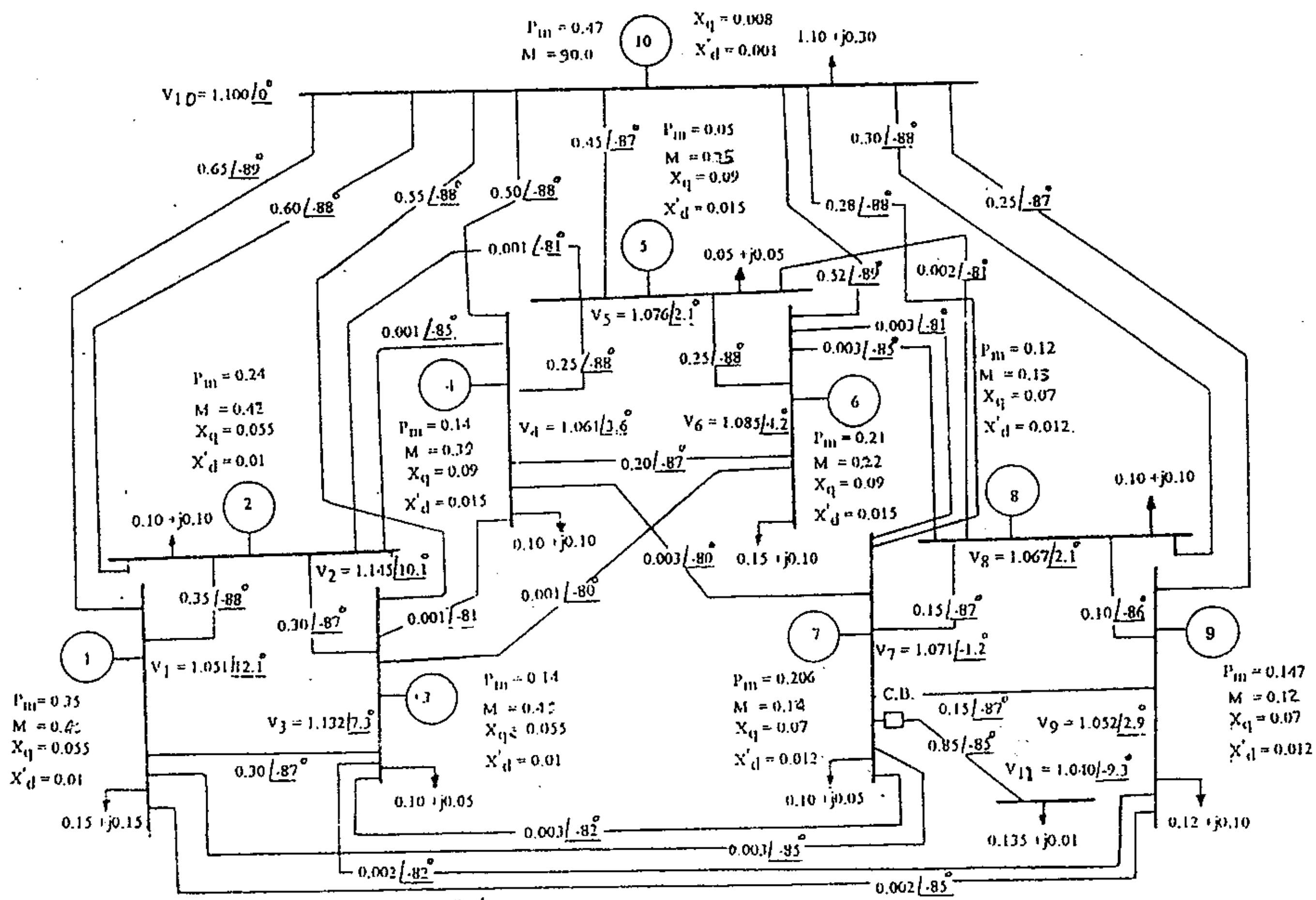

Fig. 4 10 - machine system. All values in p.u.

6	5.14	1.08643	–0.01447	1.09357
7	–0.47	1.07156	–0.01113	1.07566
8	2.46	1.06794	–0.00671	1.07289
9	3.44	1.05270	–0.00808	1.05729
10	0.23	1.10037	–0.00386	1.10303

Now, to determine an asymptotic stability domain estimate for the considered system, the stability computations are carried out as follows :

1– The reactance X'_d of each generator is inserted, and the system loads are represented by equivalent shunt admittances. Then the system nodes, except the generators internal nodes, are eliminated, and finally the reduced 10th-order (symmetric) admittance matrix **Y**, is obtained and its elements are given in Table II.

Table II. Reduced admittance matrix for post-fault system.

Arguments (deg.)										Moduli (pu)
–83.15	1.42766	0.34177	0.29362	0.00032	0.00029	0.00033	0.00313	0.00020	0.00213	0.64168
91.84	–84.67	1.31336	0.29392	0.00127	0.00124	0.00031	0.00018	0.00018	0.00016	0.59343
92.83	92.86	–83.88	1.18560	0.00125	0.00025	0.00126	0.00310	0.00017	0.00211	0.54486
92.90	94.76	97.88	–83.20	1.03273	0.24347	0.19499	0.00307	0.00017	0.00013	0.49336
93.88	98.13	95.08	91.83	–85.21	0.98216	0.24342	0.00015	0.00210	0.00011	0.44520
91.85	93.00	98.22	92.77	91.81	–81.73	1.05430	0.00308	0.00310	0.00014	0.51276
94.65	94.23	97.59	99.49	95.83	98.44	–70.70	0.69058	0.14794	0.14802	0.27767
92.99	93.82	94.14	94.56	98.57	94.72	92.77	–80.03	0.64672	0.09888	0.29725
94.78	95.10	97.66	95.09	95.93	94.18	92.75	93.83	–77.11	0.60439	0.24804
90.86	91.88	91.88	91.85	92.87	90.83	91.80	91.87	92.86	–76.50	4.42601

2– Selecting machine 10 as the reference machine, the system is decomposed, referring to the system reduced matrix **Y**, into three " four-machine " interconnected subsystems.

3– For the obtained three subsystems the following parameters are selected:

$\lambda_i = 4.0$, $T'_{doi} = 4.0$, $i = , 2, 3, \ldots, 9$; $\lambda_{10} = 9.5$, $T'_{do10} = 3.6$

$h^k_{14} = h^k_{25} = h^k_{36} = h^k_{44} = h^k_{55} = h^k_{66} = 1.0$, $k = 1, 2, 3$; $h^1_{77} = h^2_{77} = 8.0$, $h^3_{77} = 7.2$

$h^1_{88} = h^1_{99} = h^1_{10,10} = 570$, $h^1_{11,11} = 50.0$; $\varepsilon_{11} = 0.76$, $\varepsilon_{12} = 0.78$, $\varepsilon_{13} = 0.80$

$h^2_{88} = h^2_{99} = h^2_{10,10} = 310$, $h^2_{11,11} = 50.0$; $\varepsilon_{21} = 0.56$, $\varepsilon_{22} = 0.63$, $\varepsilon_{23} = 0.62$

$h^3_{88} = h^3_{99} = h^3_{10,10} = 540$, $h^3_{11,11} = 46.0$; $\varepsilon_{31} = 0.59$, $\varepsilon_{32} = 0.57$, $\varepsilon_{33} = 0.55$

Using expression (26), we compute the matrix

$$A = \begin{bmatrix} -1.497506 & 0.489385 & 0.274502 \\ 0.531260 & -0.790397 & 0.307347 \\ 0.544197 & 0.496672 & -0.675272 \end{bmatrix}$$

which is a stable matrix (it satisfies conditions (25)) . This implies the asymptotic stability of the system equilibrium.

4– It is determined (see [19], and Appendix of [10]) the system asymptotic stability domain estimate $\mathbf{E}_1$ given as,

$$\mathbf{E}_1 = \{ \mathbf{X} : [\, 3.60\, V_1(X_1) + 1.25\, V_2(X_2) + V_3(X_3)] \leq 17.83375 \} \qquad (27)$$

where, V_1, V_2 and V_3 are the free subsystem Lyapunov functions, given by eqn.22.

Now, using the developed approach, the system transient stability studies are carried out assuming the following four stability cases:

i. A sudden connection of a load of the power (0.7 + j0.3) per unit to bus 9, this load is removed after a certain time interval. This case may simulate addition of a (pulsating) load comprising large motors of a rolling mill. Applying the developed approach the longest time duration for the considered load is determined, directly, to be 0.047 sec. Note that, using the standard step-by-step method, this time is computed to be 0.059 sec.

Now, in order to rank the duration times for the considered load, the stability computations are repeated assuming the load to be connected at either bus 7, or bus 8. It is found that, the load longest duration times are 0.053 sec and 0.050 sec for buses 7 and 8, respectively.

ii. A 3-phase short circuit fault (with successful reclosure) is assumed to occur near bus 8 (at 10% of the line length) on the tie-line connecting buses 8 and 10 . The fault is cleared by switching off the faulted line, using 3–cycle circuit breakers. Now it is assumed that, just after clearing the fault, a pulsating load of the power (0.5 + j 0.2) per unit is connected to bus 8. Applying the developed approach, it is found that Eq. (27), can be satisfied if the open line is reconnected and in the same time the connected load is removed within 0.106 sec from the fault instant (note that this time is equal to 0.124 sec, by using the step-by-step method) .

iii. A 3-phase short circuit fault (with successful reclosure) is assumed to be occurred near bus 4, at 10 % length of the tie-line between buses 4 and 10. Opening two 5-cycle circuit breakers, located at both ends of the faulted line clears the fault. At the same fault clearing instant it is assumed that, due to false operation of the circuit breakers located near the fault location, the two tie-lines connecting bus 4 to buses 5 and 6 are switched off. It is found, using the developed approach, that the three lines can still open (Eq.27 is satisfied) until elapsing the time of 0.560 sec from the fault instant. However, using the step-by-step method, it is found that the critical time for reclosing the open three lines is equal to 0.726.

iv. It is required, in this case, to determine directly the critical time for clearing a 3-phase short circuit fault near bus 7, at 0.05% length of the tie-line between buses 7 and 11. Now, as a first step for the stability computation the Newton–Raphson method is used to determine the system post-fault (the fault is cleared) equilibrium state. Then, for the system under fault clearing condition, the 10-th order reduced admittance matrix is computed. Finally, it is determined for the system a new asymptotic stability domain estimate, which is given as,

$$\mathbf{E}^{**}_1 = \{ \mathbf{X} : (2.80\, V_1(X_1) + V_2(X_2) + V_3(X_3)) \leq 13.4230 \} \qquad (28)$$

Using Eq.(28), it is found that the critical time for clearing the considered fault is equal to 0.032 sec. It is to be noted that, using the step-by-step method, the critical time equals 0.042 sec .

Figs.2-5, show variations of the subsystem states, and referring to these figures it is clear that the system will regain its prefault (steady-state) condition for each of the four assumed stability cases. Note that in Figs.2-5, the time is computed just after the subsystem states enter the considered stability domain estimate.

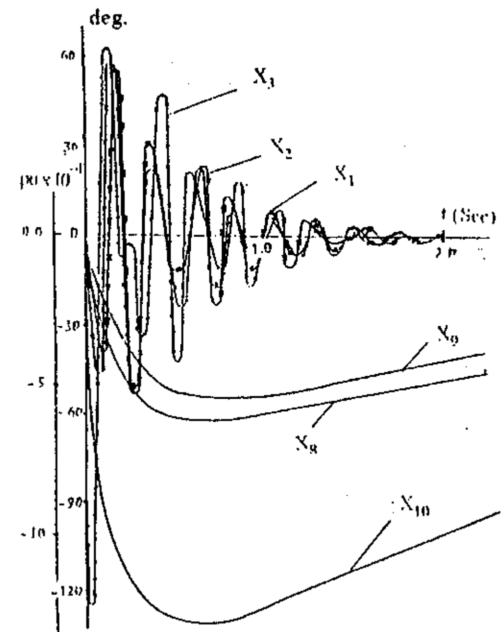

Fig. 2 Variations of the third subsystem states considering the first stability case

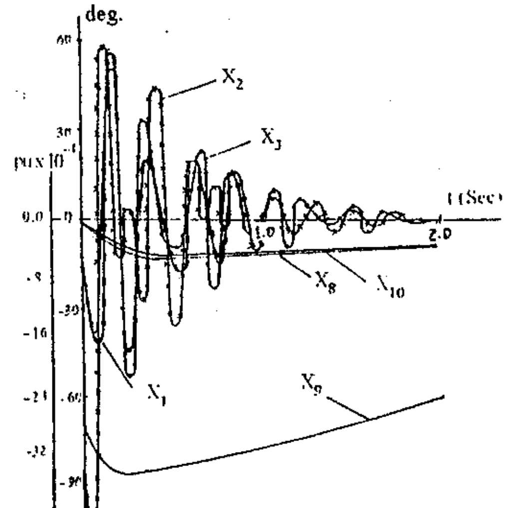

Fig. 3 Variations of the third subsystem states considering the second stability case

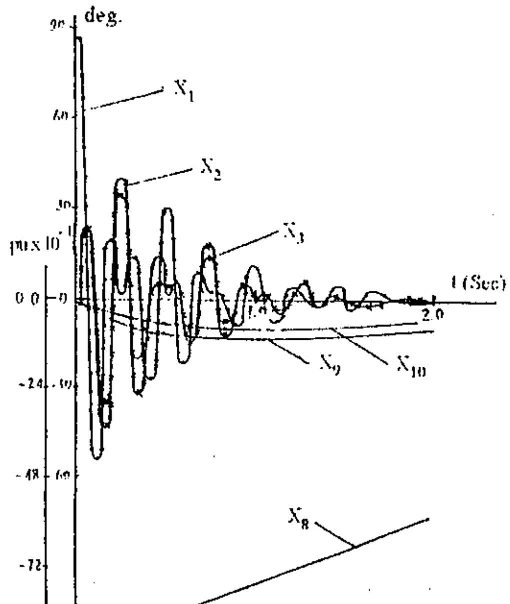

Fig. 4 Variations of the second subsystem states considering the third stability case

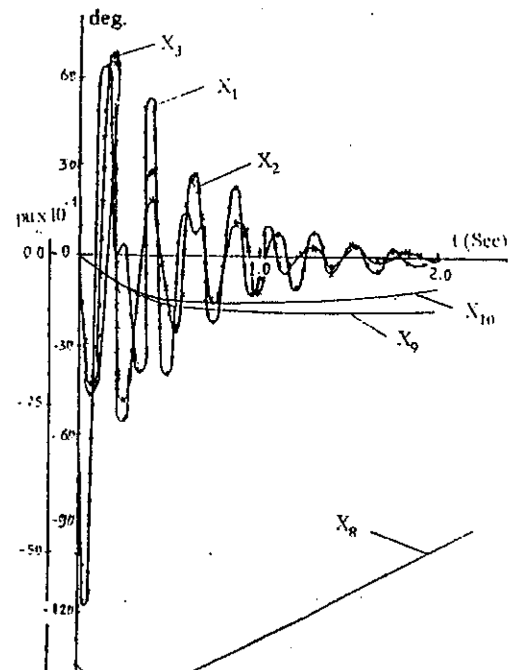

Fig. 5 Variations of the third subsystem states considering the fourth stability case

6. Conclusions

A new Lyapunov stability approach is developed, in the paper, and is used to carry out transient stability studies of a 10-machine, 11-bus power system. It is drawn the following salient conclusions:

1– The developed approach is suitable for application to real power systems. Note that, in the approach non-uniform mechanical damping case is assumed, and generators flux decay effect is considered.

2 – Order of the obtained aggregation matrix is equal to (N-1) / 3, where N is number of system machines. Hence, the matrix order is independent upon number of system buses. However, for real power systems value of N is more less than number of system buses, and hence it can be easily satisfy, for those systems, stability conditions (see Eq.25) of computed aggregation matrices.

3 – In the developed approach, all the system transfer conductances are considered, hence resistance's of the tie-lines can be taken into consideration. In addition, the system network can be greatly simplified by eliminating all system nodes, except generators internal nodes.

4 – The approach developed can be easily used to carry out transient stability studies of power systems. Note that, the approach is used to determine the critical time for clearing a 3-phase short circuit fault, the longest duration time for an added pulsating load, the critical time for reclosing three open tie-lines, and the critical time for removing a connected load with reclosing an open tie-line.

5 – The developed approach can be used for ranking contingencies according to their severity. Note that, the approach is used to find, directly, which one of three considered buses of the system is more suitable for connection of a given pulsating load.

6 - The developed approach can provide satisfactory practical results. Note that, values of the times obtained in the numerical example are about 76 % - 86 % of the exact time values computed by using the standard step-by-step method.

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List of symbols

P_{mi} = mechanical power delivered to ith machine

P_{ei} = electrical power delivered by ith machine

δ_i = rotor angle, or position of the rotor q-axis from the reference

X_{di}, X_{qi} = direct-axis, quadrature-axis synchronous reactances

X'_{di}, X'_{qi} = d-axis, q-axis transient reactances

E_{fd} =exciter voltage referred to the armature circuit

E'_i = voltage behind d-axis transient reactance

E'_{di}, E'_{qi} = d-axis, q-axis components of the voltage E'_i

E_q = armature emf corresponding to the field current

$\overset{o}{\delta}_i$, $\overset{o}{E}_{fdi}$, $\hat{E}_{qi}$, $\hat{E}_{di}$= steady state values of the angle δ_i , and the voltages E_{fdi} ,E'_{qi} and E'_{di}, respectively

ω_i - rotor speed with respect to the synchronous speed

$Y_{ij} = Y_{ji}$ = modulus of transfer admittance between internal nodes of ith and jth generators

$\theta_{ij} = \theta_{ji}$ = phase angle of transfer admittance Y_{ij}

$G_{ij} = Y_{ij} \cos\theta_{ij}$ = transfer conductance

$B_{ij} = Y_{ij} \sin\theta_{ij}$ = transfer susceptance

T'_{doi} = direct-axis transient open-circuit time constant of ith generator

D_i = mechanical damping

$\lambda_i = (D_i / M_i)$ = mechanical damping coefficient

$J_{IN} = \{iI, iI+1, iI+2, N\}$ = set introduced to denote the Ith subsystem four machines

$J_I \subset J_{IN} = \{iI, iI+1, iI+2\}$

$||X_I|| = (X_I^T X_I)^{1/2}$

$\delta_{ij} = \delta_i - \delta_j = \delta_{iN} - \delta_{jN}$; $\sigma_{ij} = \delta_{ij} - \delta^o_{ij} = \sigma_{iN} - \sigma_{jN}$

$\sigma_{kN} = \delta_{kN} - \delta^o_{kN}$, $k \varepsilon J_I$

$A_{ij} = A_{ji} = \hat{E}_{qi}\hat{E}_{qj} + \hat{E}_{di}\hat{E}_{dj}$

$\hat{A}_{ij} = -\hat{A}_{ji} = \hat{E}_{qi}\hat{E}_{dj} - \hat{E}_{di}\hat{E}_{qj}$, $i \neq j, i, j \varepsilon J_{IN}$

$K_j = (X_{dj} - X'_{dj}) / T'_{doj}$; $\Gamma j = [1.0 - (X_{dj} - X'_{dj}) B_{jj}] / T'_{doj}$

$\mu_j = 2\hat{E}_{qj} G_{jj} / M_j$, $j \varepsilon J_{IN}$

Z_2, Z_3 = two functions, defined as follows:

$Z_2(\alpha, \phi) = \min\{\sqrt{2}\max(|\alpha|, |\phi|) ; (|\alpha| + |\phi|)\}$

$Z_3(\alpha, \phi, \gamma) = \min\{2\max(|\alpha|, |\phi|, |\gamma|) ; (|\alpha| + |\phi| + |\gamma|) ; Z_2[Z_2(\alpha, \phi), \gamma] ; Z_2[Z_2(\phi, \gamma), \alpha] ; Z_2[Z_2(\gamma, \alpha), \phi]\}$

APPENDIX

Definition of the elements of the system aggregation matrix R :

$r^I_{11} = 2 a_I \{D_{iI} \varepsilon_{I1} - Đ_{iI} - m_{iI, iI+1} - m_{iI, iI+2} - \Sigma U_{iI, j}\}$

$r^I_{22} = 2 \bar{a}_I \{D_{iI+1} \varepsilon_{I2} - Đ_{iI+1} - m_{iI+1, iI} - m_{iI+1, iI+2} - \Sigma U_{iI+1, j}\}$

$r^I_{33} = 2 \hat{a}_I \{D_{iI+2} \varepsilon_{I3} - Đ_{iI+2} - m_{iI+2, iI} - m_{iI+2, iI+1} - \Sigma U_{iI+2, j}\}$

$r^I_{44} = 2(\lambda_{iI} h^I_{44} - h^I_{14})$, $r^I_{55} = 2(\lambda_{iI+1} h^I_{55} - h^I_{25})$

$r^I_{66} = 2(\lambda_{iI+2} h^I_{66} - h^I_{36})$, $r^I_{77} = 2\lambda_N h^I_{77}$, $r^I_{88} = 2\Gamma_{iI} h^I_{88}$

$r^I_{99} = 2\Gamma_{iI+1} h^I_{99}$, $r^I_{10,10} = 2\Gamma_{iI+2} h^I_{10,10}$, $r^I_{11,11} = 2\Gamma_N h^I_{11,11}$

$r^I_{12} = -\left[\left|(a_I - \bar{a}_I) n_{iI, iI+1} + a_I \bar{n}_{iI, iI+1} - \bar{a}_I \bar{n}_{iI+1, iI}\right| + a_I V_{iI, iI+1} + \bar{a}_I V_{iI+1, iI}\right]$

$r^I_{13} = -\left[\left|(a_I - \hat{a}_I) n_{iI, iI+2} + a_I \bar{n}_{iI, iI+2} - \hat{a}_I \bar{n}_{iI+2, iI}\right| + a_I V_{iI, iI+2} + \hat{a}_I V_{iI+2, iI}\right]$

$r^{I}_{14} = -b_I[Đ_{iI} + m_{iI,iI+1} + m_{iI,iI+2} + \Sigma(U_{iI,j} + \hat{U}_{iI,j})]$, $r^{I}_{15} = -\overline{b}_I m_{iI+1,iI}$

$r^{I}_{16} = -\hat{b}_I m_{iI+2,iI}$, $r^{I}_{17} = -h^{I}_{11} - a_N \tilde{D}_{iI} - |(a_N - b_I) U_{iI} - (a_N + b_I)\overline{U}_{iI}|$

$r^{I}_{18} = -a_I[d_{iI} + \overline{d}_{iI} + \tilde{V}_{iI,iI+1} + \tilde{V}_{iI,iI+2}] - c_I[\tilde{m}_{iI} + \hat{m}_{iI,N} + \hat{m}_{iI,iI+1} + \hat{m}_{iI,iI+2}]$
$-(a_I + c_I)\Sigma\overline{U}_{iI,j}$, $r^{I}_{19} = -a_I \hat{E}_{iI} Y_{iI,iI+1} - \overline{c}_I \hat{m}_{iI+1,iI}$

$r^{I}_{1,10} = -a_I \hat{E}_{iI} Y_{iI,iI+2} - \hat{c}_I \hat{m}_{iI+2,iI}$, $r^{I}_{1,11} = -a_I \hat{E}_{iI} Y_{iI,N} - c_N[m_{iI} + \hat{m}_{iI}]$

$r^{I}_{23} = -[\,|(\overline{a}_I - \hat{a}_I) n_{iI+1,iI+2} + \overline{a}_I \overline{n}_{iI+1,iI+2} - \hat{a}_I \overline{n}_{iI+2,iI+1}| + \overline{a}_I V_{iI+1,iI+2} +$
$+\hat{a}_I V_{iI+2,iI+1}]$, $r^{I}_{24} = -b_I m_{iI,iI+1}$

$r^{I}_{25} = -\overline{b}_I[Đ_{iI+1} + m_{iI+1,iI} + m_{iI+1,iI+2} + \Sigma(U_{iI+1,j} + \hat{U}_{iI+1,j})]$

$r^{I}_{26} = -\hat{b}_I m_{iI+2,iI+1}$, $r^{I}_{27} = -h^{I}_{22} - c_N \tilde{D}_{iI+1} - |(a_N - \overline{b}_I) U_{iI+1} - (a_N + \overline{b}_I)$
$\overline{U}_{iI+1}|$, $r^{I}_{28} = -\overline{a}_I \hat{E}_{iI+1} Y_{iI,iI+1} - c_I \hat{m}_{iI,iI+1}$

$r^{I}_{29} = -\overline{a}_I[d_{iI+1} + \overline{d}_{iI+1} + \tilde{V}_{iI+1,iI} + \tilde{V}_{iI+1,iI+2}] - \overline{c}_I[\tilde{m}_{iI+1} + \hat{m}_{iI+1,N} +$
$+\hat{m}_{iI+1,iI} + \hat{m}_{iI+1,iI+2}] - (\overline{a}_I + \overline{c}_I)\Sigma\overline{U}_{iI+1,j}$

$r^{I}_{2,10} = -\overline{a}_I \hat{E}_{iI+1} Y_{iI+1,iI+2} - \hat{c}_I \hat{m}_{iI+2,iI+1}$, $r^{I}_{2,11} = -\overline{a}_I \hat{E}_{iI+1} Y_{iI+1,N} -$
$-c_N(m_{iI+1} + \hat{m}_{iI+1})$, $r^{I}_{2,12} = -|(\overline{a}_I - a_I) d_{iI,iI+1} - (\overline{a}_I + a_I)\overline{d}_{iI,iI+1}|$

$r^{I}_{34} = -b_I m_{iI,iI+2}$, $r^{I}_{35} = -\overline{b}_I m_{iI+1,iI+2}$

$r^{I}_{36} = -\hat{b}_I[Đ_{iI+2} + m_{iI+2,iI} + m_{iI+2,iI+1} + \Sigma(U_{iI+2,j} + \hat{U}_{iI+2,j})]$

$r^{I}_{37} = -h^{I}_{33} - c_N \tilde{D}_{iI+2} - |(a_N - \hat{b}_I) U_{iI+2} - (a_N + \hat{b}_I)\overline{U}_{iI+2}|$

$r^{I}_{38} = -\hat{a}_I \hat{E}_{iI+2} Y_{iI+2,iI} - c_I \hat{m}_{iI,iI+2}$, $r^{I}_{39} = -\hat{a}_I \hat{E}_{iI+2} Y_{iI+1,iI+2} -$
$-\overline{c}_I \hat{m}_{iI+1,iI+2}$, $r^{I}_{3,10} = -\hat{a}_I[d_{iI+2} + \overline{d}_{iI+2} + \tilde{V}_{iI+2,iI} + \tilde{V}_{iI+2,iI+1}] -$
$-\hat{c}_I[\tilde{m}_{iI+2} + \hat{m}_{iI+2,N} + \hat{m}_{iI+2,iI} + \hat{m}_{iI+2,iI+1}] - (\hat{a}_I + \hat{c}_I)\Sigma\overline{U}_{iI+1,j}$

$r^{I}_{3,11} = -\hat{a}_I \hat{E}_{iI+2} Y_{iI+2,N} - c_N[m_{iI+2} + \hat{m}_{iI+2}]$, $r^{I}_{3,13} = -|(\hat{a}_I - a_I) d_{iI,iI+2}$
$-(a_I + \hat{a}_I)\overline{d}_{iI,iI+2}|$, $r^{I}_{3,14} = -|(\hat{a}_I - \overline{a}_I) d_{iI+1,iI+2} - (\overline{a}_I + \hat{a}_I)\overline{d}_{iI+1,iI+2}|$

$r^{I}_{47} = -h^{I}_{14}$, $r^{I}_{48} = -b_I[d_{iI} + \overline{d}_{iI} + \tilde{V}_{iI,iI+1} + \tilde{V}_{iI,iI+2} + \Sigma\overline{U}_{iI,j}]$

$r^{I}_{49} = -b_I \hat{E}_{iI} Y_{iI,iI+1}$, $r^{I}_{4,10} = -b_I \hat{E}_{iI} Y_{iI,iI+2}$, $r^{I}_{4,11} = -b_I \hat{E}_{iI} Y_{iI,N}$

$r^{I}_{57} = -h^{I}_{25}$, $r^{I}_{58} = -\overline{b}_I \hat{E}_{iI+1} Y_{iI,iI+1}$

$r^{I}_{59} = -\overline{b}_I[d_{iI+1} + \overline{d}_{iI+1} + \tilde{V}_{iI+1,iI} + \tilde{V}_{iI+1,iI+2} + \Sigma\overline{U}_{iI+1,j}]$

$r^{I}_{5,10} = -\overline{b}_I \hat{E}_{iI+1} Y_{iI+1,iI+2}$, $r^{I}_{5,11} = -\overline{b}_I \hat{E}_{iI+1} Y_{iI+1,N}$

$r^{I}_{5,12} = -|(\overline{b}_I - b_I) d_{iI,iI+1} - (\overline{b}_I + b_I)\overline{d}_{iI,iI+1}|$, $r^{I}_{67} = -h^{I}_{36}$

$r^{I}_{68} = -\hat{b}_I \hat{E}_{iI+2} Y_{iI,iI+2}$, $r^{I}_{69} = -\hat{b}_I \hat{E}_{iI+2} Y_{iI+1,iI+2}$

$r^{I}_{6,10} = -\hat{b}_I[d_{iI+2} + \overline{d}_{iI+2} + \tilde{V}_{iI+2,iI} + \tilde{V}_{iI+2,iI+1} + \Sigma\overline{U}_{iI+2,j}]$

$r^{I}_{6,11} = -\hat{b}_I \hat{E}_{iI+2} Y_{iI+2,N}$, $r^{I}_{6,13} = -|(\hat{b}_I - b_I) d_{iI,iI+2} - (\hat{b}_I + b_I)\overline{d}_{iI,iI+2}|$

$$r^{I}_{6,14} = -\left|(\dot{b}_I - \bar{b}_I)\, d_{iI+1,\, iI+2} - (\dot{b}_I + \overline{b}_I)\,\ddot{d}_{iI+1,\, iI+2}\right| \qquad r^{I}_{78} = -a_N \ddot{E}_N Y_{iI,\,N}$$

$$r^{I}_{79} = -a_N \hat{E}_N Y_{iI+1,\,N} \quad , \qquad r^{I}_{7,10} = -a_N \hat{E}_N Y_{iI+2,\,N}$$

$$r^{I}_{7,11} = -a_N \left[d_N + \hat{d}_{iI} + \hat{d}_{iI+1} + \dot{d}_{iI+2} + \Sigma \overline{U}_{N,\,j}\right]$$

$$r^{I}_{89} = -Y_{iI,\, iI+1} \sqrt{\{c_I^{\,2} + \bar{c}_I^{\,2} - c_I \bar{c}_I\, \rho_{iI,\, iI+1}\}}$$

$$r^{I}_{8,10} = -Y_{iI,\, iI+2} \sqrt{\{c_I^{\,2} + \hat{c}_I^{\,2} - c_I \hat{c}_I\, \rho_{iI,\, iI+2}\}}$$

$$r^{I}_{8,11} = -Y_{iI,\,N} \sqrt{\{c_I^{\,2} + c_N^{\,2} - c_I c_N\, \rho_{iI,\,N}\}} \quad , \qquad r^{I}_{8,12} = -c_I \widetilde{U}_{iI,\, iI+1}$$

$$r^{I}_{8,13} = -c_I \widetilde{U}_{iI,\, iI+2} \quad , \qquad r^{I}_{9,10} = -Y_{iI+1,\, iI+2} \sqrt{\{\hat{c}_I^{\,2} + \bar{c}_I^{\,2} - \bar{c}_I \hat{c}_I\, \rho_{iI+1,\, iI+2}\}}$$

$$r^{I}_{9,11} = -Y_{iI+1,\,N} \sqrt{\{\bar{c}_I^{\,2} + c_N^{\,2} - \bar{c}_I c_N\, \rho_{iI+1,\,N}\}} \quad , \qquad r^{I}_{9,12} = -\bar{c}_I \widetilde{U}_{iI+1,\, iI}$$

$$r^{I}_{9,14} = -\bar{c}_I \widetilde{U}_{iI+1,\, iI+2} \quad , \qquad r^{I}_{10,11} = -Y_{iI+2,\,N} \sqrt{\{\hat{c}_I^{\,2} + c_N^{\,2} - \hat{c}_I c_N\, \rho_{iI+2,\,N}\}}$$

$$r^{I}_{10,13} = -\hat{c}_I \widetilde{U}_{iI+2,\, iI} \ , \quad r^{I}_{10,14} = -\hat{c}_I \widetilde{U}_{iI+2,\, iI+1} \ , \quad r^{I}_{12,12} = 2\, a_I S_{iI,\, iI+1} / \widetilde{\xi}_{iI,\, iI+1}$$

$$r^{I}_{13,13} = 2\, a_I S_{iI,\, iI+2} / \widetilde{\xi}_{iI,\, iI+2} \quad , \qquad r^{I}_{14,14} = 2\, \bar{a}_I S_{iI+1,\, iI+2} / \widetilde{\xi}_{iI+1,\, iI+2} \qquad \textbf{(A-1)}$$

while the other elements of this matrix are zero.

Definition of the functions $\hat{Z}_I$ ***and*** $\widetilde{Z}_I$

In eqn.27, the two functions $\hat{Z}_I$ and $\widetilde{Z}_I$, are defined as follows (see Notation)

$$\hat{Z}_I = Z_3[\hat{Z}_{Ia}; \hat{Z}_{Ib}; \hat{Z}_{Ic}] \quad \text{,and} \quad \widetilde{Z}_I = Z_3[\widetilde{Z}_{Ia}; \widetilde{Z}_{Ib}; \widetilde{Z}_{Ic}] \qquad \textbf{(A-2)}$$

where

$$\hat{Z}_{Ia} = Z_2\{Z_3[Z_3(\beta_{iK}; \ddot{\beta}_{iK}; \hat{\beta}_{iK}); Z_3(\psi_{iK}; \overline{\psi}_{iK}; \hat{\psi}_{iK}); Z_3(\zeta_{iK}; \overline{\zeta}_{iK}; \hat{\zeta}_{iK})]; Z_2(H_{iK}; \hat{H}_{iK})\}$$

$$\hat{Z}_{Ib} = Z_2\{Z_3[Z_3(\beta_{iK+1}; \overline{\beta}_{iK+1}; \hat{\beta}_{iK+1}); Z_3(\psi_{iK+1}; \overline{\psi}_{iK+1}; \hat{\psi}_{iK+1}); Z_3(\zeta_{iK+1}; \overline{\zeta}_{iK+1}; \hat{\zeta}_{iK+1})]; Z_2(H_{iK+1}; \hat{H}_{iK+1})\}$$

$$\hat{Z}_{Ic} = Z_2\{Z_3[Z_3(\beta_{iK+2}; \overline{\beta}_{iK+2}; \hat{\beta}_{iK+2}); Z_3(\psi_{iK+2}; \overline{\psi}_{iK+2}; \hat{\psi}_{iK+2}); Z_3(\zeta_{iK+2}; \overline{\zeta}_{iK+2}; \hat{\zeta}_{iK+2})]; Z_2(H_{iK+2}; \hat{H}_{iK+2})\}$$

and where,

$$\widetilde{Z}_{Ia} = Z_2\{Z_3[Z_3(\alpha_{iK}; \bar{\alpha}_{iK}; \hat{\alpha}_{iK}); Z_3(\gamma_{iK}; \bar{\gamma}_{iK}; \hat{\gamma}_{iK}); Z_3(\eta_{iK}; \overline{\eta}_{iK}; \hat{\eta}_{iK})]; Z_2(\phi_{iK}; \hat{\phi}_{iK})\}$$

$$\widetilde{Z}_{Ib} = Z_2\{Z_3[Z_3(\alpha_{iK+1}; \overline{\alpha}_{iK+1}; \hat{\alpha}_{iK+1}); Z_3(\gamma_{iK+1}; \overline{\gamma}_{iK+1}; \hat{\gamma}_{iK+1}); Z_3(\eta_{iK+1}; \overline{\eta}_{iK+1}; \hat{\eta}_{iK+1})]; Z_2(\phi_{iK+1}; \hat{\phi}_{iK+1})\}$$

$$\widetilde{Z}_{Ic} = Z_2\{Z_3[Z_3(\alpha_{iK+2}; \overline{\alpha}_{iK+2}; \hat{\alpha}_{iK+2}); Z_3(\gamma_{iK+2}; \overline{\gamma}_{iK+2}; \hat{\gamma}_{iK+2}); Z_3(\eta_{iK+2}; \overline{\eta}_{iK+2}; \hat{\eta}_{iK+2})]; Z_2(\phi_{iK+2}; \hat{\phi}_{iK+2})\}$$

In eqns. (A-1) and (A-2), recall that Σ is given as $\sum_{K \neq I}^{S} \sum_{j \in JK}$ and the following constants are defined,

$$a_I = h^I_{14}/M_{iI}\ ,\ \bar{a}_I = h^I_{25}/M_{iI+1}\ ,\ \hat{a}_I = h^I_{36}/M_{iI+2}\ ,\ a_N = h^I_{77}/M_N$$

$$b_I = h^I_{44}/M_{iI}\ ,\ \bar{b}_I = h^I_{55}/M_{iI+1}\ ,\ \hat{b}_I = h^I_{66}/M_{iI+2}$$

$$c_I = K_{iI} h^I_{88}\ ,\ \bar{c}_I = K_{iI+1} h^I_{99}\ ,\ \hat{c}_I = K_{iI+2} h^I_{10,10}\ ,\ c_N = K_N h^I_{11,11}$$

$$D_k = (A_{kN} B_{kN} + \hat{A}_{kN} G_{kN})\ ,\ Đ_k = |A_{kN} G_{kN} - \hat{A}_{kN} B_{kN}| \hat{\xi}_k$$

$$\tilde{D}_k = |A_{kN} G_{kN} + \hat{A}_{kN} B_{kN}| \hat{\xi}_k\ ,\ m_k = |\hat{E}_{dk} B_{kN} - \hat{E}_{qk} G_{kN}| \xi_k$$

$$\hat{m}_k = |\hat{E}_{qk} B_{kN} + \hat{E}_{dk} G_{kN}| \hat{\xi}_k\ ,\ \tilde{m}_k = |\hat{E}_{dN} B_{kN} - \hat{E}_{qN} G_{kN}| \xi_k$$

$$\bar{d}_k = Y_{kN} \hat{E}_N\ ,\ \hat{d}_k = Y_{kN} \hat{E}_k\ ,\ U_k = A_{kN} B_{kN} \xi_k$$

$$\bar{U}_k = |\hat{A}_{kN} G_{kN}| \xi_k\ ,\ k \varepsilon J_I$$

$$m_{k,j} = [\hat{E}_{qk} |\hat{E}_{qj} G_{k,j} - \hat{E}_{dj} B_{k,j}| + \hat{E}_{dk} |\hat{E}_{dj} G_{k,j} + \hat{E}_{qj} B_{k,j}|] \hat{\xi}_{k,j}$$

$$\tilde{V}_{k,j} = |(\hat{E}_{qj} G_{k,j} - \hat{E}_{dj} B_{k,j}) \cos \delta^\circ_{k,j}| + |\hat{E}_{qj} B_{k,j} + \hat{E}_{dj} G_{k,j}|$$

$$V_{k,j} = \hat{E}_{qk} |\hat{E}_{qj} G_{k,j} - \hat{E}_{dj} B_{k,j}| \hat{\xi}_{k,j}\ ,\ \tilde{U}_{k,j} = |\hat{E}_{dj} B_{k,j} - \hat{E}_{qj} G_{k,j}|$$

$$n_{k,j} = |\hat{E}_{dk} G_{k,j}| \hat{E}_{dj} \hat{\xi}_{k,j}\ ,\ \bar{n}_{k,j} = |\hat{E}_{dk}| \hat{E}_{qj} B_{k,j} \hat{\xi}_{k,j}$$

$$S_{k,j} = A_{kj} B_{k,j} + \hat{A}_{kj} G_{k,j}\ ,\ k \neq j,\ k,j \varepsilon J_I$$

$$\rho_{k,j} = 2 \cos (2\,\theta_{k,j})\ ,\ d_k = \hat{E}_{qk} G_{kk}\ ,\ k \neq j\ ,\ k,j \varepsilon J_{IN}$$

$$\hat{m}_{k,j} = |\hat{E}_{qj} B_{kj} + \hat{E}_{dj} G_{kj}| \hat{\xi}_{kj}\ ,\ k \neq j,\ k \varepsilon J_I, j \varepsilon J_{IN}$$

$$\beta_j = a_I (A_{iI,j} \xi_{iI,j} + |\hat{E}_{diI}| \hat{E}_{qj} \hat{\xi}_{iI,j}) Y_{iI,j}\ ,\ \bar{\beta}_j = \beta_j (b_I / a_I)$$

$$\hat{\beta}_j = c_I (\hat{E}_{qj} \hat{\xi}_{iI,j} + |\hat{E}_{dj}| \xi_{iI,j}) Y_{iI,j}$$

$$\psi_j = \bar{a}_I (A_{iI+1,j} \xi_{iI+1,j} + |\hat{E}_{diI+1}| \hat{E}_{qj} \hat{\xi}_{iI+1,j}) Y_{iI+1,j}$$

$$\bar{\psi}_j = \psi_j (\bar{b}_I / \bar{a}_I)\ ,\ \hat{\psi}_j = \bar{c}_I (\hat{E}_{qj} \hat{\xi}_{iI+1,j} + |\hat{E}_{dj}| \xi_{iI+1,j}) Y_{iI+1,j}$$

$$\zeta_j = \hat{a}_I (A_{iI+2,j} \xi_{iI+2,j} + |\hat{E}_{diI+2}| \hat{E}_{qj} \hat{\xi}_{iI+2,j}) Y_{iI+2,j}\ ,\ \bar{\zeta}_j = \zeta_j (\hat{b}_I / \hat{a}_I)$$

$$\hat{\zeta}_j = \hat{c}_I (\hat{E}_{qj} \hat{\xi}_{iI+2,j} + |\hat{E}_{dj}| \xi_{iI+2,j}) Y_{iI+2,j}$$

$$H_j = a_N (A_{Nj} \xi_{Nj} + |\hat{E}_{dN}| \hat{E}_{qj} \hat{\xi}_{Nj}) Y_{Nj}$$

$$\hat{H}_j = c_N (|\hat{E}_{dj}| \xi_{Nj} + \hat{E}_{qj} \hat{\xi}_{Nj}) Y_{Nj}\ ,\ \alpha_j = a_I Y_{iI,j} \hat{E}_{iI}\ ,\ \bar{\alpha}_j = \alpha_j (b_I / a_I)$$

$$\hat{\alpha}_j = c_I Y_{iI,j} \xi_{iI,j}\ ,\ \gamma_j = \bar{a}_I Y_{iI+1,j} \hat{E}_{iI+1}\ ,\ \bar{\gamma}_j = \gamma_j (\bar{b}_I / \bar{a}_I)$$

$$\hat{\gamma}_j = \bar{c}_I Y_{iI+1,j} \xi_{iI+1,j}\ ,\ \eta_j = \hat{a}_I Y_{iI+2,j} \hat{E}_{iI+2}\ ,\ \bar{\eta}_j = \eta_j (\hat{b}_I / \hat{a}_I)$$

$$\hat{\eta}_j = \hat{c}_I Y_{iI+2,j} \xi_{iI+2,j}\ ,\ \phi_j = a_N Y_{N,j} \hat{E}_N\ ,\ \hat{\phi}_j = c_N Y_{N,j} \xi_{N,j}\ ,\ j \varepsilon J_K$$

$$\xi_j = |\cos \delta^\circ_{jN}|\ ,\ \hat{\xi}_j = |\sin \delta^\circ_{jN}|\ ,\ j \varepsilon J_I$$

$$\tilde{\xi}_{i,j} = |\cos \delta^\circ_{i,j}|\ ;\ \xi_{ij} = |\sin (\theta_{ij} - \delta^\circ_{ij})|\ ;\ \hat{\xi}_{ij} = |\cos (\theta_{ij} - \delta^\circ_{ij})|\ ,\ i \neq j,\ i,j \varepsilon J_I$$

$$U_{k,j} = Y_{k,j} |\hat{E}_{dk}| \hat{E}_{qj} \hat{\xi}_{k,j}\ ,\ \hat{U}_{k,j} = Y_{k,j} A_{kj} \xi_{k,j}$$

$$\bar{U}_{k,j} = Y_{k,j} (\hat{E}_{qj} \hat{\xi}_{k,j} + |\hat{E}_{dj}| \xi_{k,j})\ ,\ k \varepsilon J_I\ ,\ j \notin J_{IN}$$

تحليل الأتزان النتقالى لأنظمة القدره متعددة الألات مع الأخذ

فى الأعتبار إضمحلال مجال المولد

ملخص البحث:

تم فى البحث انجاز تحليل الأتزان النتقالى لنظام قدرة يحتوى على "ن" آله وذلك بأستخدام طريقة الفك والتراكب عن طريق دالة ليابونوف متجهة.

تم الحصول على النموذج الرياضى للنظام أخذين فى الأعتبار كل من الأخماد الميكانيكى الغير متماثل وتأثير إضمحلال مجال المولد. تم تمثيل كل مولد فى النظام بالنموذج الأحادى المحور والذى يفترض فيه أن إحدى مركبتى الجهد الداخلى للمولد تكون متغيرة مع الزمن.

تم فك نظام القدرة الى عدد { (ن – ١)/٣ } تحت نظام كل منها يشمل على أربعة ألات (إحداهما الأله المقارنه) – ومن ثم تم فك النموذج الرياضى للنظام الى عدد {(ن– ١)/٣ }تحت أنظمة كل منها من الدرجة الحادية عشر.

تم لكل تحت نظام حر أختيار دالة ليابونوف على شكل "صورة مربعة+ مجموع تكاملات ستة دوال غير خطية ". تم تكوين دالة ليابونوف متجهة , وبإستخدام هذه الدالة تم أجراء التراكب للنظام . تم الحصول على مصفوفة تراكب من الدرجة { (ن – ١)/٣ }, أتزان هذه المصفوفة يدل على الأتزان المقارب للنظام.

فى مثال عددى تم إستخدام معيار الأتزان المقدم فى أجراء دراسات الأتزان الأنتقالى لنظام قدرة مكون من عشرة آلات ويشتمل على أحد عشر قضيبا . أفترض عدة حالات لحدوث الخطأ كما يلى: قصر ثلاثى الأوجه عند نقطة قريبة من أحد قضبان النظام والذى يتصل به مولد– أضافة حمل إضافى فجائى عند أحد القضبان– فصل فجائى لخطين من النظام أثناء التشغيل العادى. لكل حاله من هذه الحالات تم بطريقة مباشرة حساب الزمن الحرج . وجد أن قيم الأزمنه الحرجه التى تم حسابها مساويه تقريبا للأزمنة التى تم حسابها بطريقة الخطوه خطوه.

وجد أيضا أن معيار الأتزان المقدم مناسبا وسهل تطبيقه على أنظمه القدرة متعددة الألات ويمكن أستخدامه لأجراء دراسات الأتزان العملية لهذه الأنظمة .