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A Method of Theodolite Installation at the Intersection of Two perpendicular lines

طريقة التوقيع الأمثل لنقطة تقاطع خطين متعامدين بواسطة جهاز التيودوليت

By

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الملخص العربي

إن مسالة توقيع نقطة تقاطع خطين متعامدين بواسطة جهاز التيودوليت في الطبيعة (مثل الحوائط ومحاور الأعمدة في أي مبنى إنشائي) مهمة جداً , وهذا التوقيع يتم بخطأ كبير خصوصا في الأطوال الكبيرة (في حدود اكبر من أربعين مترا).

في هذا البحث تم استنتاج معادلة بسيطة لتوقيع الانحراف عن نقط تقاطع محاور الأعمدة أو محاور الحوائط في المنشآت الهندسية في اتجاه المحورين السيني والصادي بدقة عالية وبسهولة .

تم استخدام المعادلة المستنتجة في هذا البحث في تطبيق عملي بواسطة تيودوليت دقيق وأخر أقل دقة وتم توقيع نقط التقاطع بدقة عالية, وأعطي نتائج جيدة.

يوصي باستخدام المعادلات المستنتجة في التطبيق العملي.

Keywords: Installation, intersection, coordinate reticulation.

Abstract

The problem of theodolite installation at the intersection of two perpendicular lines is very important to set walls and columns in any structural building. In this paper there is a simple solution for laying out a point corresponding to the crossing of principle building axes. Besides to, the accuracy evaluation of this point is available. This is by

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carrying out two examples for evaluating the most probable position of this point by theodolite 2T30 and T5.

Introduction

When carrying-out detailed stakeout operations or "as-built " surveys it is sometimes necessary to set a theodolite at a point of crossing of construction axes, paint-marked on internal walls or on the columns of a building. This problem was met by the author of the present paper while executing as-built survey of the roof covering a building. An evaluation of on-site conditions has shown that the most convenient way to carry out the measurements was to choose a point corresponding to the crossing of principal building axes paint-marked on counter-facing columns inside of the building. In such a situation it was necessary to project the position of that point to the floor of the building and to mark it. This paper presents a new solution of such a problem.

Analysis of the Problem

Suppose that the position of a point 0 at the intersection of two perpendicular lines AB and CD should be found, and that it is impossible to set a theodolite at the points A,B,C and D (see Fig. 1). First of all, the position of point T located in the vicinity of point 0 is to be determined using a cross-staff or a "by-eye" evaluation[1]. After setting a theodolite at this point the horizontal angles $\beta_1$and β_2 and distances S_1,S_2,S_3 and S_4 should be measured. Taking point 0 as the origin of coordinate system we could calculate the coordinate of point T using the following formulae based on the geometry[2] shown in Figure 1:

$$x_T \cong \frac{s_1 s_2 \alpha_1}{(s_1 + s_2)\rho}; \qquad (1)$$

$$y_T \cong \frac{s_3 s_4 \alpha_2}{(s_3 + s_4)\rho}; \qquad (2)$$

$$r_T = \sqrt{x_T^2 + y_T^2}. \qquad (3)$$

where:

S_1,S_2,S_3 and S_4 are the horizontal distances between the point T and points A,B,C and D respectively; α_1 and α_2 are the horizontal angles calculated with the use of the following formulae:

$$\alpha_1 = 180^0 - \beta_1; \qquad (4)$$

$$\alpha_2 = 180^0 - \beta_2. \qquad (5)$$

Distances S_1,S_2,S_3 and S_4 are within the limits enabling to measure them with a relative error 1:200 using stadia range finder or in any another way. Horizontal angles β_1 and β_2 are calculated as follows:

$$\beta_1 = \alpha_A - \alpha_{C,} \qquad (6)$$

$$\beta_2 = \alpha_B - \alpha_{D,} \qquad (7)$$

where:

α_A, α_B, α_C and α_D are the readings of the horizontal circle of a theodolite.
In order to improve the accuracy of β_1 and β_2 angles, it is necessary to measure them at two positions (L and R) of the telescope and to average the values to get the final results.
The values and signs of angles β_1, β_2,α_1 and α_2 enable to determine the geodetic quadrant .
To fix the point 0 we could use values x_T and y_T characterizing the relative positions of 0 and T . That could be done with the use of coordinate reticulation (CR) drawn on a dense paper sheet or on a tracing-paper at the scale of 1:1 (see fig. 2)

This coordinate reticulation is to be placed on the floor in such a way that the point T marked on the floor matches with its location on the coordinate reticulation. Rotating CR around the point T marked on the floor we could make the lines ab and cd of CR match with corresponding axes AC and BD . After that, the point 0 should be marked on the floor.

In order to control the results, a theodolite is to be installed over the newly-marked point 0 and the measurements described above are to be repeated. If the results of these measurements give the value of r_T that does not differ from the calculated with the use of formula (3) within admissible limits, it could be stated that the procedure is successful. Otherwise, it will be necessary to repeat the operation and correct the point 0 position. Admissible value of δ_T depend on a specific task and on the requirements of Construction Norms and Rules. The final position of the point 0 is then marked on the floor by paint.

To measure the horizontal angles β_1 and β_2, it is recommended to use the method of repeated angular measurements with a misclosure control. Our experience has shown that all the procedures including measurements, calculations and fixing point 0, take less than an hour.

The accuracy evaluation of point 0 positioning mainly depends on that of angles β1 and β2 and in a smaller rate – on other factors. To calculate an approximate value of δT characterizing an admissible error of point T position, a following formula could be used. By differentiating the previous equations (1) and (2), we will obtain:

$$dx_T = \frac{S_2^2\beta_1(1-S_1)}{\rho(S_1+S_2)^2}dS_1 + \frac{S_1^2\beta_1(1-S_2)}{\rho(S_1+S_2)^2}dS_2 + \frac{S_1S_2}{\rho(S_1+S_2)}d\beta_1.$$

Passing from the differentials to the finite increments, and then to the root-mean-square errors, we will obtain:

$$m_{x_T}^2 = \left.\begin{array}{l} \dfrac{S_2^4 \beta_1^2 \left(1-S_1\right)^2}{\rho^2 \left(S_1+S_2\right)^4} m_{S_1}{}^2 + \\ + \dfrac{S_1^4 \beta_1^2 \left(1-S_2\right)^2}{\rho^2 \left(S_1+S_2\right)^4} m_{S_2}{}^2 + \\ + \dfrac{S_1^2 S_2^2}{\rho^2 \left(S_1+S_2\right)^2} m_{\beta_1}{}^2 \end{array}\right\} \quad (8)$$

We will analogously have:

$$m_{y_T}^2 = \left.\begin{array}{l} \dfrac{S_4^4 \beta_2^2 \left(1-S_3\right)^2}{\rho^2 \left(S_3+S_4\right)^4} m_{S_3}{}^2 + \\ + \dfrac{S_3^4 \beta_2^2 \left(1-S_4\right)^2}{\rho^2 \left(S_3+S_4\right)^4} m_{S_4}{}^2 + \\ + \dfrac{S_3^2 S_4^2}{\rho^2 \left(S_3+S_4\right)^2} m_{\beta_2}{}^2 \end{array}\right\} \quad (9)$$

In that specific case when $S_1 = S_2$, $S_3 = S_4$ and $m_{\beta_1} = m_{\beta_2} = m_\beta$ formulae (8) and (9) are as follows:

$$m_{x_T}^2 = \left.\begin{array}{l} \dfrac{\alpha_1^2 \left(1-S_1\right)^2}{8\rho^2} m_S{}^2 + \\ + \dfrac{S_1^2}{4\rho^2} m_\beta{}^2. \end{array}\right\} \quad (10)$$

and

$$m_{y_T}^2 = \left.\begin{array}{l} \dfrac{\alpha_3^2 \left(1-S_3\right)^2}{8\rho^2} m_S{}^2 + \\ + \dfrac{S_3^2}{4\rho^2} m_\beta{}^2; \end{array}\right\} \quad (11)$$

$$m_T = \sqrt{m_{x_T}^2 + m_{y_T}^2}. \quad (12)$$

Or:

$$m_T = \frac{1}{2\rho}\left\{\begin{array}{l} \sqrt{\dfrac{\beta_1^2 \left(1-S_1\right)^2}{2} m_S^2} + \\ + \sqrt{\dfrac{\beta_2^2 \left(1-S_3\right)^2}{2} m_S^2} + \\ + \sqrt{\left(S_1^2 + S_3^2\right) m_\beta^2} \end{array}\right\}. \quad (13)$$

Error for distances is very small and may be neglected , then we have:

$$m_T = \frac{m_\beta}{2\rho} \sqrt{(S_1^2 + S_3^2)}. \quad (14)$$

Where:

m_β is a threshold error for the measurement of angles β_1 and β_2. In that specific case when $S_1 = S_2$ and $S_3 = S_4$ formula (14) looks like follows:

Example:

Theodolite: 2T30

$m_\beta = 1'$, $S_1 = S_2 = 18.0$ m, $S_3 = S_4 = 32.0$ m

Using these values in formula (9) we get:

$$m_T = \frac{1'}{\rho}\sqrt{(18^2 + 32^2)} \cong$$
$$\cong 0.005m = 5.00mm$$

For the theodolite T5 the value of m_β is equal to 10". In this case for the same values of S_1 and S_3 we have:

$$m_T = \frac{10''}{\rho}\sqrt{(18^2 + 32^2)} \cong$$
$$\cong 0.0009m = 0.90mm$$

Tables 1 and 2 show an example of field log of measurements carried out in laboratory conditions, as well as the results of their processing.

Conclusions

As accuracy analysis showed, this procedure of lay out of a point corresponding to the crossing of principle building axes ensures high accuracy and possesses high effectiveness.

This method ensures both the control in the work and will lead to further increase in the accuracy of the alignment .

References

[1] Ivanov B.E., 2006 ,"On the criterion of the accuracy of finding position ", // Geodesy and cartography", No. 1, c.5-11, Moscow, 2006.

[2] Gyuzivitsh S.N., 2006," On the inaccuracies in the formulation of the problems in geodesy and their elimination", Geodesy and cartography", No. 1, c.14-20, Moscow, 2006.

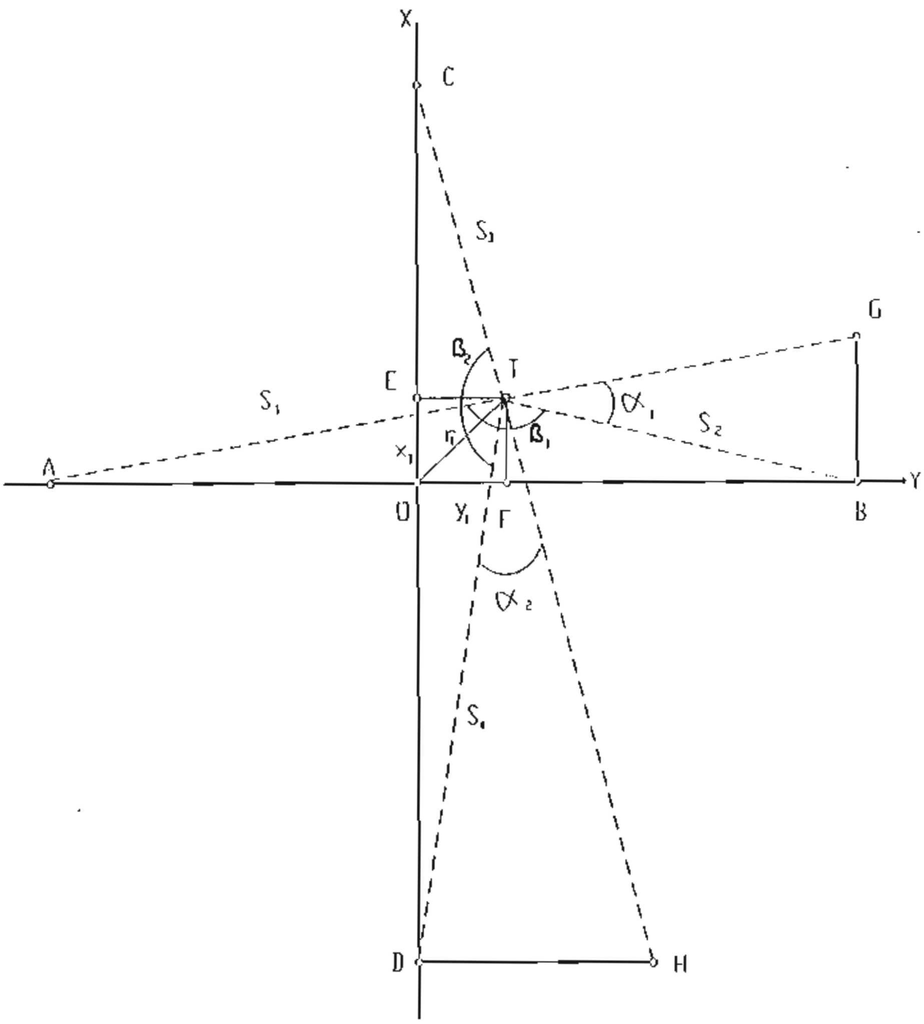

Figure 1 Geometry of measurements

Table 1

Field Measurements Log							
Date: 24.08.2007				Theodolite: 2t5k			
Station	Points	S, m	Horizontal circle reading		Average reading	Reduced direction	2C= L-R
			L	R			
T	A	3.60	0^0 06.5′	180^0 06.5′	0^0 06.6′	0^0 00.0′	-0.30
	B	2.81	90^0 00.9′	270^0 01.3′	90^0 01.1′	89^0 54.5′	-0.40
	C	3.66	178^0 50.8′	358^0 51.0′	178^0 50.9′	178^0 44.3′	-0.20
	D	3.22	269^0 00.0′	89^0 00.4′	269^0 00.2′	268^0 53.6′	-0.40
	A	3.60	0^0 06.5′	180^0 06.8′	0^0 06.6′	0^0 00.0′	0

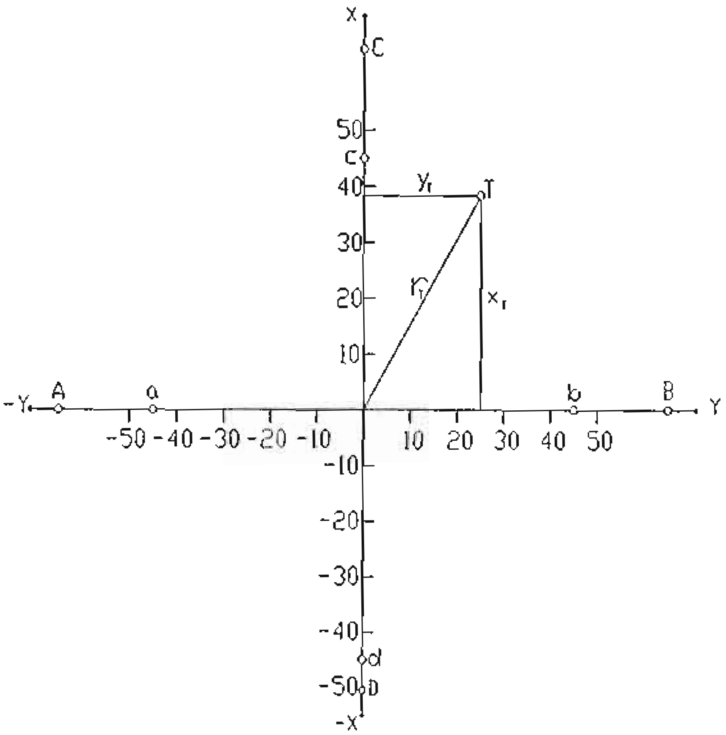

Figure 2 Fixing point 0 with the use of coordinate reticulation (CR)

Table 2

Processing Log									
Station	Points	S, m	Horizontal angles				Coordinates, m		
			β_1	β_2	α_1	α_2	x	y	r
T	A	3.60	181^0 15.7′	181^0 00.9′	-1^0 15.7′	-1^0 00.9′	-40.90	-26.60	47.5
	B	2.81			$x_T \approx 0.04000$m = 40.00 mm				
	C	3.66			$y_T \approx 0.0266$m = 26.60 mm				
	D	3.22							