Menofia University

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Academic Year : 2013- 2014

Department: Basic Eng.

Year : 2_nd Elect. Dept.

Subject : Engg.Mathematics

Time Allowed : 3 hours

Date : 5/1/2014

Answer all the following questions: [70] Marks

Q1 (40 Marks)

(i) Determine the values of a and b so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1,-1,2).

(ii) Find the directional derivative of $\nabla \cdot \bar{u}$ at the point $(4,4,2)$ in the direction of the corresponding outer normal of the sphere $x^2 + y^2 + z^2 = 36$, where $\bar{u} = x^4 i + y^4 j + z^4 k$.

(iii) Verify the Green's theorem for $\int_c (y - \sin x)\,dx + \cos x\,dy$; where c is the triangle bounded by $y = 0, x = \frac{\pi}{2}$ and $y = \frac{2}{\pi} x$.

(iv) Verify the divergence theorem for $\bar{F} = (xy + y^2) i + x^2 y\, j$ and the volume v in the first octant bounded by $x = 0, y = 0, z = 0, z = 1, x^2 + y^2 = 4$.

(v) Verify Stokes' theorem for $\bar{F} = (2y + z) i + (x - z) j + (y - x) k$ for the part of $x^2 + y^2 + z^2 = 1$ lying in the positive octant.

Q2 (30 Marks)

(a) Evaluate:

$$\Gamma(1/3)\,\Gamma(2/3)\,,\ \Gamma(5/2),\ \Gamma(-5/2),\ \int_0^{\infty} x^{10} e^{-5x}\,dx\,,\ \int_0^{4} x^{4}\sqrt[3]{64-x^{3}}\,dx$$

(b) Prove that $B(m,n) = \int_0^{\infty} \frac{y^{m-1}}{(1+y)^{m+n}}\,dy$, then find $\int_0^{\infty} \frac{x^4}{(1+x)^{15}}\,dx$.

(c) Prove that $B(m,n)=\frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$.

(d) Define the following expressions:

Extreme point solution - Basic solution - Convex set - Hyper-plane Line segment.

(e) Prove that the set of hyper-plane is a convex set.

(f) Solve the following Linear Programming Problem Using:

(i) Graphical method.

(ii) The Simplex method.

$$\min\ 3x_1 + 3x_2$$

$S.t$

$$2x_1 + \frac{1}{2}x_2 \ge 10$$

$$2x_1 \ge 4$$

$$4x_1 + 4x_2 \ge 32$$

$$x_1, x_2 \ge 0$$

Good Luck