بسم اله الرحمن الرحيم

EL-Mansoura University	**Preliminary MSc Exam**
Faculty of Engineering	**Subject: Stachastic Control Systems**
Computers & Systems Eng. Dept.	**Time Allowed: 3 Hours**

10 A.M., Wednestday, 11th Sept. 2013

Attempt the following questions: **Max. Marks (100)**

1- (a) Define the following terms:

1- Positive definite quadratic function (give example).

2- State transition matrix. (8 marks)

(b) Find the state space representation for the system represented by the following dynamic equation:

$$\dddot{Y}(t) + 20\ddot{Y}(t) + 116\dot{Y}(t) + 160Y(t) = 3u(t)$$

where $Y(t)$ = system output, $u(t)$ = system input

Have you another method for representing for the system in state space?

N.B.: Use the transformation matrix method. Then, construct the system block diagram in both cases. (22 marks)

2- (a) Derive the expression for representing a control system having the following dynamic equation:

$$y^n(t) + a_1 y^{n-1}(t) + a_2 y^{n-2}(t) + \cdots + a_{n-1}\dot{y}(t) + a_n y(t)$$
$$= b_0 u^n(t) + b_1 u^{n-1}(t) + \cdots + b_n u(t)$$

(10 marks)

(b) Define the following terms:

i- Non-Homogenous state equation. ii- Non-singular matrix

iii- Independent variables. (5 marks)

(c) A control system has the following dynamic equations:

$$\dddot{Y}(t) + 11\ddot{Y}(t) + 36\dot{Y}(t) + 36Y(t) = 4\dot{u}(t) + 5u(t)$$

Find the state space representation of the system, then construct the system block diagram . (15 marks)

3- (a) Define the following terms:

i- Completely controllable system. ii- Completely observable system.

5 marks)

(b) Investigate the complete observability for the given below system:

$$\dot{x} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

P.T.O.

$$y = \begin{bmatrix} 4 & 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$ (8 marks)

(c) Solve the folloing state equation:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Given that: $x_1(0) = 1, \; x_2(0) = -2 \; \& \; u(t) = 2$
Sketch $x_1(t)$ and $x_2(t)$. (17 marks)

4- (a) Define the following terms:

i- Performance Index.
ii- Adaptive Control System.
iii- Invariance property.
iv- Asymptotic stability in large

(8 marks)

(b) Investigate the stability for the following system using the second method of Lyapunov:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

N.B.: Use a Lyapunov function in the form:

$$V(x) = x^* P x$$

What about the asymomtotic stability in large?? (22 marks)

5- (a) State the main advantages of variable structure systems and its disadvntages (7 marks)

(b) A control system gas the state space representation:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & -3\xi \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \& \quad x(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Find the optimal value of ξ when the performance index:

$$J = \int_0^\infty x^*(t) Q x(t) dt \qquad \text{is minimized}$$

where Q is a positive definition matrix $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(23 marks)

With my best wishes,
Prof. Dr. Fayez F.G. Areed
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