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Turbulence Models for Numerical Simulation of Wall Turbulent Flows

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نماذج الاضطراب للمحاكاة العددية للجريان الاضطرابي الجداري

<u>ملخص</u>

يستعرض هذاالبحث بعض المحاولات العديدة التي تمت لتمثيل الاضطراب من اجل تحسين التنبؤات العددية للجريان الاضطرابي الجداري. وقد تم التركيز على نموذج الاضطراب k-ε الاكثر شيوعا واستعمالا. كما ونوقشت التحسينات التي اجريت على معادلات الجدار التي تستخدم مع نموذج الاضطراب k-ε الاساسي وتأثيرات عدم سوي الخواص على اللزوجة المضطربة قرب الجدار والتعديلات المختلفة المتعلقة بتأثيرات رقم رينولدز المنخفض والمطبقة على نموذج k-ε . اضافة لذلك، يشير البحث الى مدى استخدامات بعض نماذج الاضطراب الاخرى ذات الاهمية للجريان الجداري. وقد تبين بان نموذج الاضطراب k-ε الاساسي المقترن مع المعادلات الجدارية يؤدي الى السرعة والاقتصاد في استخدام الحاسوب، بينما نماذج k-ε ذات رقم رينولدز منخفض تنتج عدديا جريانا اضطرابيا جداريا اكثر دقة.

Abstract

Many turbulence models have been invalidated by the complex structure of wall turbulent flows that exhibit separation, swirl or low-Reynolds number effects. This paper, introduces some of the various attempts that have been made to model turbulence in an endeavour to improve numerical predictions of wall turbulent flows. Emphasis is given to the k-ε model of turbulence which is the most extensively used and accepted model today. Furthermore, the improvements made to the wall functions that are coupled with the standard k-ε model, and the nonisotropic effects taken on the eddy viscosity near walls have been discussed, and the various modifications applied to the k-ε model that refer to low-Reynolds number effects reviewed. In addition, the range of applicability of some of the other models having importance to wall shear flows has been pointed out. The wall functions in conjunction with the standard k-ε model provide computational expediency and economy, while the low-Reynolds number k-ε forms numerically reproduce wall turbulent flows more closely.

Introduction

Numerical simulation of physical processes has been gaining in importance recently as systematic investigations are usually expensive and time consuming under realistic conditions. By numerical simulation new model concepts and parameters are varied speedily, and comprehensive information is obtained at little cost in time and money. Unfortunately, most of the flow phenomena of importance to engineers involve turbulence, the mechanism of which has an extremely complicated nature, and is time and space dependent. Numerical predictions may not characterize the local state of turbulent flows completely, however they are practically needed for engineers to be able to predict the behaviour of such flows quantitatively.

Research workers have been actively engaged in numerical predictions of the subject of turbulence for more than four decades. In these decades engineers learned to predict laminar flows. However the desire to extend to turbulent flows is fulfilled to a considerable extent by inventing turbulence models. These consist of sets of differential equations, and associated algebraic ones and constants, the solutions of which simulate closely the averaged character of real turbulent flows. These models may consist of zero (an algebraic form of the eddy viscosity), one, two, three transport equations or more. Besides the Prandtl mixing-length model, Baldwin and Lomax [1] proposed an algebraic eddy viscosity model that has a wide variety of applications. The one equation model was first proposed by Prandtl [2], who employed the specific turbulence energy, k, while the length scale representing the macroscale of turbulence, l, was taken as proportional to the distance from the wall. Saffman [3] formulated a two-equation model, the differential equations of which were for two properties of turbulence, k, and the specific dissipation rate, ω. Harlow and Nakayama [4] proposed the first form of the k-ε model where ε is the dissipation rate of turbulence energy. A form of the k-ε model for low Reynolds number terms was first developed by Jones and Launder [5]. Rotta [6] solved simultaneously three differential equations, a shear stress, τ, one and the equations for k and for kl product. Models of more than three equations such as that of Daly and Harlow [7] in which k was replaced by the three specific energies $\overline{u'^2_i}$, $\overline{u'^2_j}$, and $\overline{u'^2_k}$, had very little use has been made of them. Other workers applied the various models to some physical processes of turbulent nature, such as Miller and Crawford [8] who employed the Prandtl mixing-length concept, Boyle [9] used a modified Baldwin and Lomax eddy viscosity model, Gibson and Spalding [10] used the k-ω model, Rodi and Spalding [11] used the k-kl model, Iacovides and Launder [12] used the standard k-ε model, whereas Rodi and Scheuerer [13], Schönung and Rodi [14] and Tafti and Yavuzkurt [15] employed the low-Reynolds number k-ε version.

The models with more than two transport equations have not yet been thoroughly tested due to their costly operation on computers. The two-equation models of turbulence are well established and there is a considerable amount of evidence in support for them. However, the k-ε model is preferred to the other two-equation models which incorporate a length scale formulation besides the turbulent kinetic energy, since the length scale relation must be adapted to the particular flow being computed, which is a major disadvantage [16]. Furthermore, the k-ε turbulence model is the most widely used and accepted two-equation model of turbulence today. It seems appropriate, therefore, to present in this work a more detailed description of the aspects of the k-ε model having importance for flows adjacent to solid walls in addition to pointing out the deficiencies and strengths of some of the other models.

Classification of Turbulence Models

Turbulence models generally fall in two main categories:

- Eddy viscosity models, and
- Reynolds stress models.

Classification of the turbulence models is given by Reynolds [17]. In general, the models can be classified as follows:

i) Zero-equation or algebraic eddy viscosity model. The model employs an algebraic form of the eddy viscosity which is based on the law of the wall or mixing length concept. The model is valid for isotropic two-dimensional flows, and its application to turbulent flows [1, 18, 19] had indicated the following conclusions:

- The model is adequate for two-dimensional compressible flows with mild pressure gradients, and can be suitable for three-dimensional boundary layers with small cross flows.
- The model is not suitable for complex shear flows, ie. for flows with separation, curvature, and rotation.

ii) One-equation model. The model employs an additional partial differential equation relating the kinetic energy of turbulence. The model is suitable to simple turbulent flows as those of the eddy viscosity models [17, 20]. However, Johnson and King [21] attempted to revive the model for complex flows to predict the two-dimensional separated flows and high-speed flows. The prediction of two-dimensional separated flows in a diffuser [21], shown in Fig. 1, indicates better mean velocity profile prediction than that of the algebraic eddy viscosity model.

iii) Two-equation model. The model employs two partial differential equations describing two properties of turbulence, k and ε, or k and ω. The k-ω model has only been used by a limited number of researchers [23, 24] compared to the extensively used k-ε one [16, 18, 19, 25, 26, etc.]. The k-ε model is much superior to the algebraic eddy viscosity or one-equation model in mildly complex flows. A good example of this is the prediction of the flow in a wing/body junction [27]. When a no-slip boundary condition was used, the significantly better predictions of the streamwise velocity by the k-ε model are shown in Fig. 2.

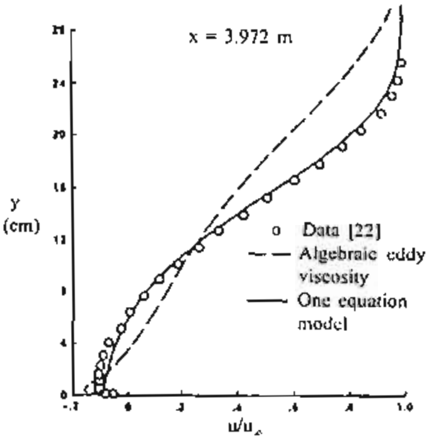

Fig. 1 Comparison of mean velocity profile for a low speed diffuser flow.

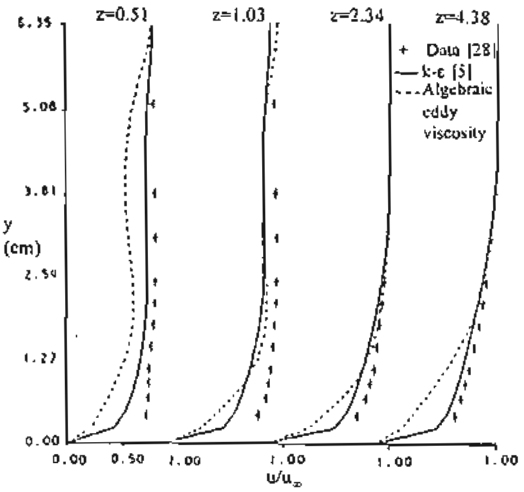

Fig. 2 Comparison of streamwise velocity for a no-slip boundary conditions at x=0.647 on a wing/ body junction.

Based on the results extracted from the various references on the standard k-ε model (without modifications) and k-ω model predictions, one may conclude the following:

- The model is adequate for attached two-dimensional boundary layer flows, two-dimensional flows with pressure gradients, and three-dimensional flows with very mild cross flows.
- The model is inadequate for separated flows, three-dimensional flows with significant cross flows and swirl, and complex flows (flows with rotation, curvature, etc.).

It seems that the two-equation models without modifications fail to capture many of the features associated with complex flows. Several attempts, therefore, have been made to modify the models to extend their range of validity to complex flow situations [5, 29, 30, etc.].

iv) Three-equation model or more (Reynolds stress model). The model employs three partial differential equations utilizing the k-ε model equations along with a Reynolds stress $\overline{u'_iu'_j}$ equation, or several partial differential equations for the components of the turbulence stresses $\overline{u'_iu'_j}$. This is one of the most complex models in use today. The model is under extensive development and is used in complex flow situations [31-35], e.g., three-dimensional flows, flows with curvature and rotation, and blowing and suction. This model is essential if details of the turbulence as well as accurate flow prediction are needed. However, numerical computation involving this model is expensive.

Generally, the zero-, and one-equation models are still used in practical engineering applications involving simple shear flows. The two-equation model, the k-ε one in particular, is employed when more accuracy and additional details on turbulence quantities are needed. For more accurate predictions of the complex flowfield, one would resort to the complicated Reynolds stress transport equation, but this would be on the expense of costly computer time and storage and even beyond the capability of present-day computers when the solution of three-dimensional flows is sought.

The advantages and simplicity of the k-ε model (as compared to the Reynolds stress model), however, should not be overlooked, and it seems that the k-ε model is the best compromise at the present time between acceptable computer outlay and accuracy of the simulation of the turbulent flow phenomena.

k-ε Model of Turbulence

The k-ε model of turbulence describes the generation, transport and dissipation of the turbulent velocity fluctuations $(-\rho\overline{u'_iu'_j})$ the additional shear stresses, and $(-\rho\overline{u'_iT'})$ the turbulent heat fluxes. The model utilizes the eddy viscosity concept that relates the turbulent stress to the mean rate of strain and involves two parameters, the turbulence kinetic energy k, and its dissipation rate ε.

According to the k-ε model the turbulent stresses may be expressed in Cartesian tensor notations [16] as:

$$-\rho\overline{u'_i u'_j} = \mu_t\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\rho\delta_{ij}k$$

where, ρ = the fluid density,
δ_{ij} = the Kronecker delta,
μ_t = the isotropic eddy (turbulent) viscosity,
$(2/3)k$ = can be thought of as the additional pressure resulting from turbulence [36].

The turbulent heat fluxes are similarly expressed as:

$$-\rho\overline{u'_i T'} = \frac{\mu_t}{Pr_t}\frac{\partial T}{\partial x_i}$$

in which Pr_t is the turbulent Prandtl number.

The eddy viscosity introduced above is related to the turbulent kinetic energy k and to its rate of dissipation ε through the Kolmogorov-Prandtl relation as:

$$\mu_t = \rho\, C_\mu \sqrt{k}\, L_\varepsilon = \rho\, C_\mu\, k^2/\varepsilon \qquad \ldots (1)$$

where $L_\varepsilon = k^{3/2}/\varepsilon$, eddy length scale, and
$C_\mu = 0.09$, empirical constant [16].

The governing transport equations for the k-ε model of turbulence may be written in tensor notation [37] as:

$$\frac{\partial k}{\partial t} + u_j\frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu_l + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] - \overline{u'_i u'_j}\frac{\partial u_i}{\partial x_j} - \varepsilon + D \qquad \ldots (2)$$

$$\frac{\partial \varepsilon}{\partial t} + u_j\frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu_l + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] - C_{\varepsilon 1}f_1\frac{\varepsilon}{k}\overline{u'_i u'_j}\frac{\partial u_i}{\partial x_j} - C_{\varepsilon 2}f_2\frac{\varepsilon^2}{k} + F \qquad \ldots (3)$$

where, ν_l and ν_t are the laminar and turbulent kinematic viscosities respectively, the empirical constants take these values as recommended in [16]; $C_{\varepsilon 1}=1.44$, $C_{\varepsilon 2}=1.92$, $\sigma_k=1.0$, $\sigma_\varepsilon=1.3$, and f_1 and f_2 are functions, and D and F are extra terms all included for the purpose of improving predictions in the wall region, or for computational expediency.

These equations are a general form of the standard high-Reynolds number form given by Launder and Spalding [16]. In the standard k-ε form viscous diffusions in equations (2) and (3) are neglected, the functions f_1 and f_2 are both assumed to be identically unity, and the extra terms D and F are ignored.

The semi-empirical transport equations for the standard k-ε model (high-Reynolds number form) as given by [16] are:

$$u_i\frac{\partial k}{\partial x_i} = \frac{\partial}{\rho\partial x_i}\left[\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right] + \frac{\mu_t}{\rho}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right]\frac{\partial u_i}{\partial x_j} - \varepsilon \qquad \ldots (4)$$

$$u_i\frac{\partial \varepsilon}{\partial x_i} = \frac{\partial}{\rho\partial x_i}\left[\frac{\mu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right] + \frac{C_{\varepsilon 1}\varepsilon\mu_t}{k\rho}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right]\frac{\partial u_i}{\partial x_j} - \frac{C_{\varepsilon 2}\varepsilon^2}{k} \qquad \ldots (5)$$

The high-Reynolds number form of the k-ε model has been applied to predict a wide variety of flows; simple flows, boundary layers, etc.. It is worth noting, however, that the standard k-ε suffers from the following simplifying assumptions:

i) The k-ε model adopts an isotropic turbulent viscosity, i.e. it assumes the mean velocity fluctuations to be of equal magnitude in all three directions of space. Although this simplification is never encountered in reality, this model generally gives a good and satisfactory predictions of turbulence properties values in simple flow and mildly complex flow situations.

ii) In the near wall region the model overestimates the generation of turbulence k [38]. Additional destruction terms in the k-transport equation near the wall region are therefore required to yield reasonable predictions to match available experimental data.

iii) The model is valid only for high-Reynolds number flows, i.e. fully turbulent flows where viscous diffusions are neglected. However, for flows close to walls, there are always viscous sublayers where the viscous effects predominate over the turbulent ones, this model is then used in conjunction with the empirical wall functions.

Wall-Function Treatment

In the standard k-ε model, viscous diffusions are neglected and empirical wall functions are used to bridge the viscous sublayer. This is accomplished by relating the velocity components at the first grid node outside this layer to the wall shear stress via the logarithmic law of the wall. A uniform shear stress prevails in this viscous layer, and generation and dissipation of energy are in balance there via the assumption that the turbulence is in a state of equilibrium.

Standard Wall Functions

The standard wall-function for turbulent flows [16] is, namely, the logarithmic law,

$$C_f = [k_v / \ln(E\ Re\ C_f^{0.5})]^2, \text{ for } Re > 132.5 \qquad \ldots (6)$$
$$C_f = 1 / Re \text{ , otherwise.}$$

where C_f is the skin-friction factor ($=\tau_w/\rho u^2$, where τ_w is the wall shear stress and u is the velocity parallel to the wall), k_v is the Von Karman constant, taken to be 0.435, and E is the smooth-wall value of 9.0. The Reynolds number, Re ($=uy/\nu_l$), is based on the resultant velocity parallel to the wall, on the distance from the wall to the grid node, y, and on the laminar kinematic viscosity, ν_l. The limit of Re of 132.5 is that at which the laminar and turbulent wall-functions intersect,. The standard laminar wall-function is that of the Blasius law, $C_f = 0.009/Re^{0.25}$.

The Stanton number, St, is given by the empirical law of Jayatillika [39],

$$St = C_f / [Pr_t(1 + P_m C_f^{1/2})], \text{ for } Re > 132.5 \qquad \ldots (7)$$

where, P_m is the smooth-wall sublayer resistance function, a semi-empirical formula given by [16],

$P_m = 9(Pr/Pr_t - 1)(Pr_t/Pr)^{1/4}$

Pr and Pr_t are the viscous and turbulent Prandtl number respectively.

For $Re \leq 132.5$, St is simply $= C_f / Pr$.

The heat flux at the wall, q_w, is then deduced from,

$$q_w = St\ \rho\ u\ (h_p - h_w)$$

where h_p is the enthalpy at the grid node in question, and h_w is the enthalpy corresponding to the prescribed wall temperature.

When local equilibrium conditions prevail in the near-wall layer, the near-wall grid node values of k and ε are fixed to the following empirical correlations via the incorporated logarithmic-law option applicable to smooth walls,

$$k_w = u_\tau^2 / 0.3, \text{ and} \qquad \ldots (8)$$
$$\varepsilon_w = u_\tau^3 / (k_v\ y) = 0.09^{3/4}\ k_w^{3/2} / (k_v\ y) \qquad \ldots (9)$$

where u_τ is the friction velocity ($=(\tau_w/\rho)^{1/2} = uC_f^{1/2}$).

The standard wall functions, however, suffer from two deficiencies:

i) they are restricted to smooth walls, but rough walls may be important as in heat exchangers where the heat transfer surfaces are intentionally roughened in order to increase the heat transfer rate, and

ii) the Stanton number is calculated via a friction factor manipulation. This treatment was found to be appropriate for boundary-layer type flows where the near-wall layer is in local equilibrium. For flows with recirculation (as, for example, at the reattachment point at the downstream end of the recirculation zone engendered by a sudden enlargement of a pipe diameter), the predicted Stanton number there was found to be negligible. Measurements, however, have indicated that the Stanton number is actually at maximum at the reattachment point. This deficiency in the standard wall functions is due to the fact that in regions where the flow separates from the wall the shear stress and hence friction velocity is zero. Since the computed Stanton number was directly proportional to the friction velocity the calculations resulted in an incorrectly predicted zero heat flux and Stanton number.

To overcome the aforementioned deficiencies in the standard wall functions, a more adequate wall-function treatment is therefore needed. Launder and spalding [16] proposed a wall-function method which can be applied to rough walls as well as smooth walls in which pressure gradient phenomena can be accounted for, and improves the predictions of heat transfer at reattachment points. This method is called the generalized wall functions.

Generalised Wall Functions

The generalised wall functions adopt the method of Launder and Spalding [16], the main feature of which is based on a modified log-law that uses the turbulent kinetic energy as the characteristic velocity scale, rather than the friction velocity.

Adoption of the practices in [16] leads to the prediction of finite values of k and of wall heat flux at a reattachment point.

The standard log-law of the wall for C_f is generalised by expressing u_τ in terms of a velocity scale calculated from the local k, thereby finite fluxes are predicted, even where the fluid velocity is zero.

The C_f log-law, eq. (6), can be written as,

$$C_f = k_v\, C_f^{1/2} / \ln(E\, Re\, C_f^{1/2})$$

since $C_f^{1/2}=u_\tau/u$, and $u=Re\, v_l/y$ it follows,

$$C_f = k_v\, (u_\tau/u) / \ln(E\, u_\tau\, y/v_l)$$

and since $u_\tau=0.3^{1/2}k^{1/2}$ from the near-wall cell value of k, eq. (8), substitution for u_τ gives the generalised log-law of the wall,

$$C_f = k_v\, 0.3^{1/2}k^{1/2} / [u \ln(E\, 0.3^{1/2}\, k^{1/2}\, y / v_l)]$$

Substitution for $C_f^{1/2}$ and u_τ in the Stanton number formula, eq. (7), gives the generalised form of St,

$$St = C_f / [Pr_t(1 + P_m\, C_f\, u / 0.3^{1/2}\, k^{1/2})]$$

The value of k at the near-wall grid cells is not fixed in this option, and is calculated from its regular transport equation. However, in the source term for k (eq. 4), the dissipation rate for the near-wall cells is fixed to,

$$\varepsilon = 0.09^{3/4}\, k^{1.5} \ln(E\, 0.3^{1/2}\, k^{1/2}\, y / v_l) / (2\, k_v\, y)$$

When the turbulence is in local equilibrium, away from separated regions, the above expression recovers the near-wall empirical correlation of k in the standard wall functions.

The wall roughness is allowed for by using Jayatillika [39] empirical formulae for E, which expresses E as functions of the roughness Reynolds number, Re_r, defined as:

$$Re_r = u_\tau e / \nu_l$$

in which e is the absolute roughness height.

The formulae for E are as follows:

when $Re_r < 3.7$, E = the smooth wall value of 9.0;
when $3.7 < Re_r < 100$, $E = 1/[a(Re_r/29.7)^2 + (1-a)/81]^{0.5}$;
and when $Re_r > 100$, $E = 29.7 / Re_r$;

where, $a = (1+2X^3-3X^2)$; and
$X = 0.02248\,(100-Re_r)/Re_r^{0.564}$.

Allowance for rough walls in the calculation of Stanton number is accounted for by replacing the sublayer resistance function for smooth walls P_m by P, an empirical formula appropriate to rough walls [39], in which

$$P = 3.15\, Pr^{0.695}(1/E - 1/9)^{0.359} + P_m(E/9.0)^{0.6}$$

The effect of introducing the generalized wall functions in place of the standard wall functions is demonstrated by Rosten and Worrell [40]. Their predictions for a flow with addition of heat transfer over a backward-facing step in a channel bounded by walls are given in Fig. 3. The superiority of the generalized. wall functions to the standard wall functions is shown in the figure, for the peaks in k and ε with the improvement by the generalized wall functions are clearly visible at the reattachment point. A significant increase in the heat flux at the reattachment point can also be noticed. Elsewhere, the flow properties are little affected by the improvement to the wall functions.

Wall functions have been widely used since they economize computer time and storage. However, Nagano and Hishida [41] cited that the Evaluation Committee at the 1980-1981 Stanford Conference on Complex Turbulent Flows pointed out that wall functions are not well established in many situations, and thus the methods which include integration right up to the wall are better than those assuming the wall functions. This is because the low-Reynolds number models are valid throughout the fully turbulent, semilaminar and laminar regions. But, the incorporation of low-Reynolds number terms for the calculation to be carried out right up to the solid wall increases the computing time (by at least a factor of three [42]) due to the fine mesh required to resolve the immediate near-wall region adequately.

An improvement in predictions of turbulent wall flows behaviour may be attained by considering a nonisotropic eddy viscosity in the standard k-ε turbulance model.

Nonisotropic Form of the k-ε Model

The standard k-ε model adopts an isotropic turbulent viscosity. But, in flows with swirl, and in three-dimensional flows generally a more adequate level of viscosity for each active stress component is needed to include non-isotropic effects. For instance, in a flat- plate boundary layer, the normal and lateral stresses are approximately equal near the outer edge of the layer. However, as the wall is approached, the lateral stress rises more steeply than the normal stress in the fully turbulent region. Therefore, the eddy diffusivity in the lateral direction would be several times greater than that in the normal direction close to the wall, but that the diffusivities should become equal as the outer edge of the boundary layer is approached. This in turn suggests that a coefficient f_v should be employed to account for the anisotropy effects on the eddy viscosity. Bergeles et al [42] derived a relation for f_v depending on the distance from the wall. The relation was obtained from empirical data and represents a linear decay of the anisotropy from the wall to the edge of the boundary layer, and accordingly, the eddy viscosity and diffusivity for the turbulent transport in the lateral direction is increased over that in the normal direction in the boundary-layer region. They [42] have adopted the following simple fit to empirical data:

$$\mu_{t,j} = \mu_t = \rho\, C_\mu\, k^2 / \varepsilon$$
$$\mu_{t,k} = \mu_t\, f_v$$
$$\text{where } f_v = (1 + 3.5(1 - y/\Delta)) \quad \text{when } y \le \Delta$$
$$= 1 \quad \text{when } y > \Delta$$

in which Δ is the local boundary-layer thickness.

Using this nonisotropic k-ε viscosity model, Bergeles et al [42] predicted in almost perfect agreement with experimental data [43] the film cooling effectiveness for a film injection through a single hole into a free stream as shown in Fig. 4a, whereas Fig. 4b shows the better predictions of effectiveness trends by the nonisotropic k-ε viscosity model.

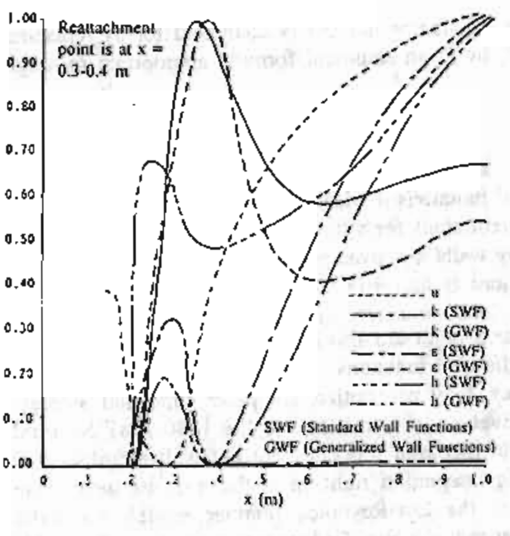

Fig. 3 Comparison of turbulence properties, enthalpy, and velocity computed using the k-ε model coupled once with SWF, and another with GWF for a flow over a backward-facing step in a channel bounded by walls.

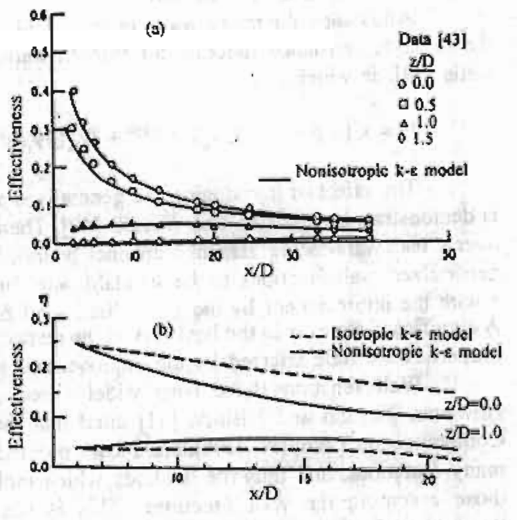

Fig. 4 Comparison of film cooling effectiveness for injection through a single hole into a free stream.

Bergles et al [44] employed somewhat a similar form for the anisotropy coefficient f_v,

$$f_v = 3.5 - 2.5\, y/\Delta \quad \text{when } y \le \Delta$$
$$= 1 \quad \text{when } y > \Delta$$

for the approximation of the stresses across the boundary layer. Demuren et al [45] employed the Bergeles et al [42] nonisotropic eddy-viscosity relation for the three-dimensional film-cooling injection over a flat surface, and have shown a fairly good agreement of the predictions of velocity and temperature distributions with measurements. However, discrepancies were found regarding the film cooling effectiveness, in particular behind the injection where a very complex flow field was established. If disagreements are found between measurements and predictions, it may be difficult to judge whether the weakness of the method lies in the wall function formulae or in the basic model equations. Then it may be desirable to consider using an appropriate improved k-ε turbulence model in which low-Reynolds number form is used that allows calculations right up to the solid wall.

Low-Reynolds Number k-ε Model of Turbulence

All of the two-equation models of turbulence which are pertinent to low-Reynolds number phenomena, equations (2) and (3), are mainly dependent upon the modeling of the eddy viscosity μ_t, and the incorporation of extra terms to improve near-wall behaviour. The influence of μ_t is due to the predominant effect of molecular viscosity on the flow structures in the immediate neighborhood of the wall, and the influence of wall proximity is due to the preferential damping of velocity fluctuations in the direction normal to the wall. Models, therefore have been proposed to include the influence of the molecular viscosity and wall proximity. In all of the proposed models, f_μ, f_1, f_2 and the extra terms D and F are either a function of one, or two of the following parameters, the turbulence Reynolds number $R_t(= k^2/\nu_l\varepsilon \approx \mu_t/\mu)$, the dimensionless distance $R_k(= \sqrt{k}y/\nu_l)$ and the dimensionless distance (local Reynolds number) $R_\tau(=u_\tau y/\nu_l = y^+)$.

Development of the Function f_μ

The production of k depends upon μ_t (see equation (4)), therefore correct modelling of μ_t is important in obtaining correct turbulence values near the wall. This implies that the modification of the eddy viscosity is achieved by the multiplication of the Kolmogorov-Prandtl relation, equation (1), by a damping function f_μ that expresses viscous effects on the stress $-\rho\overline{u'_iu'_j}$ so that, $\mu_t = \rho C_\mu f_\mu k^2/\varepsilon$. The function f_μ should account for the two separate influences mentioned above. Hence, it should be consistent with the physical argument that simulates the influence of the molecular viscosity near the wall, and approaches to unity in the fully turbulent region.

The proposed function f_μ by Jones and Launder [5], Hoffman [29] and Hassid and Poreh [46], given in Table 1, are apparantly a unique function of turbulence Reynolds number R_t, and thus the influence of the presence of a wall on ν_t is not taken into consideration directly. On the other hand, the formula f_μ by Chien [30] and Nagano and Hishida [41] take into account the effect of wall proximity directly via the dimensionless distance (local Reynods number), R_τ. The effect of R_t is not included in f_μ via the assumption that ν_t is already modelled as a function of R_t. The f_μ formula of Lam and Bremhorst [38], however, is dependent upon both the turbulence Reynolds number R_t and the dimensionless distance from the wall R_k. Thus, the two influences are considered directly in their formulation.

Table (1) Constants and functions in the k-ε models [41].

Model	ε_w -BC	C_μ	$C_{\varepsilon1}$	$C_{\varepsilon2}$	σ_k	σ_ε	f_μ	f_1	f_2	D	E
Standard high-Re	WF	0.09	1.44	1.92	1.0	1.3	1.0	1.0	1.0	0	0
Jones - Launder	$\varepsilon = 0$	0.09	1.45	2.0	1.0	1.3	$\exp\left[\frac{-2.5}{1+R_t/50}\right]$	1.0	$1-0.3\exp(-R_t^2)$	$-2\nu\left[\frac{\partial\sqrt{k}}{\partial y}\right]^2$	$2\nu\nu_t\left[\frac{\partial^2 U}{\partial y^2}\right]$
Hassid - Poreh	$\varepsilon = 0$	0.09	1.45	2.0	1.0	1.3	$1-\exp(-0.006R_t)$	1.0	$1-0.3\exp(-R_t^2)$	$-2\nu\frac{k}{y^2}$	$-2\nu\left[\frac{\partial\sqrt{k}}{\partial y}\right]^2$
Hoffmann	$\varepsilon = 0$	0.09	1.81	2.0	2.0	3.0	$\exp\left[\frac{-1.75}{1+R_t/50}\right]$	1.0	$1-0.3\exp(-R_t^2)$	$-\frac{\nu}{y}\frac{\partial k}{\partial y}$	0
Chien	$\varepsilon = 0$	0.09	1.35	1.8	1.0	1.3	$1-\exp(-0.0115R_\tau)$	1.0	$1-0.22\exp[-(R_t/6)^2]$	$-2\nu\frac{k}{y^2}$	$-2\nu\frac{\varepsilon}{y^2}\exp(-0.5R_\tau)$
Reynolds	$\varepsilon = \nu\frac{\partial^2 k}{\partial y^2}$	0.084	1.0	1.83	1.69	1.3	$1-\exp(-0.0198R_k)$	1.0	$\left[1-0.3\exp\left\{-\left(\frac{R_t}{3}\right)^2\right\}\right]f_\mu$	0	0
Lam - Bremhorst	$\varepsilon = \nu\frac{\partial^2 k}{\partial y^2}$	0.09	1.44	1.92	1.0	1.3	$\left\{1-\exp(-0.0165R_k)\right\}^2\left[1+\frac{20.5}{R_t}\right]$	$1+(0.05/f_\mu)^3$	$1-\exp(-R_t^2)$	0	0
Nagano - Hishida	$\varepsilon = 0$	0.09	1.45	1.9	1.0	1.3	$\left[1-\exp\left[-\frac{R_\tau}{26.5}\right]\right]^2$	1.0	$1-0.3\exp(-R_t^2)$	$-2\nu\left[\frac{\partial\sqrt{k}}{\partial y}\right]^2$	$\nu\nu_t(1-f_\mu)\left[\frac{\partial^2 U}{\partial y^2}\right]^2$

Development of the Functions f_1, f_2, D and F

The high-Reynolds k-ε form suggests that f_1 is approximately unity remote from the wall. Near the wall it is found that f_1 assumes larger values in order to increase the predicted dissipation rate thereby reducing the predicted turbulence level to match available experimental data. Otherwise, additional destruction terms would be required in the k-transport-equation to yield reasonable predictions as have been proposed by [5, 29, 30, 41, 46] in their low-Reynolds number forms of the k-ε model. Lam and Bremhorst [38], however, didn't include additional terms in either of the k and ε equations, instead they proposed a formula for f_1 being a function of f_μ only, see Table 1, so that when the turbulence level is high, f_μ and hence f_1 will be approximately unity. Close to a wall, f_μ will be small but finite and f_1 will become large.

As for f_2, all of the low Reynolds number forms of the k-ε model given in Table 1 employ a formula for f_2 dependent upon R_t, except Nagano and Hishida [41] where f_2 is dependent upon R_t, such that as R_t (or R_t) tends to zero at a wall f_2 must tend to zero.

In most of the low-Re forms, ε which is finite at the wall, is made zero for computational expediency [5]. Since this makes the k equation inconsistent at the wall, the term D is added in the k equation. whereas, the extra term F is added for the necessity of improving near-wall behaviour [41].

To assess the performance of these models, the comparison made by [41] of the predictions of the major low Reynolds number k-ε models with measurements of basic test cases covering wall flows are presented. In Fig. 5, predictions and experimental data [47-49] are compared for mean velocity profiles in a pipe. Fig. 6 comparison of predictions with measurements [50] are made for a relaminarizing turbulent boundary layer flow. It seems that in general the low-Reynolds number k-ε models reproduce the physical phenomena in such a good and satisfactory manner, although, not exact.

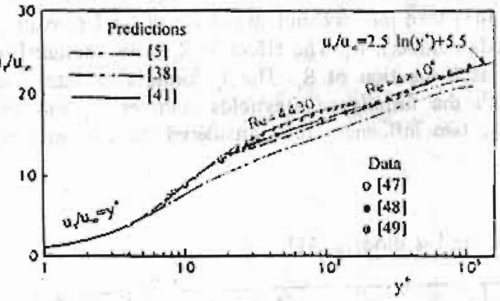

Fig. 5 Comparison of models for mean velocity profiles in a pipe.

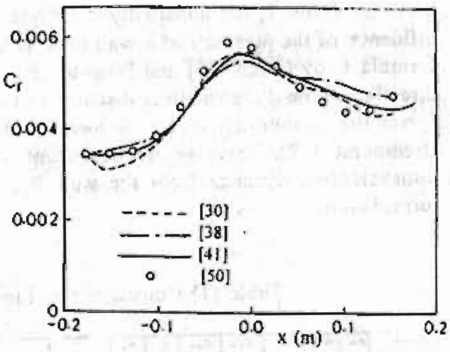

Fig. 6 Comparison of models for relaminarizing flow.

Concluding Remarks

It is evident from this review that if interest lies with predicting wall turbulent flows and computational economy, the standard k-ε model in conjunction with the standard wall functions is adequate but limited to translate simple boundary layer flows. However, the k-ε model coupled with the generalized wall functions provide fairly adequate predictions of mildly complex wall turbulent flowfields. Further improvement could be obtained by taking into account the nonisotropic effects on the eddy viscosity near walls. But even with this consideration, predictions were only slightly improved for complex wall turbulent flows.

The various low-Reynolds number k-ε forms have shown considerable improvements in reproducing wall effects that are in good agreement with experimental data. But this is achieved at the cost of computer time and storage which is justifiable if greater accuracy is desired.

It is recommended that the Reynolds stress models are essential when detailed and more accurate predictions of the complex flow problems are desired. Because of the recent big and rapid advances in today's computer technology, the Reynolds stress models will and have become more widely used in application to complex flow situations.

Nomenclature

C_f	skin-friction factor
C_μ, $C_{\epsilon 1}$, $C_{\epsilon 2}$, σ_k, σ_ϵ	empirical constants in k-ε turbulence model
D	extra term in k-equation, or injection hole diameter
E	constant for the law of the wall
e	wall absolute roughness height
F	extra term in ε-equation
f_1, f_2	functions in low Reynolds number k-ε turbulence model
f_v	coefficient accounting for anistropy effects on the eddy viscosity
f_μ	damping function on the eddy viscosity
h	specific enthalpy
k	kinetic energy of turbulence
k_v	Von Karman constant
L_ϵ	eddy length scale
P	rough wall empirical formula
P_m	smooth-wall sublayer resistance function
Pr	Prandtl number
q	heat flux
Re	Reynolds number
R_k	dimensionless distance $= \sqrt{k}y/v_l$
R_t	turbulence Reynolds number $= k^2/v_l\epsilon$
R_τ	dimensionless distance $= u_\tau y/v_l = y^+$
St	Stanton number
T	temperature
t	time
u	velocity component in x-direction
u'	instantaneous velocity fluctuations about the mean
u_τ	friction velocity
x	streamwise distance along the wall from an arbitrary reference point
y	vertical distance from wall
y^+	dimensionless distance $= u_\tau y/v_l$
z	lateral distance from an arbitrary reference point
x_i, x_j, x_k	Cartesian coordinates
Δ	thickness of boundary layer
ε	dissipation rate of turbulence
μ	viscosity
ν	kinematic viscosity
ρ	density
τ	shear stress
ω	specific dissipation rate of turbulence

Subscript

l	laminar
p	grid cell node
r	roughness
t	turbulent
w	wall
∞	mainstream

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