



**Attempt all questions**

**Question 1 [18 Marks]**

A) In the figure is shown a tank in which flow is supplied at rate of  $Q$ . Construct the fluid circuit representation for this system. Determine the equation of the pressure  $P$  (head  $H=P/\rho$ ) as a function of  $P_1$  and  $Q$ . **(8 Marks)**

B) The oven shown in Figure is supplied with heat from an electric source. The rate of heat supplied  $Q$  is proportion to the voltage  $Q_s = K_1 E$ . The rate at which the heat lost  $Q$  is proportional to the difference in the oven temperature  $T$  and the ambient temperature  $T_a$ , thus  $Q = k_2(T - T_a)$ . The rate of change of temperature of the oven is  $Dt = k_3(Q_s - Q)$ . Construct the block diagram representation for this oven. Determine the differential equation which relates the temperature  $T$  to the voltage  $E_s$  and the ambient temperature  $T_a$ . What is the time constant  $\tau$ . **(10 Marks)**

**Question 2 [20 Marks]**

A) For subsonic flow of air through a restriction, the mass rate of flow is  $M = 1.05A[(P_1 - P_2)P_2/T]^{0.5}$ . The area  $A$  of the restriction and temperature  $T$  are constant. Determine the linear approximation for  $m$  due to a change  $p_1 - p_2$  in the pressure drop across the restriction and due to a change  $p_2$  in the downstream pressure. Express this variation  $m$  in terms of the reference value  $M_i$ .

B) For the mechanical system shown in figure

i) Draw the grounded chi diagram

ii) determine the equation relating  $f$  with  $x$  and  $f$  with  $y$ .

**Question 3 [20 Marks]**

A system for controlling flow is shown in figure. Increasing the desired flow setting increases the compression of spring  $K_1$ , which causes  $x$  and the position  $e$  of the balanced valve to move up. This in turns causes the flow valve to move down, which increases the flow. The amount of flow out is measured by a venture-type flow meter, so that the pressure drop  $P_1 - P_2$  is a function of  $G_o$ . The diaphragm prevents leakage from the high pressure  $P_1$  to the low pressure  $P_2$ , but it permits motion, just as a piston would. The effective area of the diaphragm is  $A_d$ . The flow  $Q_o$  is seen to be a function of the flow valve opening  $Y$  and the supply pressure  $P_s$ . Determine the overall block diagram representation for this system.

**Question 4 [16 Marks]**

A) The steady state operating curves for a unity feed back system ( $K_H=1$ ) are shown in figure. Construct the block diagram that described the steady state operation of this system. **(9 Marks)**

B) The block diagram for an industrial temperature control is shown in figure. At the reference operating point,  $V_i = C_i = 100$ ,  $M_i = 40$ , and  $U_i = 20$ . The  $U_i = 20$  load line is shown in the figure. Insert values on the  $C$  axis of the figure where the question marks appear such that it has the correct slope. Draw the  $U = 30$  load line. Determine the value of  $K_1$  such that when  $V = V_i$  the change of controlled variable  $c$  will be 1 unit when  $U$  changes from 20 to 30. Determine  $A$  such that the coefficient of  $v$  term in the equation for steady state operation is unity. **(7 marks)**

**Question 5 [16 Marks]**

Determine the solution of the following differential equation where  $f(t) = 2e^{-t}$  and all initial conditions are zero  $(D^3 + 4D^2 + 5D + 2)y(t) = f(t)$

Laplace transform pairs		Laplace transform properties	
$f(t)$	$F(s)$	Time function	Laplace transform
$\delta(t)$	1	$kf(t)$	$kF(s)$
$u(t)$	$\frac{1}{s}$	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
$t$	$\frac{1}{s^2}$	$f'(t)$	$sF(s) - f(0)$
$e^{at}$	$\frac{1}{s-a}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$t^n$	$\frac{n!}{s^{n+1}}$	$f^n(t)$	$s^nF(s) - s^{n-1}f(0) - \dots - f^{n-1}(0)$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$	$f^{(-1)}(t)$	$\frac{F(s)}{s} + \frac{f^{(-1)}(0)}{s}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	$f^{(-n)}(t)$	$\frac{F(s)}{s^n} + \frac{f^{(-1)}(0)}{s^{n-1}} + \dots + \frac{f^{(-n)}(0)}{s}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$	$e^{at} f(t)$	$F(s-a)$
$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$	$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n} F(s)$
		$f(\tau) = f(t-t_0)$	$e^{-t_0 s} F(s)$
		$\int_0^t f(\lambda) g(t-\lambda) d\lambda$	$F(s)G(s)$

