

## TRAJECTORY ANALYSIS FOR AN INDUSTRIAL ROBOTIC SYSTEM

المسار الأمثل لمنظومة ذراع آلي صناعية

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ملخص البحث :

يقدّم البحث تصوراً عاماً عن الدور المتنامي لمنظومات الذراع الآلي الصناعية في الصناعات الحديثة. ويصنّف البحث على النموذج الرياضي لمنظومة ذراع آلي صناعية وقد تمت دراسة الخواص الديناميكية للمنظومة والتي تبين مدة تذبذبها نتيجة لوجود كثير من العناصر اللاخطية بها. ولاختيار المسار المناسب للمنظومة التحرّعت طريقة لهذا الاختيار تتلخّص فيما يلي :

أولاً : يتم اختيار أكثر من مسار مستقر لكل ذراع في المنظومة

ثانياً : يجب أن يستوفى المسار المختار شروط الاستقرار في بداية ونهاية الحركة.

ثالثاً : يتم التعويض بكل مسار مع الاختيار في النموذج الرياضي للمنظومة وانتنتاج عزم الدوران الواجب الحصول عليه من المحرك الخاص بكل ذراع.

رابعاً : يكون المسار الأمثل (الناسب) هو المسار الذي يعطى عزمًا مقبولاً (ذو استجابة لخطية ملبّولة).

تم تطبيق الطريقة المقترحة بمثال عددي و تبين نتائج المصاغة فعالية الطريقة المقترحة في اختيار المسار المناسب.

**ABSTRACT :** The present paper introduces a general scope about the growing role of industrial robot system in modern industries. A dynamic model for a two links robot is introduced. This dynamic model is obtained using, the basics of Lagrangian mechanics. The dynamic behaviour of such robotic system is oscillatory. That is, due to the different types of nonlinearities acting on the system. A computer program based on Runge-kutta (R-K) fourth order method with variable step, is designed to study the dynamic behaviour of such robotic system.

A suggested technique for choosing the most suitable control input (controlling torque) for such nonlinear system is introduced. The steps of the suggested technique are classified as follows:-

1. For each link different stable trajectories are assumed. All these trajectories must fulfill the stability conditions.
2. A back substitution for each of the chosen trajectories and its derivatives, into the dynamic equations of the robotic system is carried-out.
3. For each chosen trajectory, back substitution results in, the controlling torque, necessary for each link.
4. The most suitable (optimal) trajectory for the corresponding link will be the trajectory, which produces, the most suitable controlling torque, that must be obtained by the actuator.

An illustrative numerical example is introduced in the paper, to show the effectiveness of the suggested technique. Computational results insure the effectiveness & validity of suggested technique.

INTRODUCTION :- In the present time, Industrial robotic systems play an increasing & important role in modern industries. For example, in the last decade, they are intensively used in mobile industry, in which mass production with lower cost is required. Also, in dangerous industries such as Iron & Steel industries, such robotic systems are required to deal with blast furnaces, having high temperatures [1,2]. In massive production of machine tools with accurate standard specifications and in chemical & Medical Industries, such robotic systems are also required.

A robotic system, as an advanced control engineering application stands as a large challenge, facing control engineers. Severe nonlinearities in such systems, make it difficult to model, simulate & control them [2,3].

#### MODELING OF INDUSTRIAL ROBOT :

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For the two links manipulator shown in fig, (1) & assuming that, the masses  $m_1$  &  $m_2$  are concentrated at the ends of the first and second links respectively, and using Lagrange-Euler technique [5], the dynamic model of such system can be written as [6,7] :

$$\dot{X}_1 = X_2 \quad (1)$$

$$\dot{X}_2 = (T_1 - m_1 g d_1 \sin X_1) / m_1 d_1 \quad (2)$$

$$\dot{X}_3 = X_4 \quad (3)$$

$$\dot{X}_4 = \frac{T_1}{m_1 d_1} - \frac{(m_2 d_2 + m_2 d_1 d_2 \cos X_3)}{m_2 d_2} - \frac{T_1 - m_1 d_1 g \sin X_1}{m_1 d_1}$$

$$+ \frac{m_2 d_1 d_2 \sin X_3}{m_2 d_2} X_2 X_4 - m_2 g d_2 \sin(X_1 + X_3) / m_2 d_2 \quad (4)$$

where,

$X_1 = \theta_1$ ,  $X_2 = \dot{\theta}_1$ ,  $X_3 = \theta_2$ ,  $X_4 = \dot{\theta}_2$ ,  $g$  = gravity acceleration

$T_1$  = torque applied to the first link,

$T_2$  = torque applied to the second link,

#### DYNAMIC BEHAVIOUR ANALYSIS :-

We shall assume the following numerical values for the mentioned above dynamic model :

$m_1 = 2$  kg ,  $d_1 = 1$  m ,  $m_2 = 5$  kg ,  $d_2 = 1.5$  m

$\theta_1(0) = 0$  rad.,  $\theta_2(0) = 0.1$  rad.,  $T_1(0) = 0.00266$  Nm,  $T_2(0) = 0.00244$  Nm

$\theta_1(f) =$  Final angular position of first link  $= 0.5$  rad. &  $\theta_2(f) = 0.3$  rad

Using Runge-Kutta (R-K), fourth method with variable step, the above equations (1-4) are solved through a computer program.

Results of computations are illustrated in figures (2-a,b), (3-a,b) & (4-a,b). It is clear from these figures that, the angular positions & angular velocities have a severe oscillatory behaviour. Also, the applied torque could not be easily obtained by the actuators.

#### A SUGGESTED TECHNIQUE FOR CHOOSING THE OPTIMAL TRAJECTORY:

The optimal trajectory here means the suitable trajectory, which produces a reasonable torque, that can be obtained easily from the actuator.

For this purpose, we shall assume three different trajectories for each link. All the chosen trajectories must fulfill the following stability conditions :-

a- at the starting of motion (i.e; at  $t = 0$ )  $\dot{\theta}_1 = \ddot{\theta}_1 = \dot{\theta}_2 = \ddot{\theta}_2 = 0$ ,  
and  $\theta_1 = \theta_1(0)$  ,  $\theta_2 = \theta_2(0)$

b- at the end of motion (i.e; for  $t$  large enough)  $\dot{\theta}_1 = \ddot{\theta}_1 = \dot{\theta}_2 = \ddot{\theta}_2 = 0$   
and  $\theta_1 = \theta_1(f)$  ,  $\theta_2 = \theta_2(f)$

the three suggested trajectories are assumed as follows :-

First Trajectory :

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$$\text{For the first link:- } \theta_1 = \theta_1(f) - (\theta_1(f) - \theta_1(0)) e^{-t^3} \quad (5)$$

$$\text{For the second link:- } \theta_2 = \theta_2(f) - (\theta_2(f) - \theta_2(0)) e^{-t^3} \quad (6)$$

Second Trajectory :

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$$\text{For the first link :- } \theta_1 = \theta_1(f) - (\theta_1(f) - \theta_1(0)) e^{-t^4} \quad (7)$$

$$\text{For the second link:- } \theta_2 = \theta_2(f) - (\theta_2(f) - \theta_2(0)) e^{-t^4} \quad (8)$$

Third Trajectory:

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$$\text{For the first link : } \theta_1 = \theta_1(f) - (\theta_1(f) - \theta_1(0)) e^{-t/\sin t} \quad (9)$$

$$\text{For the second link: } \theta_2 = \theta_2(f) - (\theta_2(f) - \theta_2(0)) e^{-t/\sin t} \quad (10)$$

All the above chosen trajectories, equations (5-10), fulfill the stability conditions mentioned above.

Back substitution of each of the above trajectories into the system equations, produces the torque assumed to be obtained from the system actuator. A computer program is designed to calculate these torques for a time base = 4 sec.

#### COMPUTATIONAL RESULTS & ANALYSIS:-

Applying the designed program for back substitution, the torques  $T_1$  &  $T_2$  required for the first & second links respectively, when the first Trajectory is assumed, are shown in figure (5-a,b).

For the second Trajectory, the torques  $T_1$  &  $T_2$  are illustrated in fig; (6-a,b), and for the third Trajectory  $T_1$  &  $T_2$  are illustrated in fig; (7-a,b).

It is clear from the comparative study of the torques produced by the third assumed Trajectory are much smoother and represent a satisfactory transient response for the actuator. Hence; We can deduce that the third assumed trajectory is a suitable trajectory for such robotic system.

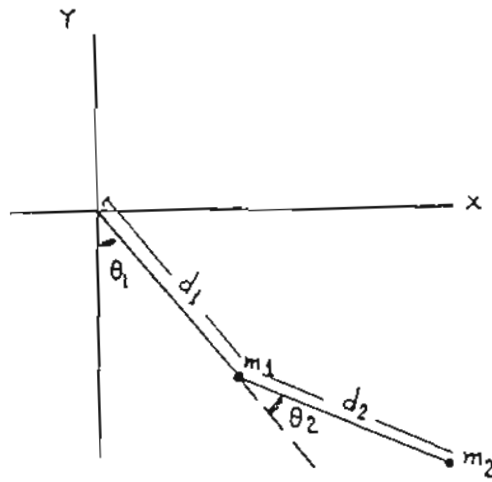
## CONCLUSION :

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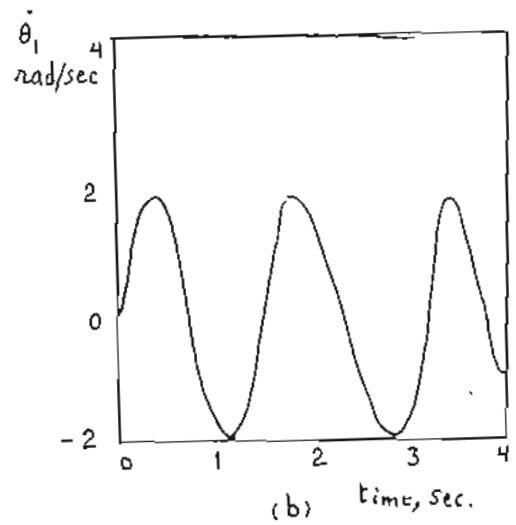
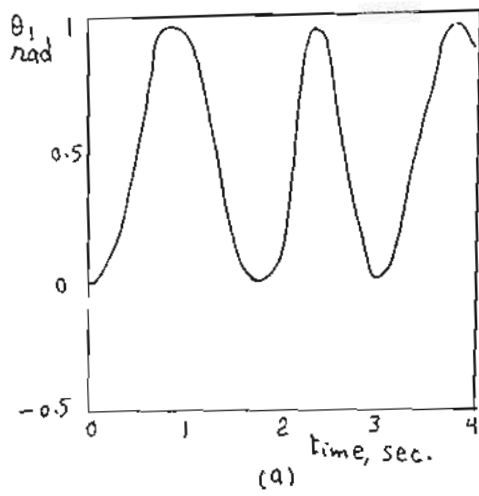
In the paper, a suggested numerical technique for choosing the most suitable (Optimal) trajectory for an industrial robotic system is introduced. The technique depends on the choice of different stable trajectories, then, a comparative study between them is carried-out, to choose the most suitable one. Computational results insure the validity and effectiveness of the suggested technique.

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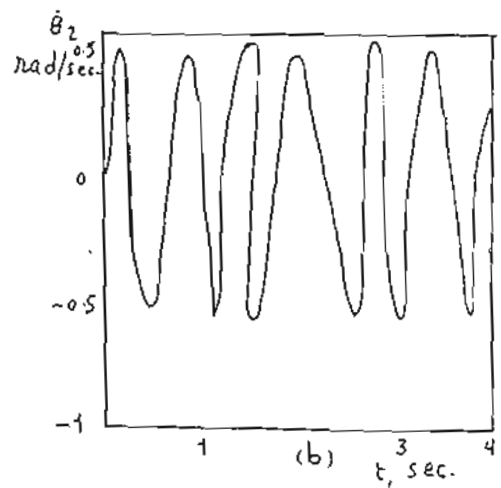
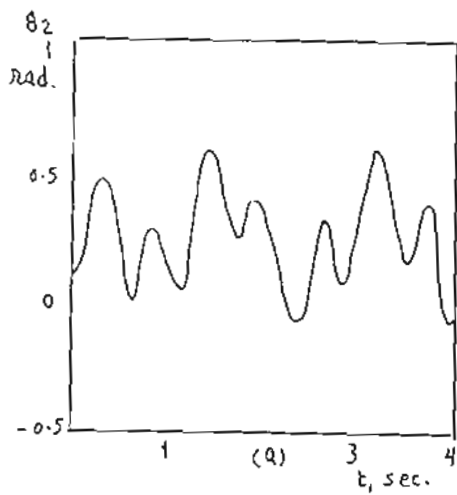
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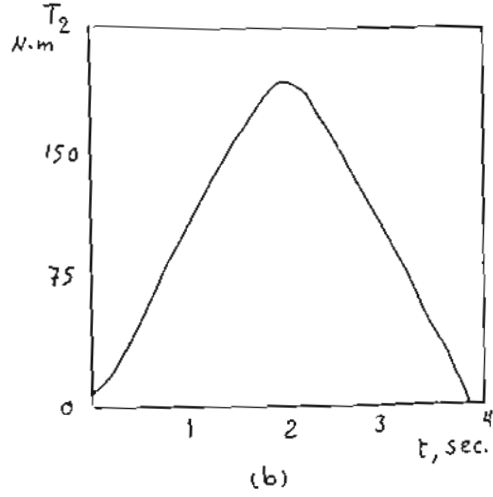
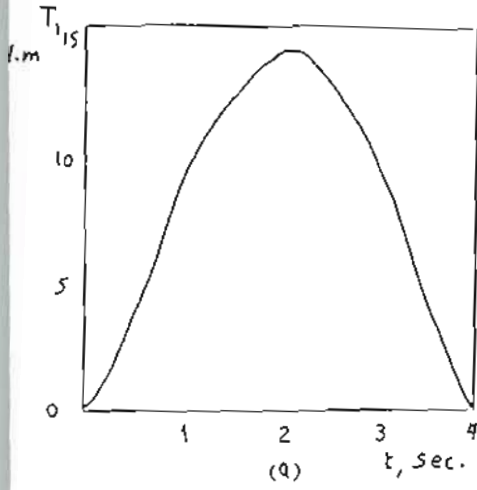
Fig(1)



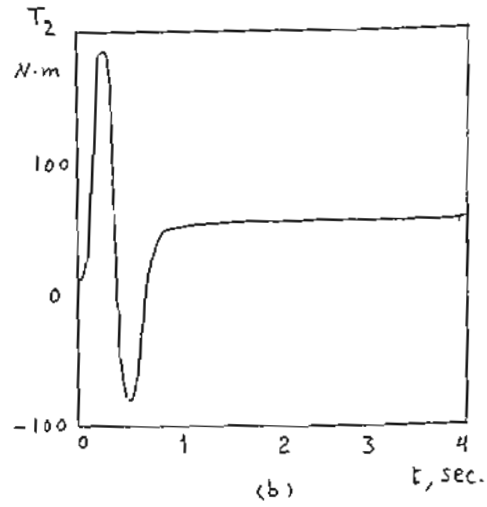
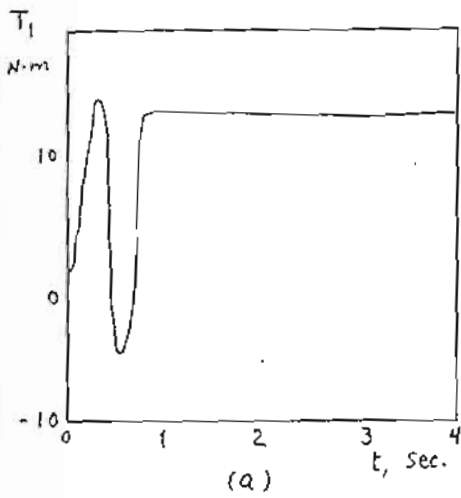
Fig(2)



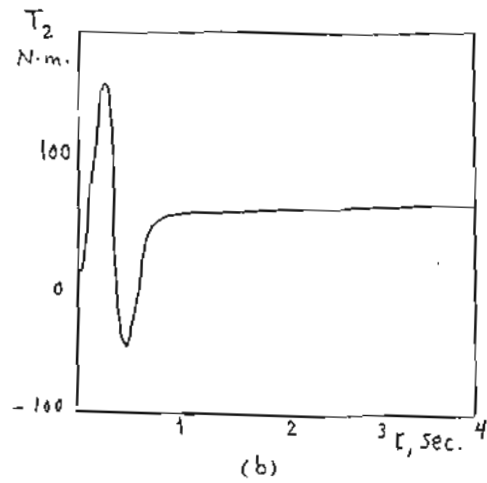
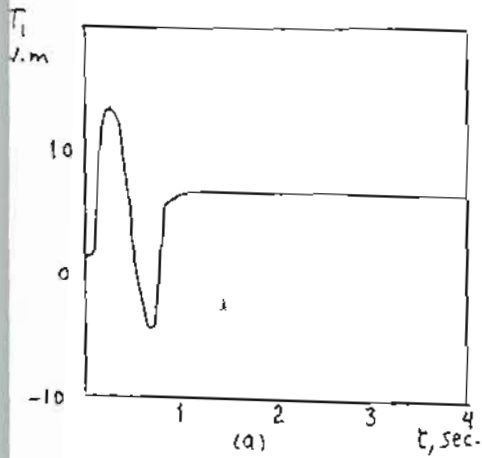
m.s.c.



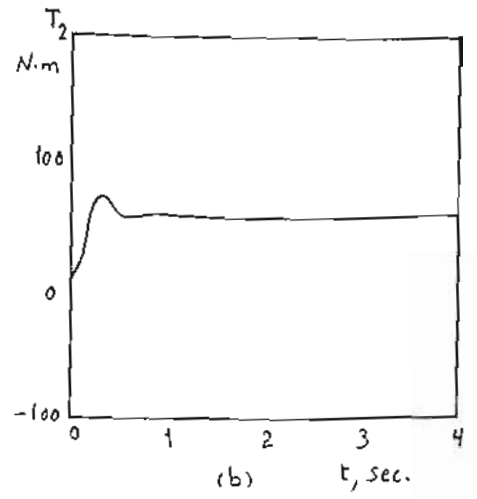
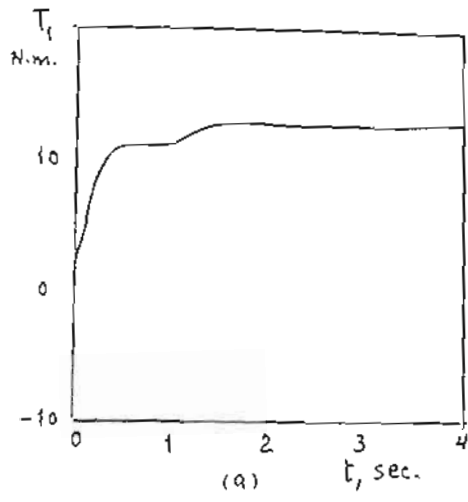
Fig(4)



Fig(5)



Fig(6)



Fig(7)