



**Answer the following questions**

**Question 1**

(30 marks)

a) Solve the following differential equations:

(10 marks)

i)  $\cos x \frac{dy}{dx} + y \sin x = 2x \cos^2 x$       ii)  $y'' = e^{2x} + 3x$

b) Test the convergence of the following series:

(10 marks)

i)  $\sum n^2 e^{-n^3}$       ii)  $\sum \frac{3n^3 - 2n^2 + 4}{n^7 - n^3 + 2}$

c) Draw and compute the Fourier series of the function:

(10 marks)

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x^2 & 0 < x < \pi \end{cases}$$

**Question 2**

(25 marks)

a) Solve the initial value problem by using Laplace transform:

(10 marks)

$y'' + 2y' - 3y = 6e^{-2t}$        $y(0) = 2, y'(0) = -14$

b) Find the interval of convergence :  $\sum \frac{(2x-3)^n}{4^{2n}}$

(5 marks)

c) Find the Laplace transform of the functions:

(10 marks)

i)  $f(t) = (t+1)^2 e^{-3t}$       ii)  $u(t-5)e^{2(t-5)} \cosh(t-5)$

**Question 3**

(30 marks)

a) Find the moments of inertia  $I_x, I_y, I_0$  for the lamina that occupies the region D, where D is bounded by

$y = e^x, y = 0, x = 0, \text{ and } x = 1$  ;  $\rho(x, y) = y$

(10 marks)

b) Find the orthogonal trajectories of the curve:  $y^2 = Cx^3$

(10 marks)

c) Find the inverse Laplace transform of the functions:

(10 marks)

i)  $\frac{1 + e^{-2s}}{s^2 - 9}$       ii)  $\ln \frac{(s-1)^2}{s^2 + 1}$

**Question 4****(30 marks)**

a) Solve the system of simultaneous differential equations:

**(10 marks)**

$$\frac{d^2x}{dt^2} + \frac{dy}{dt} = e^{-t} \quad , \quad \frac{d^2y}{dt^2} - 2\frac{dx}{dt} = \cos 4t$$

b) Evaluate the triple integral  $\iiint_D z e^y dx dz dy$  where D is given by**(10 marks)**

$$D: 0 \leq x \leq \sqrt{1-z^2}, \quad 0 \leq y \leq 3, \quad 0 \leq z \leq 1$$

c) Solve the differential equation:  $(x^2 D^3 + 4xD^2 - 5D - \frac{15}{x})y = \frac{1}{x^3}$ **(10 marks)****Question 5****(25 marks)**

a) Solve the ordinary differential equations:

**(10 marks)**

$$i) \left(\frac{3y^2}{x^2 + 3x}\right)dx + \left(2y \ln \frac{x}{x+3} + 3 \sin y\right)dy = 0 \quad ii) p^2 - 2xp - 8x^2 = 0$$

b) Use the definition of the Laplace transform to obtain the Laplace transformation for function:  $f(t) = \sin at$ **(5 marks)**

c) Find the solution of the initial value problem:

**(10 marks)**

$$\frac{d^2x}{dt^2} - 16x = 32 \quad x = 0, \quad \frac{dx}{dt} = 2 \quad \text{when } t = 0$$

*With my best wishes**Dr. Eng. Rizk Masoud*