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AUTOREGRESSIVE MODELING OF GEOMAGNETIC DATA

نمذجة بيانات المغناطيسية الأرضية

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الخلاصة- فى هذا البحث تمت نمذجة اختلافات المغناطيسية الأرضية المسجلة كل ساعة في المدى القصير (الفترة شهر) كعملية ارجاع ذاتى لها دخل على صورة ضوضاء بيضاء. وقد استخدم النموذج الناتج فى انتاج عدد من التسجيلات المحاكية للتسجيلات الحقيقية باستعمال التوزيعات الاحتمالية لمعاملات نموذج الارجاع الذاتى والضوضاء المعينة من التسجيلات الحقيقية. وقد قورنت التسجيلات الحقيقية والمحاكية لكل شهر بعدة طرق منها تعيين قيمة الانحراف المعيارى للقيم، وعدد التقاطعات الصفرية وعدد القيم القصوى فى كل تسجيل، والتوزيع الاحتمالى للسعات لهذة الاشارات. وأظهرت النتائج انه لايوجد اختلافات معنية فى هذة المتغيرات. وفائدة هذة التسجيلات المحاكية انها تسمح باختبار خوارزمات التنبو بدقة وهو غير متوفر مع قلة البيانات المسجلة حاليا.

ABSTRACT- Geomagnetic (GM) data generally appear to have significant organization or structure. We attempted to determine if GM records could be modeled as an autoregressive process with a white noise excitation during the short-time period of one month. The autoregressive (ER) model is then used to synthesize GM signals using AR coefficients and a white noise excitation with the same propability distribution function as those determined from the autoregressive model of an ensemble of monthly records. Both the original and synthesized monthly records are then compared using the root-mean-square (rms) amplitude, the number of zero crossings per month, the number of peaks, and the amplitude distributions of the signals. The results of examining the synthesized GM records indicate that there are no significant differences in the values of these parameters. The use of such synthesized GM records may allow more through testing of forecasting algorithms than is possible with the present limited number of GM records.

I. INTRODUCTION

Studies of Earth's magnetic field have continued at a vigorous pace because the magnetic field is a fundamental parameter of great significance to many other applications such as mining exploration, archaeomagnetism, telecommunications, space research...., etc. [1]

The time-varying component of the geomagnetic field at the earth's surface is composed of a primary field that is produced in the ionosphere and magnetosphere and a secondary field due to geomagnetic induction in the crust and upper mantel [2]. Later measurements at magnetic observatories showed many changes in the field that have shorter periods than these originally observed. The variations may be resolved into secular changes, solar-diurnal changes, lunar-diurnal changes, and abrupt changes resulting from magnetic storms.

Recently, the present authors, have initiated research work that provided some rational basis for characterizing and identifying the underlying features of the geomagnetic variations. The project is being carried out at the National Research Institute of Astronomy and Geophysics and Mansoura University. It aims at the development of an automated system that records, processes, analyses, and forecasts the geomagnetic activity at Misallate, Helwan. Isolation and quantification of three identifiable components contributing to the total geomagnetic field records using appropriate statistical and signal processing techniques was reported [3].

The present work is an extension of the previous research. We seek to more fully characterize the hourly geomagnetic activity records to determine if they contain any underlying organization or structure and if they can be well modeled as some type of autoregressive process. In this paper, we model geomagnetic signals as autoregressive (AR) processes during the time period of one month. An AR process is a simple linear model in which the output amplitude at each time instant depends linearly on the past output and the additive noise excitation. After modeling the geomagnetic signals as an AR process, new synthetic geomagnetic (GM) records are generated by using the information derived from the distributions of the relevant AR parameters and the prediction error of the available records. Such synthetic records may be useful for a through investigation of the geomagnetic variability and for forecasting purposes which are not possible with limited number of records.

In this paper, we first test for stationarity of the mean and root-mean-square (rms) amplitude. Subsequently, the appropriateness of the AR model order selection criteria is tested by determining if the residual errors (estimated excitation noise) represent a white noise process. Next the models are compared using the variance of the residual errors to the variance of the original signal. Finally, the autoregressive model and empirical estimate of the probability distribution functions of the determined coefficients and the excitation noise are used to synthesize geomagnetic records. The results of comparing characteristics of the synthesized geomagnetic records to those of the true records are then discussed.

II. ELEMENTS AND VARIABILITY OF THE GEOMAGNETIC FIELD

II.1 Geomagnetic Elements

At every point along the earth's surface, a magnetic needle free to orient itself in any direction around a pivot at its center will assume a position in space determined by the direction of the earth's magnetic field F at that point.

Normally, this direction will be at an angle with the vertical, and its horizontal projection will make an angle with the north-south direction [4]. Since measuring instruments conventionally installed in magnetic observatories respond only to the horizontal or vertical components of the actual field, it is customary to resolve the total field F into its horizontal component H (separated into X and Y projections) and its vertical component Z (Fig. 1). The angle which F makes with its horizontal component H is the inclination I, and the angle between H and X (which by convention points north) is the declination D.

The quantities $X, Y, Z, D, I, H,$ and F, known as magnetic elements, are related as follows:

$$H = F\cos I \qquad Z = F\sin I = H\tan I$$

$$X = H\cos D \qquad Y = H\sin D \qquad (1)$$

$$X^2 + Y^2 = H^2 \qquad X^2 + Y^2 + Z^2 = H^2 + Z^2 = F^2$$

All these relations are derivable from the diagram. The vertical plane through F and H is called the local magnetic meridian.

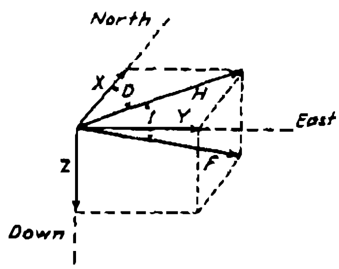

Fig. 1 The magnetic elements

II.2 Geomagnetic Variability

In the early days of navigation with the compass, it was recognized that the earth's magnetic field intensity changes its direction slowly and irregularly. Later measurements at magnetic observations showed many changes in the field that have shorter periods than those originally observed. The variations may be resolved into secular changes, solar-diurnal changes, lunar-diurnal changes, and changes resulting from magnetic storms [4].

1) Secular variations: Slow changes in the earth's field which take place progressively over decades or centuries are known as secular variations. Such changes are noted in all the magnetic elements at magnetic observatories everywhere in the world. The rates of change vary with time [4].

2) Diurnal variations: The records generally show two types of variations, the quiet day and the disturbed day. The quiet-day variation is smooth, regular, and low in amplitude. The disturbed-day variation is less regular and is associated with magnetic storms. Analysis of magnetically quiet days shows a definite 24-h periodicity that depends on a close approximation, only on local time and

geographic latitude. Because of its correlation with the period of the earth's rotation as referred to the sun, this portion of the variation is referred to as the solar-diurnal variation.

Another component in the periodic variation of the earth's magnetic elements has about one-fifteenth the amplitude of the solar-diurnal variation and a periodicity of about 25 h corresponding to the length of the lunar day. This component of the variation has been related to the earth's rotation with respect to the moon and is referred to as lunar-diurnal variation [5].

3) Magnetic storms: In addition to the predictable short-term variations in the earth's field, there are transient disturbances which by analogy with their meteorological counterparts are called magnetic storms. Such variations in the magnetic field can be so rapid, unpredictable, and of large amplitude, that normally no corrections can be made.

III. DATA COLLECTION AND STATIONARITY ANALYSIS

For the present study, geomagnetic data for the period of 1984-1993 were obtained from the National Research Institute of Astronomy and Geophysics at Helwan. Measurements were made at the magnetic observatory at Missalate station using the magnetometer Type La cour. This instrument records the changes in the geomagnetic field on a sensitive paper of size 30 X 40 cm. The magnetometer records the daily variations as three traces which are the *Z*, *H* and *D* geomagnetic-field components. The records were then examined, calibrated and then sampled for further data processing and analysis. The sampling rate used for this study is 1 sample/hour. Fig. 2 illustrates a typical example of the recorded hourly geomagnetic elements for one month.

Before applying any model to GM records, we first examined the stationarity of the signal over the time span we intend to model. This test for stationarity was used to determine if there were any trend present in the data, such as increasing or decreasing amplitudes. The stationarity of the mean and rms amplitude was tested using a " run test " [6]. The data were partitioned into 10 consecutive segments (of 72 hour each). Geomagnetic data show often recurrent nature and daily cyclic variations and each segment had at least three daily cycles.

For each segment i, the average ($\bar{x}_i$) and rms. amplitude (A_i) is estimated. A median value for all segments is then determined for each of the statistics, and the deviation D from the median for each segment is calculated as :

$$D_{x_i} = \bar{x}_{median} - \bar{x}_i$$

$$D_{A_i} = A_{median} - A_i \qquad (2)$$

The number of sign changes (observed runs) of the D_{x_i} and D_{A_i} are then compared with the range of sign changes that would occur in a truly stationary process for a level of significance of 5% and it is concluded that there is no evidence of nonstationarity.

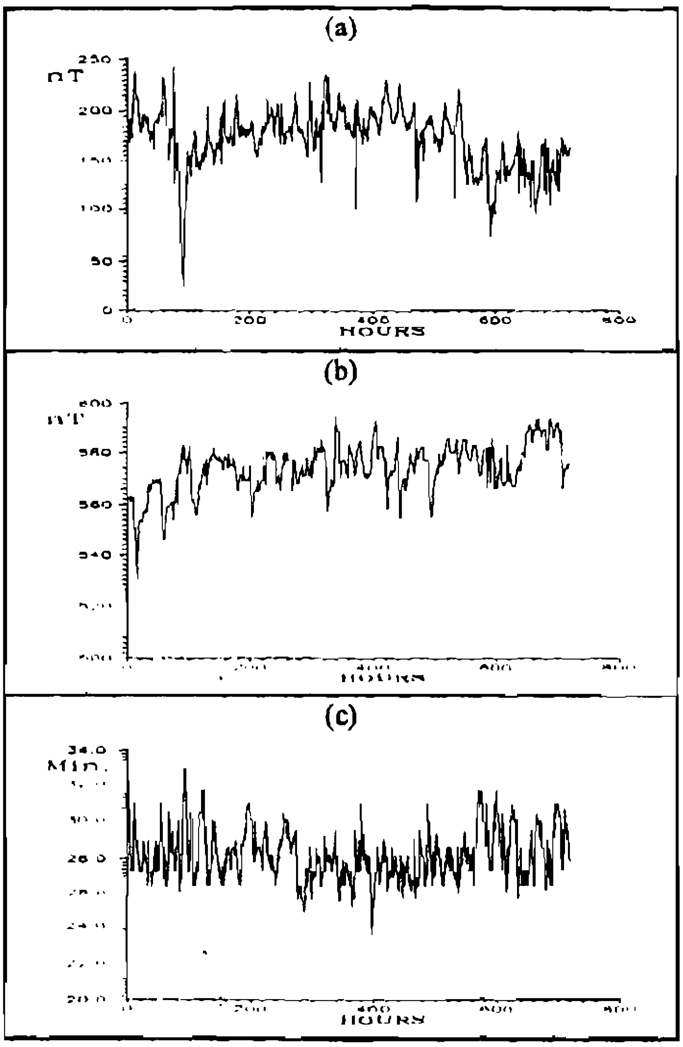

Fig. 2 A typical hourly geomagnetic record (November 1986)
(a) *H* element, (b) *Z* element, and (c) *D* element

IV. AUTOREGRESSIVE (AR) MODELING

In an autoregressive process of order p, the output x_n at time n depends linearly, via coefficients a_i on the previous p outputs and an additive white noise excitation, e_n. Specifically, the process is modeled as

$$x_n = -\sum_{i=1}^{p} a_i x_{n-i} + e_n \qquad (3)$$

The noise factor e_n is referred to as the prediction error. The model represented by (3) can be used in a backward fashion (retrospective regression analysis); the signal at time n is considered as being a linear combination of p future values. In this case, the noise term is referred to as the retrospection error.

IV.1 Determination of the Coefficients

There are various proposed methods for determing the coefficients in an autoregressive process. Two methods will be outlined briefly in the following subsections [7].

IV.1.a Yule-Walker Equations

This technique of estimating the model coefficients is based on finding a least-square of the autoregressive model to the recorded signal by finding those coefficients for which the squared error is least. As a result, one obtains a set of linear equations for the coefficients in terms of the autocorrelation coefficients. These equations are usually called the Yule-Walker equations [7], and can be expanded in matrix form as

$$\begin{bmatrix} r(0) & r(-1) & \cdots & r(-(p-1)) \\ r(1) & r(0) & \cdots & r(-(p-2)) \\ \vdots & \vdots & \ddots & \vdots \\ r(p-1) & r(p-2) & \cdots & r(0) \end{bmatrix} \begin{bmatrix} a(1) \\ a(2) \\ \vdots \\ a(p) \end{bmatrix} = - \begin{bmatrix} r(1) \\ r(2) \\ \vdots \\ r(p) \end{bmatrix} \qquad (4)$$

where

$$r(k) = \frac{1}{N} \sum_{n=0}^{N-k} x(n)x(n+k) \qquad k = 0, 1, 2, \ldots, p-1$$

denote the autocorrelation coefficients for lag i
The matrix in (4) is hermitian, Toeplitz, and positive definite.

The Levinson-Durbin algorithm provides an efficient recursive solution to the Yule-Walker equations. It computes recursively the (AR) coefficients.

IV.1.b Covariance Method

The covariance method is based on minimizing the estimate of the prediction error power [8]. This minimization yields the AR coefficients as the solution of the equations

$$\begin{bmatrix} c(1,1) & c(1,2) & \cdots & c(1,p) \\ c(2,1) & c(2,2) & \cdots & c(2,p) \\ \vdots & \vdots & \ddots & \vdots \\ c(p,1) & c(p,1) & \cdots & c(p,p) \end{bmatrix} \begin{bmatrix} a(1) \\ a(2) \\ \vdots \\ a(p) \end{bmatrix} = - \begin{bmatrix} c(1,0) \\ c(2,0) \\ \vdots \\ c(p,0) \end{bmatrix} \qquad (5)$$

where

$$c(j,k) = \frac{1}{N-P}\sum_{n=P}^{N-1} x(n-j)x(n-k)$$

Hence, the matrix is Hermitian and positive semidefinite. The equations may be solved using the Cholesky decomposition [7].

IV.2 Model Order Selection

In selecting the appropriate model order, there is a tradeoff between computational requirements and minimizing the residual errors. although higher model orders will reduce the error, they require more computation.

A method of determing the order of an AR model was described by Akaike [9] and has been adopted in the present work. In this procedure, the mean-square error of the one-step-ahead prediction error is computed for $p = 0, 1, \ldots, L$ where L is some upper limit. The appropriate order of the model is given by p which gives the minimum mean square error.

IV.3 Whiteness of the Residuals

One method of determining whether the model actually fits the data is to choose a model order p and then compute the residuals $\hat{e}_k$ between the two values x_k and the predicted values $\hat{x}_k$ at each sample k, i.e. $\hat{e}_k = x_k - \hat{x}_k$. If the residuals are indeed a white noise process, then the AR model is a reasonable fit to the data and GM signal has a random excitation. After choosing the model order, the whiteness of the residual was tested using a Kolmogorov-Smirnov one-sample test [10]. First the power spectrum was determined, then the normalized cumulative distribution is constructed and compared with that of a flat spectrum for a level of significance of 5%. Acceptable deviations for an underlying white noise signal are given in [10].

V. DIGITAL SIMULATION OF GEOMAGNETIC RECORDS

V.1 Construction of Synthetic GM Records

For each hourly record analyzed, the model order selected was used and the appropriate coefficients a_p were determined. The residuals or errors, between the predicted values and the observed values were then computed at each sample i as $\hat{e}_i = x_i - \hat{x}_i$ The empirical probability distribution function F of these $\{\hat{e}_i\}$ and $\{a_p\}$ were then determined. To generate the white noise process with estimated F, we used a random number generator to generate a series of gaussian random numbers of zero mean and variance σ^2. The value of the variance σ^2 is determined by calculating the residual signal power. The probability distributions for each coefficient a_p were found to be unimodal. Assuming that these distributions exhibit nearly a first-order Gamma shape with the appropriate mode , the different distributions can be simulated as follows. The Gamma probability density function can be simulated by adding two identically distributed Poisson distributions. These magnitudes can be generated from a uniformly distributed set of random numbers, RN, transformed as $(-1/\lambda)$

ln(RN) where λ is the ensemble mean of each coefficient (The precise form of these distributions is unlikely to be very significant).

A set of random number representing each of these (AR) coefficients a_p were generated using the mean value of each coefficient ensemble. In order to generate synthetic geomagnetic records, a set of initial values is needed to get the AR process started. We initiated each synthetic record with the same p consecutive x_n values from the original geomagnetic signal. However, if the AR process represents a stable filter, any transients due to the specific initial conditions will decay and any initial conditions, including all $x_n = 0$, will be acceptable. In the instances where the time series was corrected for trends in an attempt to produce white residuals, the synthesized signals were then modified to include the trends which were initially removed. An ensemble of 50 hourly records of period one month were generated using different realization of the white noise excitation as input and different sets of AR coefficients.

V.2 Evaluation Of Synthetic Signals

In order to assess whether the synthetic GM records have a structure similar to true GM records, various methods were used for evaluation of both the true and synthetic GM signals. These methods are already used here and likely to be used for evaluation [11].

A- Root-Mean-Square Amplitude

The rms amplitude is computed as

$$rms = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2} \tag{6}$$

B- Amplitude Distribution

Amplitude distribution is a measure of the amount of time a signal spends at certain level. Therefore, the amplitude distributions of the original and simulated signals were constructed and compared using a K-S two-sample test [10] at a 5% level of significance.

C- Zero Crossings

We used a simple measure of zero crossing. A zero crossing occurred whenever $x_i < 0$ and $x_{i+1} > 0$ or whenever $x_i > 0$ and $x_{i+1} < 0$. There was no effort to see if the zero crossing was in any way sustained (i.e. by seeing if the previous m points were on one side of zero and the next m points on the other side of zero.) The number of zero crossings was determined and expressed as the number of zero crossings per month.

D- The Number Of Peak Values

While GM records a generally have a disorganized appearance, it is typically organized sufficiently to find individual rise and fall patterns and count them. A threshold is set equal to the mean value plus one standard deviation of the signal and the number of maxima detected in the simulated signals is compared to that obtained from the true records.

VI. RESULTS

For the GM records used for constructing the AR models, the number of sign changes in $D_{\bar{x}_i}$ and D_{A_i} for the segments ranged from three to seven (average 5.0). This indicates the mean and rms values were within limits of 95% of all stationary processes and there is no statistically discernible trend in these values over the time span examined.

The optimal model order for the two methods used in the determination of the AR coefficients is eleven. This optimal model order produced white residuals using the K-S test. The range of the normalized residual error (the ratio of the variance of the residual errors to the variance of the original signal) lies between 0.0092 and 0.2515 This confirms the model fitness.

Fig. 3 and 4 display portions of the true GM records (the top tracing) and two synthesized tracings (the bottom two tracings). As these figures indicate, the synthesized GM signals appear to be in good agreement with the real signal.

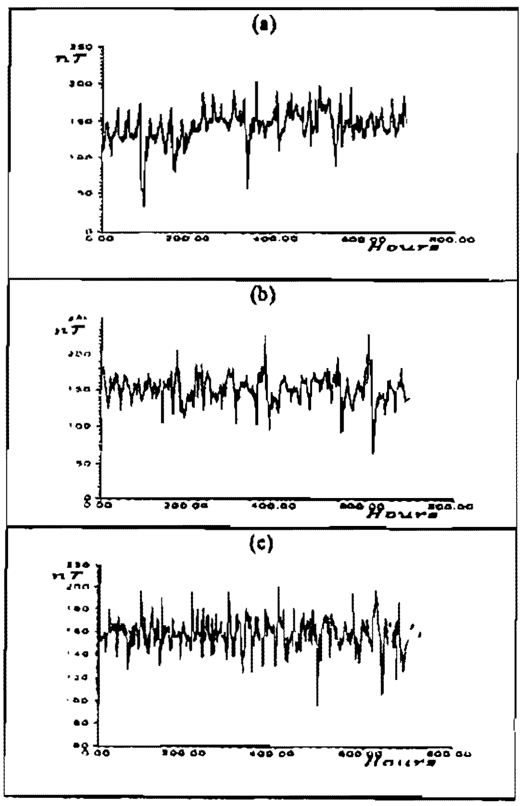

Fig 3 An hourly record (a) the original GM signal (H-element)
(b) and (c) two synthesized GM records (period = one month)

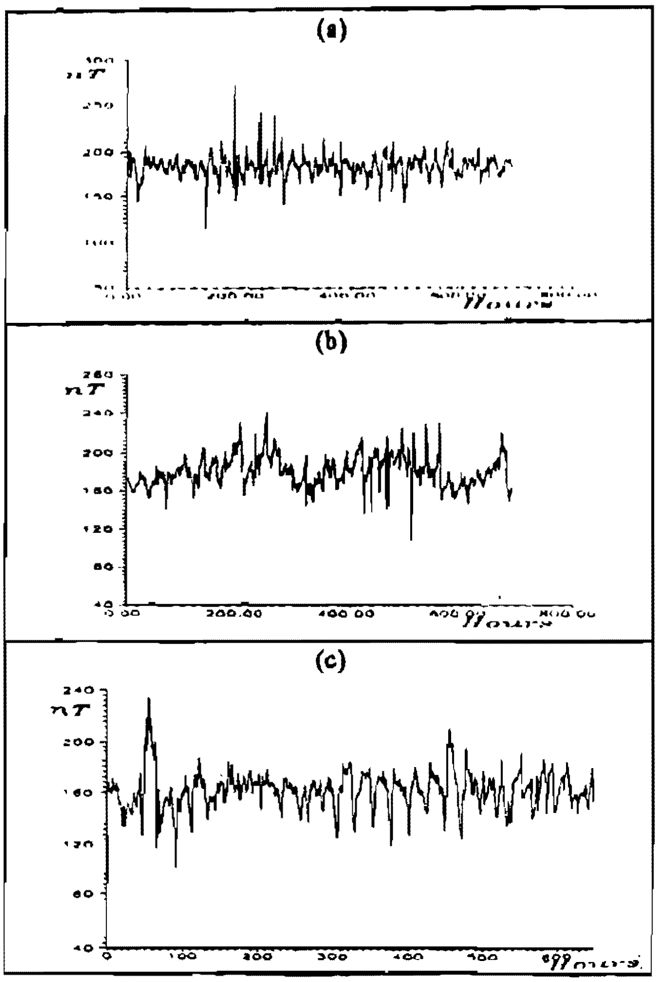

Fig.4 An hourly record (a) the original GM signal (Z-element)
(b) and (c) two synthesized GM records (period = one month)

Table I displays the results of the rms amplitude and each of the three algorithms used for evaluating the synthesized GM signals. There are two rows associated with each geomagnetic element. The first row (actual) indicates the value of the measured statistic for the true GM records used to reconstruct the AR model. The second row (synthesized) indicates the mean ± standard deviation for a number of five records synthesized for one month. The results in this row represent the AR model which most closely matched the true GM signal using Yule-Walker method.

As displayed in the table, the rms amplitudes of the synthesized and true GM signals used to generate them are very similar for all geomagnetic elements. Column 3 in the table displays the results of applying the amplitude distribution algorithm to both true and synthesized GM signals. The values of the deviation D resulting from the K-S test are within the range of acceptable values at 5% level of significance, of the true signals.

The results of applying the simple zero-crossing algorithm to both the true and synthetic signals are displayed in column 4 of the same table. The number of zero crossing per month for the synthetic signals is generally within the range of the number of zero crossing for the true signals used to generate the model. The number of the individual rise and fall patterns (peak values) occurring in both the true and synthetic GM records is reported in column 5 of Table 1. Most of the synthesized GM signals produce a number of rise and fall patterns near the number existed in the true signals.

Table 1 Comparison of computed values of real and simulated GM records

Element	GM records	RMS	K-S test Deviation D $\alpha = 0.05$	Number of zero-crossings	Number of peaks
H	Real	18.1	0.0245	55	18
	Simulated	17.0 ± 4.26		56 ± 3	16 ± 2
D	Real	3.01	0.0239	57	20
	Simulated	2.83 ± 1.31		58 ± 3	23 ± 2
Z	Real	21.5	0.0251	56	17
	Simulated	20.3 ± 4.52		57 ± 3	25 ± 2

VII. CONCLUSION

For the short-term variations typically exhibited by the hourly geomagnetic (GM) records, the mean and rms amplitude appear to be stationary with no discernible trends. Rather than being totally random and without structure, the hourly GM records over a short-time period (one month) can be modeled as a deterministic element with a white noise excitation. Both FFT and the Chaos theory do not provide a method for separating the deterministic component from the excitation noise that time-series modeling allows. The decomposition of the signal using an AR model of sufficient order produces white residuals, this allowing synthesis of statistically equivalent signals. The results of comparing the characteristics of the synthesized GM records to those of the true GM signals have indicated that there was generally no significant difference in the values of the rms amplitude, the amplitude distributions, the number of zero crossings per month, and number of rise and fall patterns.

By decomposing each hourly record of the geomagnetic elements into an autoregressive process with a white noise input, it is possible to study in detail and forecast the geomagnetic variability. Synthetic signals may allow developing and testing algorithms using models based on an existing small data set while the process of acquiring new data continues.

It is concluded that geomagnetic records can be modeled as autoregressive processes for at least the short period of one month typically used in the present study. A practical application of this model is in forecasting purposes. Using the statistics of the available records of successive years, it would be possible to generate synthetic records of an ensemble of hourly GM signals, each of one-year length. This would help in testing and training the forecasting algorithm which is currently being developed.

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