LYAPUNOV STABILITY ANALYSIS OF LARGE-SCALE POWER SYSTEMS USING THE DECOPMOSITION-AGGREGATION METHOD

H .Shaaban

Assist. Prof. . Faculty of Engineering and Technology. Menoufia University.

Shebin El-Kom, Egypt.

Abstract:

The aim of this paper is to carry out transient stability analysis of an N-machine power system using the decomposition-aggregation method, and considering a more sophisticated generator model. Each of the system generators is represented by the so-called 2-axis model [1], in which the two components E'_q and E'_d of the generator internal voltage E' are considered to change with time. This is instead of assuming the voltage E' , or the voltage component E'_d , to be constant as usually considered, for simplicity, in power system stability analysis using the direct methods.

The system network. in which the loads are represented by constant impedances. is reduced to the generators internal nodes. Describing each generator by a fourth-order dynamic model,and considering uniform mechanical damping, the system mathematical model (the transfer conductances are included) is obtained and decomposed into (N-1) interconnected subsystems by using the pair-wise decomposition. A square aggregation matrix of the order (N-1) is obtained , and stability of this matrix implies asymptotic stability of the system equilibrium.

The developed stability approach is applied to a 3-machine,4-bus power system example and an estimate for the system asymptotic stability domain is determined. A 3-phase short circuit fault. with successful reclosure. is assumed near a system bus. and the stability computations are carried out. A reclosure time for the faulted line is determined such that the system can regain its prefault (normal) conditions. It is shown that the developed approach is suitable and applicable in practical and on-line stability studies of power systems .

1 Introduction

With the advent of large power systems came a renewed interest in the stability properties of such systems. Indeed, the tendency of a power system to lose synchronism appears to be much more prevalent for large systems than for relatively isolated groups. Most stability investigations of large power systems are based on direct simulation of the system and integration of the differential equations of the system for various initial conditions. and to observe if the various machines tend to lose or maintain synchronism. However. this method becomes cumbersome and very costly for very large systems involving a great number of generators. This explains the need for direct methods for stabil-

MANUSCRIPT RECEIVED FROM DR. H.SHAABAN AT: 8/11/1995 .
ACCEPTED AT: 24/12/1995, PP 1 - 15 .
ENGINEERING RESEARCH BULLETIN, VOL.19, NO. 1, 1996
MENOUFIYA UNIVERSITY, FACULTY OF ENGINEERING,
SHEBIEN EL-KOM, EGYPT . ISSN. 1110-1180

ity investigations. These methods determine stability without explicitly solving the differential equations describing the system dynamics. Obviously, the direct methods advantage over the standard numerical integration procedure is their rapidity and the resulting saving of computing time [2].

However, the direct methods of stability analysis are acknowledged to provide satisfactory practical results, as far as the use of a simplified mathematical system description may be acceptable. It is to be noted that only classical generator model(that is .constant internal voltage behind the generator transient reactance) can be used, and the effects of control and stability aids can not be represented [3].

Because of the high efficiency of the Lyapunov's direct method, it has important applications in power system design and operation. It can be used, for example, for estimating critical fault clearing time, for on-line security assessment, and for emergency control. This method has come, recently, to possess accuracy well consistent with results predicted by simulations for relatively simplified system representations [4].

In the last two decades the decomposition-aggregation method, which is based on Bellman's concept of vector Lyapunov functions[5], has been used for stability analysis of large-scale power systems[6-17].

In Ref.[18],a matrix Lyapunov function was constructed and used for the system aggregation.

However, the expected advantages of the decomposition-aggregation method are numerous[19]. It is obvious that the Lyapunov function of a low-order disconnected (free)subsystem can handle more sophisticated generator and transmission models. Furthermore , exact estimates of the overall system stability domain may be defined.

It is to be noted that , the power system stability analysis was carried out in the papers[6-18], considering the generator classical model. This is equivalent to neglecting the effect of generators flux decays.

In the papers[4,20-22], the transient stability analysis of multimachine power systems have been carried out considering the generator third-order dynamic model ,that is, the generator internal voltage component E'_q ,. is changed with time, while the voltage component E'_d , is kept constant during the transient period. The authors applied the scalar Lyapunov function approach, and they introduced different forms for the used (scalar) Lyapunov functions which were constructed under the assumption that all transfer conductances G_{ij} , are neglected.

In the present paper an N-machine power system is considered, and the two internal voltage components E'_q and E'_d ,of each machine are assumed to change with time. Assuming the uniform mechanical damping case and applying the pair-wise decomposition (each subsystem including two machines, one of them is the comparison machine) the system mathematical model (the transfer conductances are taken into consideration) is obtained, and it is decomposed into N-1 interconnected subsystems. Then, each subsystem is decomposed into free (disconnected) subsystem contains three (the largest number) nonlinearities, and interconnections. Finally, a square aggregation matrix of the order (N-1) is obtained .and stability of this matrix implies asymptotic stability of the system equilibrium.

2 Power system model

Consider an N-machine power system (the transfer conductances are included)with mechanical damping, and let us assume that the machine parameters M_i and P_{mi} are constant.

Now assume that each machine (the stator resistance is neglected) is represented by the two-axis model [1], in which the two components E'_q and E'_d of the internal voltage

E' (see Fig. 1) are considered to be time variables. The absolute motion of the ith machine is described by the equations,

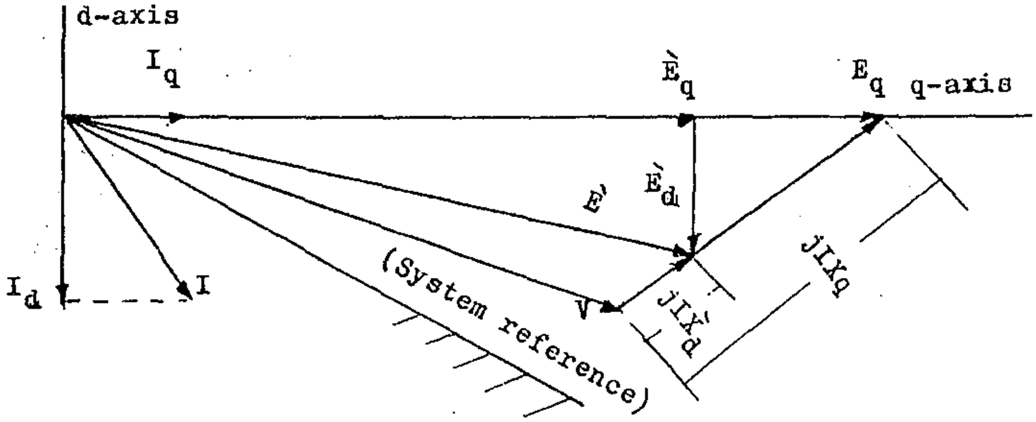

Fig. 1 Phasor diagram of system generator

$$M_i \ddot{\delta}_i + D_i \dot{\delta}_i = P_{mi} - P_{ei}$$
$$T'_{d0i} \dot{E}'_{qi} = E_{fdi} - E'_{qi} + (X_{di} - X'_{di}) I_{di}$$
$$T'_{q0i} \dot{E}'_{di} = - E'_{di} - (X_{qi} - X'_{qi}) I_{qi} \qquad (1)$$

where

$$P_{ei} = E'_{di} I_{di} + E'_{qi} I_{qi} - (X'_{qi} - X'_{di}) I_{di} I_{qi}, \quad i = 1,2,\ldots\ldots,N \qquad (2)$$

It is to be noted that, the dynamics of the automatic voltage regulator (AVR) are not considered ,for simplicity , and hence the voltage E_{fdi} will be equal to its pretransient value E_{fdi}.

Under the assumption $X'_{di} = X'_{qi}$ (machines with solid cylindrical rotors are considered), we get [1],

$$P_{ei} = \sum_{j=1}^{N} Y_{ij} \{ E'_{qi} [E'_{qj} \cos(\theta_{ij} - \delta_{ij}) - E'_{dj} \sin(\theta_{ij} - \delta_{ij})] + E'_{di} [E'_{dj} \cos(\theta_{ij} - \delta_{ij}) + E'_{qj} \sin(\theta_{ij} - \delta_{ij})] \} \quad i = 1,2,\ldots\ldots,N \qquad (3)$$

where δ_i, is the rotor angle of the i-th machine, *or position of the rotor q-axis from the* common reference frame.

Now, let us introduce the following (4N-1) state variables (the Nth machine is selected as a comparison machine)

$$\sigma_{iN} = \delta_{iN} - \overset{o}{\delta}_{iN} \qquad , i = N$$
$$\omega_i = \dot{\delta}_i \qquad , i = 1,2,\ldots\ldots,N$$
$$E_{Qi} = E'_{qi} - \overset{o}{E}'_{qi} \quad ; \quad E_{Di} = E'_{di} - \overset{o}{E}'_{di} \quad , i = 1,2,\ldots\ldots,N \qquad (4)$$

where $\overset{o}{\delta}_{iN}$, $\overset{o}{E}'_{di}$ and $\overset{o}{E}'_{qi}$ *are the pretransient values of* δ_{iN} ,E'_{di} and E'_{qi} ,respectively.

Assuming the uniform damping case, that is, when

$$(D_i / M_i) = \lambda \qquad , i = 1,2,\ldots\ldots,N \qquad (5)$$

we can derive the mathematical model of the whole system as

$$\dot{\sigma}_{iN} = \omega_i - \omega_N = \omega_{iN}$$

$$\dot{\omega}_i = -\lambda\ \omega_i - (1/M_i)\, G_{ii}[E^2_{Qi} + 2E_{Qi}\overset{o}{E}'_{qi} + E^2_{Di} + 2E_{Di}\overset{c}{E}'_{di}] - (1/M_i)$$
$$\Sigma Y_{ij}\,[\{A_{ij}\, f_{ij}(\sigma_{ij}) + A^*_{ij}\, g_{ij}(\sigma_{ij})\} + (\overset{c}{E}'_{qi}E_{Qj} + \overset{o}{E}'_{di}E_{Dj})\cos(\theta_{ij} - \delta_{ij}) +$$
$$+ (\overset{o}{E}'_{di}E_{Qj} - \overset{o}{E}'_{qi}E_{Dj})\sin(\theta_{ij} - \delta_{ij}) + \{E_{Qi}(E_{Qj} + \overset{o}{E}'_{qj}) + E_{Di}(E_{Dj} + \overset{o}{E}'_{dj})\}$$
$$\cos(\theta_{ij} - \delta_{ij}) + \{E_{Di}(E_{Qj} + \overset{c}{E}'_{qj}) - E_{Qi}(E_{Dj} + \overset{c}{E}'_{dj})\}\sin(\theta_{ij} - \delta_{ij})]$$

$$T'_{d0i}\dot{E}_{Qi} = -[1 - (X_{di} - X'_{di})B_{ii}]E_{Qi} + (X_{di} - X'_{di})[G_{ii}E_{Di} + \Sigma Y_{ij}\{\overset{o}{E}'_{dj} f_{ij}(\sigma_{ij}) -$$
$$- \overset{o}{E}'_{qj}\, g_{ij}(\sigma_{ij}) + E_{Qj}\sin(\theta_{ij} - \delta_{ij}) + E_{Dj}\cos(\theta_{ij} - \delta_{ij})\}]$$

$$T'_{q0i}\dot{E}_{Di} = -[1 - (X_{qi} - X'_{qi})B_{ii}]E_{Di} - (X_{qi} - X'_{qi})[G_{ii}E_{Qi} + \Sigma Y_{ij}\{\overset{o}{E}'_{qj} f_{ij}(\sigma_{ij}) +$$
$$+ \overset{o}{E}'_{dj}\, g_{ij}(\sigma_{ij}) + E_{Qj}\cos(\theta_{ij} - \delta_{ij}) - E_{Dj}\sin(\theta_{ij} - \delta_{ij})\}] \quad , i = 1,2,\ldots, N \quad (6)$$

where, Σ is defined as $\sum_{j\neq i}^{N}$, and the nonlinear functions f_{ij} and g_{ij} are given as

$$f_{ij}(\sigma_{ij}) = \cos(\sigma_{ij} + \overset{o}{\delta}_{ij} - \theta_{ij}) - \cos(\overset{c}{\delta}_{ij} - \theta_{ij})$$
$$g_{ij}(\sigma_{ij}) = \sin(\sigma_{ij} + \overset{o}{\delta}_{ij} - \theta_{ij}) - \sin(\overset{o}{\delta}_{ij} - \theta_{ij}) \qquad (7)$$

3 Power system decomposition

Decomposition of the considered N-machine system is carried out , in the paper, as follows:

1- All the system loads are represented by constant impedances to ground (those impedances are obtained from the pretransient conditions in the system).

2- All the system nodes, except the generators internal nodes, are eliminated . Hence, we obtain the system Nth-order reduced admittance matrix **Y** .

3- Referring to the obtained **Y**-matrix, and using the pair-wise decomposition [7-9,11-14] the system is decomposed into (N - 1) "two-machine" subsystems.

Now, defining the state vector X_I in the form

$$X_I = [\sigma_{iN}, \omega_{iN}, E_{Qi}, E_{Di}, E_{QN}, E_{DN}]^T = [X_{I1}, X_{I2}, X_{I3}, X_{I4}, X_I, X_{I6}]^T \qquad (8)$$

we can decompose the mathematical model of the whole system (eqn. 6) into S = N - 1 sixth-order interconnected subsystems. Each subsystem can be written in the general form

$$\dot{X}_I = P_I\, X_I + B_I F_I(\sigma_I) + h_I(X) \quad , \sigma_I = C^T_I\ X_I \quad , I = 1,2,\ldots\ldots, S \qquad (9)$$

where P_I , B_I and C_I are constant matrices with appropriate dimensions, and $F_I(\sigma_I)$ is a nonlinear vector function, whose elements are arbitrary chosen.

Referring to eqn. 8, we derive the matrix P_I in the form

$$P_I = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\lambda & -2G_{ii}\overset{o}{E}'_{qi}/M_i & -2G_{ii}\overset{c}{E}'_{di}/M_i & 2G_{NN}\overset{o}{E}'_{qN}/M_N & 2G_{NN}\overset{c}{E}'_{dN}/M_N \\ 0 & 0 & [-1+(X_{di}-X'_{di})B_{ii}]/T'_{d0i} & K_i G_{ii} & 0 & 0 \\ 0 & 0 & -L_i\, G_{ii} & [-1+(X_{qi}-X'_{qi})B_{ii}]/T'_{q0i} & 0 & 0 \\ 0 & 0 & 0 & 0 & [-1+(X_{dN}-X'_{dN})B_{NN}]/T'_{doN} & K_N G_{NN} \\ 0 & 0 & 0 & 0 & -L_N G_{NN} & [-1+(X_{qN}-X'_{qN})B_{NN}]/T'_{qoN} \end{bmatrix} \qquad (10)$$

Now, in order to obtain a larger stability domain estimate [13-17], it is assumed that the following three (the largest number) nonlinear functions (see eqn. 7) are included in the vector F_I.

$$f_{I1}(\sigma_{I1}) = \cos(\sigma_{iN} + \overset{o}{\delta}_{iN} - \theta_{iN}) - \cos(\overset{o}{\delta}_{iN} - \theta_{iN})$$
$$f_{I2}(\sigma_{I2}) = \sin(\sigma_{iN} + \overset{o}{\delta}_{iN}) - \sin\overset{o}{\delta}_{iN}$$
$$f_{I3}(\sigma_{I3}) = \cos(\sigma_{Ni} + \overset{o}{\delta}_{Ni} - \theta_{iN}) - \cos(\overset{o}{\delta}_{Ni} - \theta_{iN}) \qquad (11)$$

Note carefully that the three functions given by eqn.11 , satisfy the following conditions

$$f_{Ik}(0) = 0 \; ; \quad 0 \le \sigma_{Ik} f_{Ik}(\sigma_{Ik}) \le \xi_{Ik}\sigma^2_{Ik} \qquad , k = 1,2,3 \qquad (12)$$

on the bounded intervals which are defined for the three functions, respectively, as follows

$$-2(\pi - \theta_{iN} + \overset{o}{\delta}_{iN}) \le \sigma_{iN} \le 2(\theta_{iN} - \overset{o}{\delta}_{iN})$$
$$-(\pi + 2\overset{o}{\delta}_{iN}) \le \sigma_{iN} \le (\pi - 2\overset{o}{\delta}_{iN})$$
$$-2(\pi - \theta_{iN} - \overset{o}{\delta}_{iN}) \le \sigma_{Ni} \le 2(\theta_{iN} + \overset{o}{\delta}_{iN}) \qquad (13)$$

In eqn.12, the positive constants ξ_{Ik} may be determined as

$$\xi_{Ik} = \partial f_{Ik}(\sigma_{Ik}) / \partial\sigma_{Ik} \Big|_{\sigma_{Ik}=0}, \; k = 1,2,3 \qquad (14)$$

Note also that there exist positive constants, $\varepsilon_{Ik} \in (0, \xi_{Ik})$, for which the following condition

$$\sigma_{Ik} f_{Ik}(\sigma_{Ik}) \ge \varepsilon_{Ik}\sigma^2_{Ik} \qquad , k = 1,2,3 \qquad (15)$$

is satisfied on the compact interval of σ_{Ik}.

$$U_{Ik} = [\underline{U}_{Ik}, \overline{U}_{Ik}] \qquad , k = 1,2,3 \qquad (16)$$

where $\underline{U}_{Ik}$, $\overline{U}_{Ik}$ are the negative and positive solutions, respectively, of the equation

$$f_{Ik}(\sigma_{Ik}) = \varepsilon_{Ik}\sigma_{Ik} \qquad , k = 1,2,3 \qquad (17)$$

Now, referring to eqn. 6 , we define the following matrices

$$F_I(\sigma_I) = [f_{I1}(\sigma_{I1}), f_{I2}(\sigma_{I2}), f_{I3}(\sigma_{I3})]^T \qquad (18)$$

$$C^T_I = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (19)$$

$$B_I = \begin{bmatrix} 0 & 0 & 0 \\ -A_{iN}Y_{iN}/M_i & [(1/M_i) - (1/M_N)]A^*_{iN}G_{iN} & A_{iN}Y_{iN}/M_N \\ K_i Y_{iN}\overset{o}{E}'_{dN} & -K_i\overset{o}{E}'_{qN}G_{iN} & 0 \\ -L_i Y_{iN}\overset{o}{E}'_{qN} & -L_i\overset{o}{E}'_{dN}G_{iN} & 0 \\ 0 & K_N\overset{o}{E}'_{qi}G_{iN} & K_N\overset{o}{E}'_{di}Y_{iN} \\ 0 & L_N\overset{o}{E}'_{di}G_{iN} & -L_N\overset{o}{E}'_{qi}Y_{iN} \end{bmatrix} \qquad (20)$$

Let us , for simplicity , write the (vector) matrix $h_I(X)$, as the sum of two (vector) matrices,

$$h_I(X) = h_I(X_I) + h^*_I(X) \qquad (21)$$

where

$$h_I(X_I) = [0, h_{I2}(X_I), h_{I3}(X_I), h_{I4}(X_I), h_{I5}(X_I), h_{I6}(X_I)]^T$$

$$h^*_I(X) = [0, h^*_{I2}(X), h^*_{I3}(X), h^*_{I4}(X), h^*_{I5}(X), h^*_{I6}(X)]^T \quad (22)$$

The elements of the (vector) matrix $h_I(X_I)$, are given as

$$h_{I2}(X_I) = -[G_{ii}(X^2_{I3} + X^2_{I4})/M_i] + [G_{NN}(X^2_{I5} + X^2_{I6})/M_N] - [(1/M_i) + (1/M_N)] A^*_{iN} B_{iN} f^*_{I4}(X_{I1}) - Y_{iN}[(1/M_i)\cos(\theta_{iN} - \delta_{iN}) - (1/M_N \cos(\theta_{iN} - \delta_{Ni})] \{\mathring{E}'_{qN} X_{I3} + \mathring{E}'_{qi} X_{I5} + \mathring{E}'_{dN} X_{I4} + \mathring{E}'_{di} X_{I6} + X_{I3} X_{I5} + X_{I4} X_{I6}\} - Y_{iN}[(1/M_i)\sin(\theta_{iN} - \delta_{iN}) + (1/M_N)\sin(\theta_{iN} - \delta_{Ni})] \{-\mathring{E}'_{dN} X_{I3} + \mathring{E}'_{qN} X_{I4} + \mathring{E}'_{di} X_{I5} - \mathring{E}'_{qi} X_{I6} + X_{I3} X_{I6} + X_{I4} X_{I5}\}$$

$$h_{I3}(X_I) = K_i \mathring{E}'_{qN} B_{iN} f^*_{I4}(X_{I1}) + K_i Y_{iN}[X_{I5} \sin(\theta_{iN} - \delta_{iN}) + X_{I6} \cos(\theta_{iN} - \delta_{iN})]$$

$$h_{I4}(X_I) = L_i \mathring{E}'_{dN} B_{iN} f^*_{I4}(X_{I1}) + L_i Y_{iN}[X_{I6} \sin(\theta_{iN} - \delta_{iN}) - X_{I5} \cos(\theta_{iN} - \delta_{iN})]$$

$$h_{I5}(X_I) = -K_N \mathring{E}'_{qi} B_{iN} f^*_{I4}(X_{I1}) + K_N Y_{iN}[X_{I3} \sin(\theta_{iN} + \delta_{iN}) + X_{I4} \cos(\theta_{iN} + \delta_{iN})]$$

$$h_{I6}(X_I) = L_N \mathring{E}'_{di} B_{iN} f^*_{I4}(X_{I1}) + L_N Y_{iN}[X_{I4} \sin(\theta_{iN} + \delta_{iN}) - X_{I3} \cos(\theta_{iN} + \delta_{iN})]$$

and the elements of the matrix $h^*_I(X)$, are defined as

$$h^*_{I2}(X) = -(1/M_i)[\Sigma Y_{ij} \{[A_{ij} f_{ij}(\sigma_{ij}) + A^*_{ij} g_{ij}(\sigma_{ij})] + [\mathring{E}'_{qj} X_{I3} + \mathring{E}'_{dj} X_{I4} + \mathring{E}'_{qi} X_{J3} + \mathring{E}'_{di} X_{J4} + X_{I3} X_{J3} + X_{I4} X_{J4}] \cos(\theta_{ij} - \delta_{ij}) - [\mathring{E}'_{dj} X_{I3} - \mathring{E}'_{qj} X_{I4} + \mathring{E}'_{qi} X_{J4} - \mathring{E}'_{di} X_{J3} + X_{I3} X_{J4} - X_{I4} X_{J3}] \sin(\theta_{ij} - \delta_{ij})\}] + (1/M_N)[\Sigma Y_{Nj} \{[A_{jN} f_{Nj}(\sigma_{Nj}) + A^*_{jN} g_{Nj}(\sigma_{Nj})] + [\mathring{E}'_{qN} X_{J3} + \mathring{E}'_{dN} X_{J4} + \mathring{E}'_{qj} X_{I5} + \mathring{E}'_{dj} X_{I6} + X_{I5} X_{J3} + X_{I6} X_{J4}] \cos(\theta_{Nj} - \delta_{Nj}) - [\mathring{E}'_{qN} X_{J4} - \mathring{E}'_{dN} X_{J3} + \mathring{E}'_{dj} X_{I5} - \mathring{E}'_{qj} X_{I6} + X_{I5} X_{J4} - X_{I6} X_{J3}] \sin(\theta_{Nj} - \delta_{Nj})\}]$$

$$h^*_{I3}(X) = K_i \Sigma Y_{ij} \{[\mathring{E}'_{dj} f_{ij}(\sigma_{ij}) - \mathring{E}'_{qj} g_{ij}(\sigma_{ij})] + X_{J3} \sin(\theta_{ij} - \delta_{ij}) + X_{J4} \cos(\theta_{ij} - \delta_{ij})\}$$

$$h^*_{I4}(X) = -L_i \Sigma Y_{ij} \{[\mathring{E}'_{qj} f_{ij}(\sigma_{ij}) + \mathring{E}'_{dj} g_{ij}(\sigma_{ij})] + X_{J3} \cos(\theta_{ij} - \delta_{ij}) - X_{J4} \sin(\theta_{ij} - \delta_{ij})\}$$

$$h^*_{I5}(X) = K_N \Sigma Y_{Nj} \{[\mathring{E}'_{dj} f_{Nj}(\sigma_{Nj}) - \mathring{E}'_{qj} g_{Nj}(\sigma_{Nj})] + X_{J3} \sin(\theta_{Nj} - \delta_{Nj}) + X_{J4} \cos(\theta_{Nj} - \delta_{Nj})\}$$

$$h^*_{I6}(X) = -L_N \Sigma Y_{Nj} \{[\mathring{E}'_{qj} f_{Nj}(\sigma_{Nj}) + \mathring{E}'_{dj} g_{Nj}(\sigma_{Nj})] + X_{J3} \cos(\theta_{Nj} - \delta_{Nj}) - X_{J4} \sin(\theta_{Nj} - \delta_{Nj})\} \quad (23)$$

Note that Σ is defined as $\sum_{j \neq i}^{N-1}$, and the nonlinear function $f^*_{I4}(X_{I1})$, is given in the form

$$f^*_{I4}(X_{I1}) = \cos(X_{I1} + \mathring{\delta}_{iN}) - \cos \mathring{\delta}_{iN}$$

4 Power system aggregation

Let us, as a first step, decompose each of the interconnected subsystems of eq. 9, into the free (disconnected) subsystem, described by the equations

$$\dot{X}_I = P_I X_I + B_I F_I(\sigma_I) \quad ; \quad \sigma_I = C^T_I X_I \quad , I = 1,2,\ldots\ldots,S \qquad (25)$$

and the interconnections $h_I(X)$.

Next, we accept a free subsystem Lyapunov function in the form [7-10, 13 - 17],

$$V_I(X_I) = X_I^T H_I X_I + \sum_{m=1}^{3} \gamma_{Im} \int_0^{\sigma_I} f_{Im}(\sigma_{Im})\, d\sigma_{Im} \quad , I = 1,2,\ldots,S \qquad (26)$$

where H_I is sixth-order symmetric positive definite matrix, γ_{Im} are arbitrary positive numbers, and the nonlinear functions f_{Im} are given by eqn. 11. Finally, following the aggregation procedure in Reference 23, an aggregation matrix, $A = [\alpha_{IJ}]$, is constructed. The elements (real numbers) of this matrix obey the inequality

$$\dot{V}_I(X_I) \leq \sum_{J=1}^{S} \alpha_{IJ} U_I(X_I) U_J(X_J) \quad , I = 1,2,\ldots\ldots,S \qquad (27)$$

where $\dot{V}_I(X_I)$, is the total time derivative of the function $V_I(X_I)$, along the motion of the i-th interconnected subsystem of eqn. 9.

It is to be noted that the left-hand side of eqn. 27, can be written as

$$\dot{V}_I(X_I) = \dot{V}_I(X_I)_f + [\operatorname{grad} V_I(X_I)]^T h_I(X) \qquad (28)$$

where $\dot{V}_I(X_I)_f$, is the total time derivative of the function V_I, along the motion of the i-th free subsystem.

In eqn. 27, the comparison functions U_I and U_J, are chosen in the form [7,9]

$$U_k(X_k) = \|X_k\| = (X^T_k X_k)^{1/2} \quad \text{for } k = 1,2,\ldots\ldots, S \qquad (29)$$

4.1 Stability criterion

According to theorem 1 of Reference 23, stability of the aggregation matrix, $A = [\alpha_{jk}]$, or, equivalently, if it is satisfied the Hick's conditions

$$(-1)^k \begin{vmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1k} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2k} \\ \vdots & \vdots & & \vdots \\ \alpha_{k1} & \alpha_{k2} & \cdots & \alpha_{kk} \end{vmatrix} > 0 \qquad k = 1,2,\ldots\ldots,S \quad (30)$$

implies asymptotic stability of the system equilibrium.

4.2 Aggregation matrix

As a first step for determining the system aggregation matrix, the two terms in the right-hand side of eqn. 28, are computed. Then the following majorizations are introduced,

$$|f^*_{I4}(X_{I1})| \leq \eta_i |X_{I1}| \qquad , \quad \eta_i = |\sin \overset{o}{\delta}_{Ni}|$$

$$|f_{ij}(\sigma_{ij})| \leq \xi_{ij} (|X_{I1}| + |X_{J1}|) \quad , \quad \xi_{ij} = |\sin(\theta_{ij} - \overset{o}{\delta}_{ij})|$$

$$|g_{ij}(\sigma_{ij})| \leq \xi^*_{ij} (|X_{I1}| + |X_{J1}|) \quad , \quad \xi^*_{ij} = |\cos(\theta_{ij} - \overset{o}{\delta}_{ij})|$$

$$|f_{Nj}(\sigma_{Nj})| \leq \xi_{Nj} |X_{J1}| \qquad , \quad \xi_{Nj} = |\sin(\theta_{jN} + \overset{o}{\delta}_{jN})|$$

$$|g_{Nj}(\sigma_{Nj})| \leq \xi^*_{Nj} |X_{J1}| \qquad , \quad \xi^*_{Nj} = |\cos(\theta_{jN} + \overset{o}{\delta}_{jN})|$$

$$a \sin(\theta - \delta) + b \cos(\theta - \delta) \leq \sqrt{a^2 + b^2} \qquad (31)$$

where, a and b are any given (positive,negative, or even zero)numbers.
Finally, the right-hand side of eqn.28, is majorized as,

$$\dot{V}_I(X_I) \leq -\lambda^*_I \| X_I \|^2 + \sum_{K \neq I}^{S} 2\ Z_2(Z^*_I ; Z^\sim_I)\ \| X_I \| \| X_K \| \quad , I = 1,2,\ldots\ldots,S \qquad (32)$$

where λ^*_I is the minimal (positive) eigenvalue of the sixth-order symmetric matrix R_I, whose elements are given by eqn. 35 , and the elements Z^*_I and $Z^\sim_I$ are defined by eqn. 37 (see Appendix) .

Comparing eqns. 27 and 32 , the system aggregation matrix , $A = [\alpha_{IK}]$, of order (N-1) is derived, and its elements are defined as

$$\alpha_{IK} = \begin{cases} -\lambda^*_I & , K = I \\ \\ 2 Z_2(Z^*_I ; Z^\sim_I) & , K \neq I \qquad K, I = 1,2,\ldots, S = N-1 \end{cases} \qquad (33)$$

It is of importance to note that, stability of the aggregation matrix A (see condition 30), can be easily ensured for larger values of the eigenvalue λ^* ,and / or smaller values of the off-diagonal elements α_{ij} . However, smaller values of the elements α_{ij} ,can be obtained by decomposing the system , referring to the reduced admittance matrix **Y** , so that *only weak interconnections among internal nodes of the system machines* appear as subsystem couplings.

<u>*5 Numerical example*</u>

Fig. 2 shows the one-line diagram of the 3-machine, 4-bus power system which is chosen, in this example, for an application of the developed stability approach. The system stability computations are carried out as follows:

1 - The reactances X'_d and X'_q *are inserted at the respective buses of the* system, and we copute

$\overset{o}{E}'_{q1} = 1.01926$, $\overset{o}{E}'_{d1} = -0.01615$, $\delta_1 = -3.93^o$; $\overset{o}{E}'_{q2} = 1.00532$,
$\overset{o}{E}'_{d2} = -0.00362$, $\delta_2 = -2.76$ $\overset{o}{E}'_{q3} = 1.03389$, $\overset{o}{E}'_{d3} = -0.01097$, $\delta_3 = 0.72^o$

2 - The equivalent impedances of the system loads are computed and inserted in the network.Then the system nodes, except the machines internal nodes, are eliminated, and the system reduced third-order symmerric admittance matrix **Y** , is determined as,

$Y_{11} = 0.30553 \underline{/-73.04^o}$; $Y_{12} = 0.00113 \underline{/93.64^o}$; $Y_{13} = 0.23709 \underline{/91.27^o}$

$Y_{22} = 0.11819 \underline{/-79.52^o}$; $Y_{23} = 0.10514 \underline{/90.85^o}$; $Y_{33} = 0.57099 \underline{/-58.93^o}$

3 - Referring to the system matrix **Y**, given in step 2, the system is decomposed (machine 3 is chosen as the comparison machine) into two "two-machine" subsystems.. Then, selecting the following parameters

$M_1 = M_2 = 0.20$, $M_3 = 14.0$; $\lambda = 6.0$

$\varepsilon_{11} = 0.60$, $\varepsilon_{21} = 0.44$; $\varepsilon_{i3} = 0.001$, $i = 1,2$;

$h^i_{12} = 1.0$, $i = 1,2$; $h^1_{33} = 300$, $h^1_{55} = 150$; $h^2_{33} = 200$,

$h^2_{55} = 40$; $K_1 = 0.45$, $K_2 = 0.15$

and using expression (33), we compute the matrix

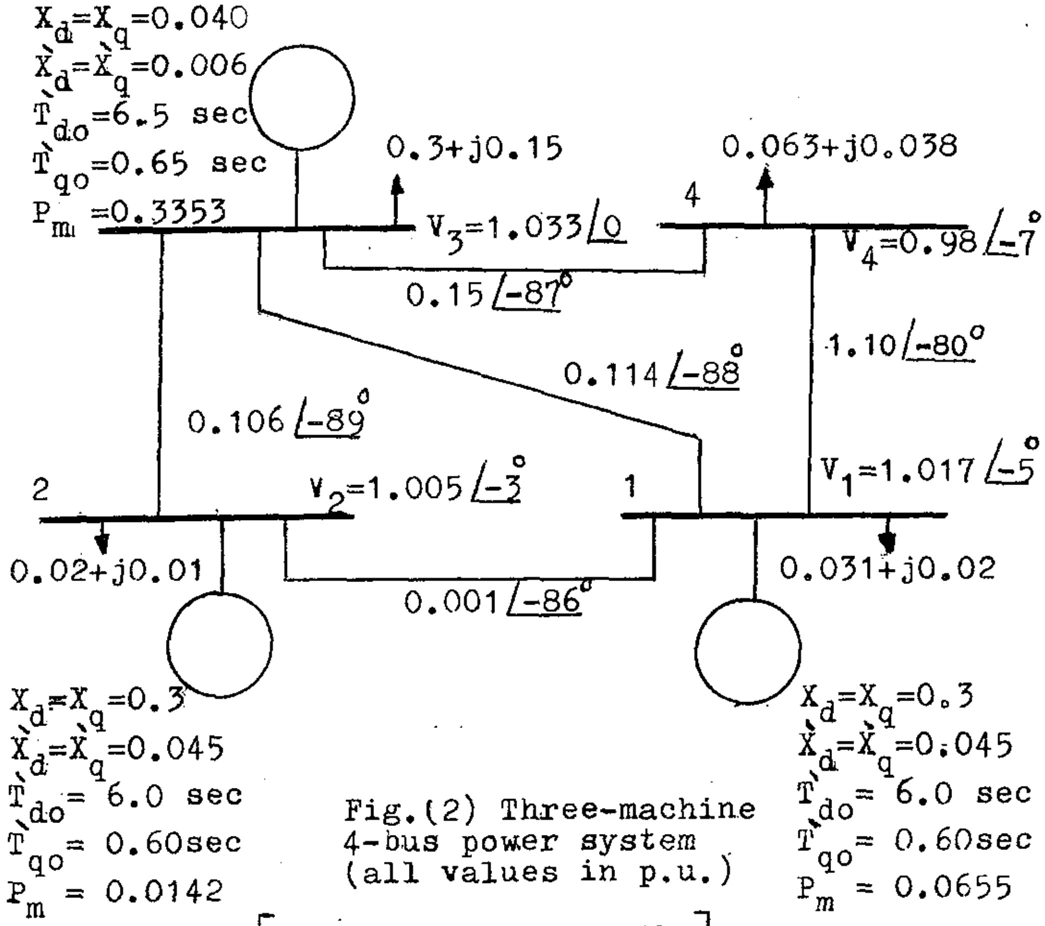

Fig.(2) Three-machine 4-bus power system (all values in p.u.)

$$A = \begin{bmatrix} -0.745154 & 0.449899 \\ 0.280617 & -0.178351 \end{bmatrix}$$

which is a stable matrix and satisfies conditions (30). This implies asymptotic stability of the system equilibrium. Then, according to theorem 4 of Reference 23, and referring to the *Appendix in Reference 16,* we compute

$$E_1 = \{ X : (V_1 (X_1) + 1.5 V_2 (X_2)) \leq 4.680125 \} \qquad (34)$$

as an estimate of the system asymptotic stability domain.

4 - As an application of the developed approach to practical stability studies of the considered system, it is assumed that a 3-phase short circuit fault, with successful *reclosure, occurred near bus* 4, *at 5 % of the* distance between the buses 1 and 4. The fault is cleared, by switching off the faulted line, after 0.24 second from the fault instant. *Considering* the *fault* and *fault-clearing conditions, the system equations (see eqn.* 1), are solved. For each time interval the Lyapunov function, given by eqn.26, is computed for each subsystem. Substituting the two computed Lyapunov functions into eqn.34, it is found that this equation is satisfied ($V_1 = 4.63527$, and $V_2 = 0.00395$, are computed) at 0.450 second from the fault-clearing instant.

Fig. 3, shows variations of the six *state* variables (the time is measured from the instant at which the open line is reclosed) of the first subsystem, which includes the machines 1 and *3*.

It is obvious, referring to Fig.3, that the system will regain its prefault (normal) condition after reclosing (the fault is disappeared) the faulted line.

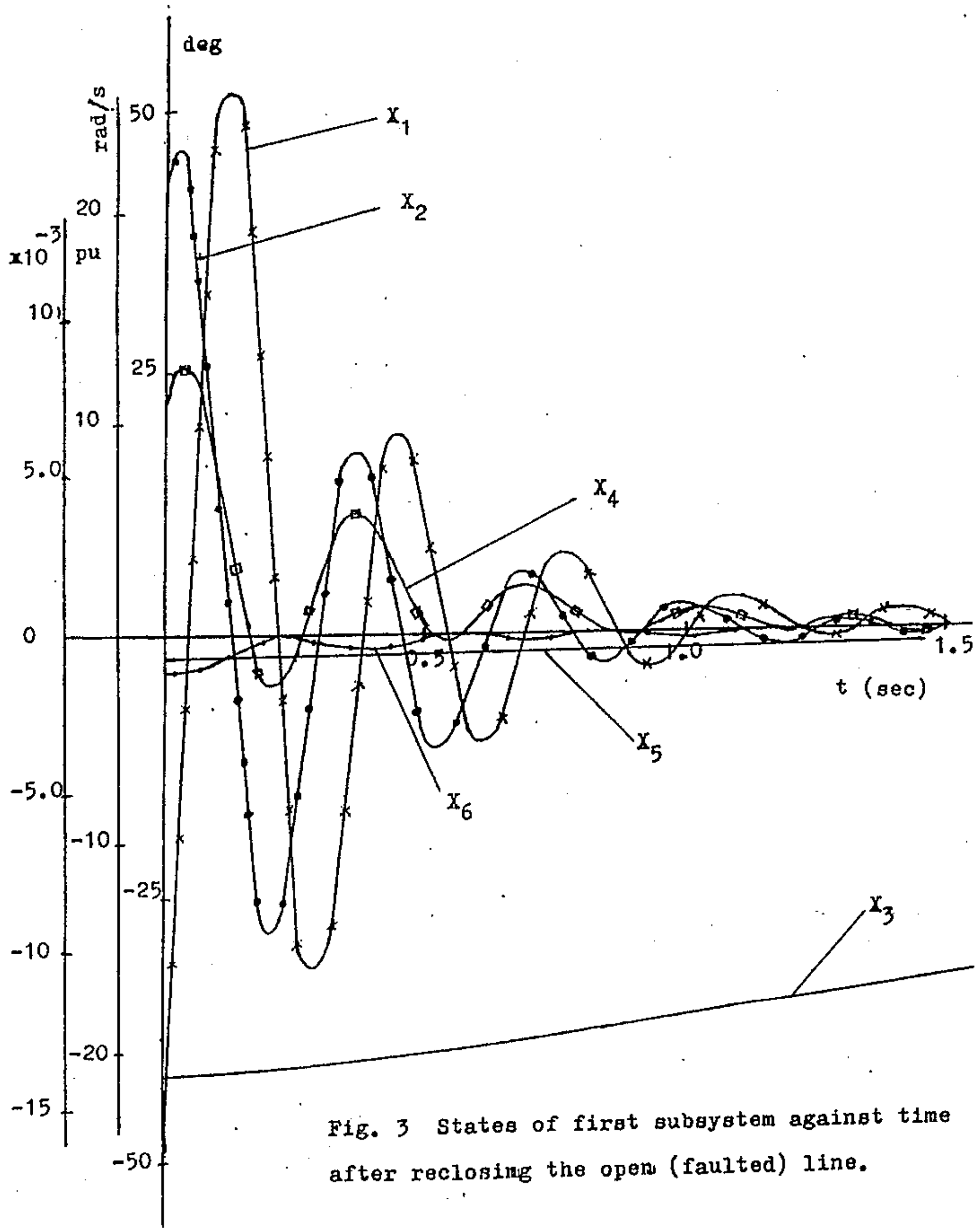

Fig. (3) States of the first subsystem against time after reclosing the open (faulted) line.

6 Conclusions

A transient stability approach is developed, in the paper, for multi machine power systems considering the 2-axis generator model instead of the one-axis model, or the classical model, which are usually considered for transient stability studiess using the direct methods. Thus each generator is described by a fourth-order dynamic model.

The approach developed is applied to a 3-machine, 4-bus power system, and an estimate for the system asymptotic stability domain is determined. A 3-phase short circuit fault ,with successful reclosure, is assumed to occurr near one of the system buses ,and a reclosure time for the faulted line is determined such that the system can regain its prefault (normal) conditions. It is found that the stability approach developed is suitable and can be easily used for practical and on-line stability studies of multimachine power systems in which number of the machines may be more than three .

7 *References*

1- ANDERSON, P.M., and FOUAD, A .A.: 'Power system control and stability', (Book) Iowa State University Press,1977

2- RIBBENS-PAVELLA, M., CALVAER, A., and GHEURY, J.:' Transient stability index for on-line evaluation', IEEE PES Winter Meeting, New York, Feb. 3-8, 1980

3- KUNDUR, P.:' Evaluation of methods for studying power system stability', International Symps. on Power syst. Stability, AMES,IOWA,May 13-15,1985

4- KAKIMOTO, N., OHNOGI, Y., MATSUDA, H., and SHIBUYA, H.:' Transient stability analysis of large-scale power system by Lyapunov's direct method', IEEE Trans.,1984,PAS-103,No1, pp. 160-167

5- BELLMAN, R.:' Vector Lyapunov functions', SIAM J. Control & Optimiz., 1962,No.1, pp. 32-34

6- PAI, M.A., and NARAYANA, C.L.:' Stability of large-scale power systems', Proc. of 6th IFAC World Congress, Boston, 1975, pp. 1-10

7 - GRUJIC, LJ.T., and RIBBENS-PAVELLA, M.:' New approach to stability domain estimate of large-scale power systems', Revue E,1977,Vol.8.No.10, pp.241-249

8 - JOCIC, LJ.B., and RIBBENS-PAVELLA, M., and SILJAK, D.D.:' Multimachine power systems : stability, decomposition, and aggregation', IEEE Trans.,1978, AC-23, No.2, pp. 325-332

9 - GRUJIC, LJ.T., RIBBENS-PAVELLA, M., and BOUFFIOUX, A.:' Asymptotic stability of large-scale systems with application to power systems. Part 2: transient analysis', Int. J. Electr. Power Energy Syst., 1979 , Vol.1, No.3, pp. 158-165

10- ARAKI, M., METWALLY, M.M., and SILJAK, D.D.:' Generalized decompositions for transient stability analysis of multimachine power systems'. Proc. Joint Automatic Control Conf., California, August,1980, pp. 1-7

11- MAHALANABIS, A.K., and SINGH, R.:' On the analysis and improvement of the transient stability of multimachine power systems', IEEE Trans. 1981,PAS-100,No.4, pp. 1574-1579

12 - MICHEL, A.N., NAM, B.H., and VITTAL, V.:' Computer generated Lyapunov functions for interconnec- ted systems: improved results with applications to power systems', IEEE Trans.,1984,CAS-31.No.2,pp. 189-198

13 - SHAABAN, H., and GRUJIC, LJ.:' An improved pair-wise decomposition aggregation approach to stability analysis of multimachine power systems', R.A.I.R.O. Automatique/Systems Analysis and Control,1984,Vol.18,No.1, pp. 63-77

14 - SHAABAN, H., and GRUJIC, LJ.:' Transient stability analysis of power systems via aggregation on subsets', Int.J.Control,1994,Vol.59,No.6,pp.1401-1419

15 - SHAABAN, H., and GRUJIC, LJ.:' Transient stability analysis of large-scale power systems with speed governor via vector Lyapunov functions'.Proc. IEE, Vol. 132,pt.D,No.2,pp 45-52,1985.

16 - SHAABAN, H., and GRUJIC, LJ.:' Improvement of large-scale power systems decomposition-aggregation approach', Int. J. Electr. Power Energy Syst. Vol .8,No.4,pp 211-220, 1986.

17 - SHAABAN, H.:` New decomposition-aggregation approach applied to power system with speed governor', IEE-Proc.-C,1991,Vol.138,No.5,pp. 434-444

18 - GRUJIC, LJ., and SHAABAN, H.:` On transient stability analysis of large-scale power systems', Computing and Computers for Control systems, J.C. Baltzer AG,Scientific Publishing Co. IMACS,1988,pp.249-254

19 - XUE, Y., VAN CUTSEM, TH., and RIBBENS-PAVELLA, M.:` A new decomposition method and direct criterion for transient stability assessment of large-scale electric power systems', IMACS Symp. Modelling and Simulation for Control of Lumped and Distributed Parameter Systems,Lille,France,3rd-6th June,1986

20 - KAKIMOTO, N., OHSAWA, Y., and HAYASHI, M.:` Transient stability analysis of multimachine power systems with field flux decays via Lyapunov's direct method', IEEE Trans.,1980,PAS-99,No.5, pp. 1819-1827

21 - PAI, M.A.:` Power system stability analysis by the direct method of Lyapunov', (Book),North-Holland publishing Co.,1981

22 - BERGEN, A.R., HILL, D.J., and DE MARCOT, C.L.:` Lyapunov function for multimachine power systems with generator flux decay and voltage dependent loads', Int. J. Elect. Power Energy Syst.,1986, pp.2-10

23 - GRUJIC, LJ., and RIBBENS-PAVELLA, M.:` Asymptotic stability of large-scale systems with application to power systems. Part 1 : domain estimation', Int. J. Elect. Power Energy Syst.,1979,1, pp. 151-157

Nomenclature

P_{mi} = mechanical power delivered to ith machine

P_{ei} = electrical power delivered by ith machine

δ_i = rotor angle, or position of the rotor q-axis from the reference

X_{di} , X_{qi} = direct-axis,quadrature-axis synchronous reactances

X'_{di} , X'_{qi} = d-axis, q-axis transient reactances

E_{fd} =exciter voltage referred to the armature circuit

E'_i = voltage behind d-axis transient reactance

E'_{di} , E'_{qi} = d-axis, q-axis components of the voltage E'_i

E_q = armature emf corresponding to the field current

$\mathring{E}_{fdi}$, $\mathring{E}'_{qi}$, $\mathring{E}'_{di}$= pre-transient(or steady-state)values of the voltages E_{fdi} ,E'_{qi} and E'_{di} ,respectively

ω_i = rotor speed with respect to the synchronous speed

$Y_{ij} = Y_{ji}$ = modulus of transfer admittance between internal nodes of ith and jth generators

$\theta_{ij} = \theta_{ji}$ = phase angle of transfer admittance Y_{ij}

D_i = mechanical damping

$\lambda_i = (D_i / M_i)$= mechanical damping coefficient

$\delta_{ij} = \delta_i - \delta_j = \delta_{iN} - \delta_{jN}$

$\sigma_{ij} = \delta_{ij} - \mathring{\delta}_{ij} = \sigma_{iN} - \sigma_{jN}$, $\sigma_{kN} = \delta_{kN} - \mathring{\delta}_{kN}$, $k = i , j$

$A_{kN} = \mathring{E}'_{qk} \mathring{E}'_{qN} + \mathring{E}'_{dk} \mathring{E}'_{dN}$, $A^*_{kN} = \mathring{E}'_{qN} \mathring{E}'_{dk} - \mathring{E}'_{qk} \mathring{E}'_{dN}$, $k = i , j$

$A_{ij} = \mathring{E}'_{qi} \mathring{E}'_{qj} + \mathring{E}'_{di} \mathring{E}'_{dj}$; $A^*_{ij} = \mathring{E}'_{qi} \mathring{E}'_{dj} - \mathring{E}'_{di} \mathring{E}'_{qj}$

$G_{ij} = Y_{ij} \cos \theta_{ij}$ = transfer conductance

$B_{ij} = Y_{ij} \sin\ \theta_{ij}$ = transfer susceptance

T'_{doi}= direct-axis transient open-circuit time constant of ith generator

T'_{qoi}= quadrature-axis transient open-circuit time constant of ith generator

$K_j = (X_{dj} - X'_{dj}) /\ T'_{doj}$; $L_j = (X_{qj} - X'_{qj}) /\ T'_{qoj}$, $j = i, N$

Z_2, Z_3 = two functions, defined as follows:

$$Z_2(\alpha, \phi) = \min \{\sqrt{2}\ \max(\ |\alpha|, |\phi|\ )\ ; (\ |\alpha| + |\phi|\ )\}$$

$$Z_3(\alpha,\phi,\mu) = \min \{\ 2 \max\ (|\alpha|, |\phi|, |\mu|); (\ |\alpha| + |\phi| + |\mu|\ ) ; Z_2[Z_2(\alpha, \phi), \mu]; Z_2[Z_2(\phi, \mu), \alpha] ; Z_2[Z_2(\mu, \alpha), \phi]\ \}$$

8 APPENDIX : *Definition of the elements of the matrix R_I*

Elements of the sixth-order symmetric matrix R_I (see eqn.33), are defined as follows:

$$r^I_{11} = 2\, h^I_{12}\{A_{iN} Y_{iN} [(1/M_i)\, \varepsilon_{I1} + (1/M_N)\, \varepsilon_{I3}] - |[(1/M_i) - (1/M_N)] A^*_{iN} G_{iN}|\, \xi_{I2} - [(1/M_i) + (1/M_N)]\, |A^*_{iN}|\, B_{iN}\, \eta_i - (1/M_i) \Sigma Y_{ij} |A^*_{ij}|\, \xi^*_{ij}\}$$

$$r^I_{12} = -h^I_{22}\ \{\ |(1/M_i)[A^*_{iN} - |A^*_{iN}|] - (1/M_N) A^*_{iN}|\, |G_{iN}|\, \xi_{I2} + [(1/M_i) + (1/M_N)]\, |A^*_{iN}|\, B_{iN}\, \eta_i + (1/M_i)\ \Sigma Y_{ij} [A_{ij}\ \xi_{ij} + |A^*_{ij}|\ \xi^*_{ij}]\ \}$$

$$r^I_{13} = -\ [\max\{\ [(1/M_i) G_{ii} \mathring{E}'_{qi} h^I_{12} + K_i h^I_{33} \{Y_{iN} |\mathring{E}'_{dN}|\, \xi_{I1} + \Sigma Y_{ij} |\mathring{E}'_{dj}|\, \xi_{ij}\}] ; K_i h^I_{33} \mathring{E}'_{qN} |G_{iN}|\, \xi_{I2}\} + \Lambda_I h^I_{12} + C_i \mathring{E}'_N Y_{iN} h^I_{12} + K_i h^I_{33} B_{iN} \mathring{E}'_{qN}\, \eta_i + (1/M_i)\, h^I_{12}\ \Sigma Y_{ij}\ \mathring{E}'_j + K_i h^I_{33}\ \ \Sigma Y_{ij} \mathring{E}'_{qj} \xi^*_{ij}]$$

$$r^I_{14} = -\ [\max\{\ L_i h^I_{44} [Y_{iN} \mathring{E}'_{qN}\, \xi_{I1} + \mathring{E}'_{dN} G_{iN}\ \xi_{I2} + \Sigma Y_{ij} \mathring{E}'_{qj}\ \xi_{ij}] ; (1/M_i)\, G_{ii} |E'_{di}|\ h^I_{12}\} + \Lambda_I\ h^I_{12} + C_i\ \mathring{E}'_N Y_{iN}\ h^I_{12} + L_i h^I_{44} B_{iN} |\mathring{E}'_{dN}|\, \eta_i + (1/M_i)\, h^I_{12}\ \Sigma Y_{ij}\ \mathring{E}'_j + L_i h^I_{44}\ \ \Sigma Y_{ij}\ |\mathring{E}'_{dj}|\, \xi^*_{ij}]$$

$$r^I_{15} = -\ [\ \max\{\ [(1/M_N) h^I_{12}\ \mathring{E}'_{qN} G_{NN}\ \ + K_N h^I_{55} |\mathring{E}'_{di}|\, Y_{iN}\, \xi_{I3}] ; K_N \mathring{E}'_{qi} h^I_{55}\ |G_{iN}|\, \xi_{I2}\ \} + \Lambda_I\ h^I_{12} + C_i\ \mathring{E}'_i Y_{iN}\ h^I_{12} + K_N\ h^I_{55} B_{iN} \mathring{E}'_{qi}\, \eta_i + (1/M_N)\, h^I_{12}\ \Sigma Y_{Nj}\ \mathring{E}'_j\ ]$$

$$r^I_{16} = -[\ \max\{L_N h^I_{66} [|\mathring{E}'_{di} G_{iN}|\, \xi_{I2} + Y_{iN} \mathring{E}'_{qi}\, \xi_{I3}] ; (1/M_N)\, G_{NN}\ |\mathring{E}'_{dN}|\ h^I_{12}\} + \Lambda_I\ h^I_{12} + C_i\ \mathring{E}'_i Y_{iN}\ h^I_{12} + L_N\ h^I_{66} B_{iN} |\mathring{E}'_{di}|\, \eta_i + (1/M_N)\, h^I_{12}\ \Sigma Y_{Nj}\ \mathring{E}'_j\ ]$$

$$r^I_{22} = 2\ K_I\ \ h^I_{12}$$

$$r^I_{23} = -\, h^I_{22}\ [\ (1/M_i)\, G_{ii}\ \mathring{E}'_{qi} + \Lambda_I + C_i\ \mathring{E}'_N Y_{iN} + (1/M_i)\, \Sigma Y_{ij} \mathring{E}'_j]$$

$$r^I_{24} = -\, h^I_{22}\ [\ (1/M_i)\, G_{ii} |\mathring{E}'_{di}| + \Lambda_I + C_i\ \mathring{E}'_N Y_{iN} + (1/M_i)\, \Sigma Y_{ij} \mathring{E}'_j]$$

$$r^I_{25} = -h^I_{22}\ \ [\ (1/M_N)\, G_{NN} \mathring{E}'_{qN} + \ \Lambda_I + C_i\ \mathring{E}'_i Y_{iN} + (1/M_N)\, \Sigma Y_{Nj} \mathring{E}'_j]$$

$$r^I_{26} = -h^I_{22}\ \ [\ (1/M_N)\, G_{NN} |\mathring{E}'_{dN}| + \Lambda_I + C_i\ \mathring{E}'_i Y_{iN} + (1/M_N)\, \Sigma Y_{Nj} \mathring{E}'_j]$$

$$r^I_{34} = r^I_{56} = 0 \quad ; \quad r^I_{35} = r^I_{36} = r^I_{45} = r^I_{46} = C^*_i Y_{iN}$$

$$r^I_{33} = 2\ (1/T'_{d0i})\, [1 - (X_{di} - X'_{di})\, B_{ii}]\ h^I_{33}$$

$$r^I_{44} = 2\ (1/T'_{q0i})\ [1-(X_{qi} - X'_{qi})B_{ii}]\ (K_i / L_i)\, h^I_{33}$$
$$r^I_{55} = 2\ (1/T'_{d0N})\ [1-(X_{dN} - X'_{dN})B_{NN}]\ h^I_{55}$$
$$r^I_{66} = 2\ (1/T'_{q0N})\ [1-(X_{qN} - X'_{qN})B_{NN}]\ (K_N / L_N)\, h^I_{55} \qquad (35)$$

It is to be noted that Σ is defined as $\sum_{j \neq i}^{N-1}$, and the following constants are given

$$h^I_{22} = \{(1+K_I)/\lambda\}\, h^I_{12} \quad , \quad h^I_{44} = (K_i/L_i)\ h^I_{33} \ ,$$
$$h^I_{66} = (K_N/L_N)\ h^I_{55}$$

where, K_I, h^I_{12}, h^I_{33} and h^I_{55}, are arbitrary positive constants.

In eqn. 35, the elements C_i and C^*_i are defined as

$$C_i = \sqrt{(1/M_i)^2 + (1/M_N)^2 - 2\,(1/M_i)(1/M_N)\cos 2\theta_{iN}}$$
$$C^*_i = \sqrt{(K_i h^I_{33})^2 + (K_N h^I_{55})^2 - 2\, K_i K_N\, h^I_{33}\, h^I_{55} \cos 2\theta_{iN}}$$

and Λ_I, is magnitude of the maximal eigenvalue of the fourth-order symmetric matrix Q_i, whose elements are given as

$$q^i_{11} = q^i_{22} = -(1/M_i)\ G_{ii} \ ;\ q^i_{33} = q^i_{44} = (1/M_N)\ G_{NN}$$
$$q^i_{13} = q^i_{14} = q^i_{23} = q^i_{24} = 0.5\ C_i\ Y_{iN} \ ;\ q^i_{12} = q^i_{34} = 0 \qquad (36)$$

Definition of the elements Z^*_I and $Z^\sim_I$

The elements Z^*_I and $Z^\sim_I$, given in eqn.33, are defined as follows :

$$\begin{aligned} Z^*_I = Z_3\, [& Z_2\,(h^I_{12} ; h^I_{22})\{\max[(1/M_i)Y_{ij} A_{ij}\, \xi_{ij} ; (1/M_N)Y_{Nj} A_{jN}\, \xi_{Nj}] + \\ & + (1/M_i)\, Y_{ij} |A^*_{ij}| \xi^*_{ij} + (1/M_N)\, Y_{Nj} |A^*_{jN}|\ \xi^*_{Nj}\} \ ; \\ & ; K_i h^I_{33} Y_{ij}\ \ Z_2 \{[\,|\mathring{E}'_{dj}|\, \xi_{ij} + \mathring{E}'_{qj}\, \xi^*_{ij}] ; [\mathring{E}'_{qj}\, \xi_{ij} + |\mathring{E}'_{dj}|\ \xi^*_{ij}]\} \ ; \\ & ;\ K_N h^I_{55} Y_{Nj} Z_2 \{[\,|\mathring{E}'_{dj}|\, \xi_{Nj} + \mathring{E}'_{qj}\, \xi^*_{Nj}] ;\ [\mathring{E}'_{qj}\ \xi_{Nj} + |\mathring{E}'_{dj}|\, \xi^*_{Nj}]\}] \end{aligned}$$

$$\begin{aligned} Z^\sim_I = Z_3\, [& Z_2\,(h^I_{12} ; h^I_{22})\ \{(1/M_i)Y_{ij}\, \mathring{E}'_i + (1/M_N)Y_{Nj}\, \mathring{E}'_N + Z_2\,[(1/M_i)Y_{ij} ; \\ & ;\ (1/M_N)Y_{Nj}]\}\ ;\ \sqrt{2}\ \ K_i h^I_{33} Y_{ij} \ ;\ \sqrt{2}\ \ K_N h^I_{55} Y_{Nj}\ ] \end{aligned} \qquad (37)$$

" تحليل اتزان ليابونوف لأنظمة القوى كبيرة المقيـــــاس
باستخدام طريقة الفـك والتراكـــــــــم "

- الغرض من هذا البحث هو انجاز تحليل الاتزان الأنتقالى لنظام قوى، يحتوى على ن من المولدات، وذلك بأستخدام طريقة الفك والتراكـم .
- تم تمثيل كل مولد فى النظام بما يسمى نموذج المحورين ، وفى هذا النموذج اعتبرت مركبتى الجهد بأنها تتغير مع الزمن . وذلك بدلا من أفتراض أن مقياس الجهد يكون مقدارا ثابت مع الزمن ، أو افتراض نموذج المحور الواحد. يلاحظ أن افتراض ثبـات مقياس الجهد أو أستخدام المحور الواحد يفترض عادة عند اجراء تحليل الاتزان لأنظمة القوى باستخدام الطرق المباشـرة ؛ وذلك للسهوله .
- تم تمثيل أحمال نظام القوى بأنها معاوقات ثابته ، بعد ذلك تم أزاله جميع عقـــد الأحمال فقط وبذلك أمكن الحصول على مصفوفه السماحيات للنظام وهى من الدرجـــه ن . ثم وصف كل مولد بنموذج ديناميكـى من الدرجــة الرابعـــه .
- تم افتراض حاله الأخماد الميكانيكى المتماثل ، ثم تم الحصول على النموذج الرياضـــى للنظام . تم فك النظام الرياضى الى عدد (ن-١) تحت نظام مرتبط وذلك باستخدام الفك الثنائى ، والذى فيه يكون كل تحت نظام مشتملا على مولدين فقط أحدهمـــا مولـــد دليــل .
- لكل تحت نظام تم افتراض داله ليابونوف وهى تتكون من صوره مربعة + مجمـــــوع تكاملات ثلاثه دوال غير خطيه. هذة الدوال تم استخدامها لتكوين متجه دالـــــــه ليابونوف تم اجراء التراكم للنظام بأستخدام متجه داله ليابونوف وبذلك أمكن الحصـــول على مصفوفه التراكم (المربعة) وهى من الدرجه (ن-١).
- تم تطبيق أسلوب الاتزان المقدم فى البحث على نظام قوى يشتمل على ثلاثه مولدات وأربعة قضبان . أمكن الحصول على حيز أتزان للنظـــام .
- تم أفتراض حدوث قصر ثلاثى الأوجه فى موضع قريب من أحد القضبان، ثم أجريــــت حسابات الأتزان . ثم تحديد زمن لأرجاع الخط، الذى تم فصله لعزل منطقـــــه حدوث الخطأ، المفصول بحيث يستطيع النظام العوده الى نفس خالته قبل حـــــدوث الخطـــأ .
- وجد أن أسلوب الأتزان المقدم فى البحث مناسبا ويمكن بسهوله استخدامه فـــــــــى الدراسات العمليه لأنظمه القوى والتى تحتوى على عدد من المولـــدات .