

**ALGORITHMIC APPROACH FOR THE FOURIER
ANALYSIS WITH APPLICATION TO THE ANNUAL
SMOOTHED SUNSPOTS NUMBER FROM THE
YEAR 1700.5 TO 2000.5**

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Abstract

In this paper, algorithmic approach for Fourier analysis of smoothed data is developed and applied to the annual sunspots number from the years 1700.5 to 2000.5. The precision criteria of the representation is very satisfactory.

Introduction

A growing mass of evidence suggests that the solar activity affects our weather and long-term variations of the sun's energy output affects our climate. The literature on this subject covers a period of 300 years, and many distinguished scientists have contributed (see, e.g. the extraordinary number of articles on the site: <http://adsabs.harvard.edu>). Moreover almost every large solar-terrestrial symposium now includes at least one session on sun weather/climate investigations. On the other hand, the basic measure of the solar activity is the number of the sunspots visible on the solar disk at any given time; the more spots, the more active is the sun[3].

Now, if the sunspots are a key factor, that is, a good usable indicator of solar activity for sun-weather relationships, an obvious

condition must be met before sunspot's number can be used to predict changes in weather and climate: The sunspots themselves must be predictable. In fact it is very important to have a full understanding of sunspot predictability for sun weather purposes.

In the present paper ,we started the first phase towards sunspot predictability by developing algorithmic approach for Fourier analysis of smoothed data. As an application of the algorithm we considered the annual sunspot's number from the years 1700.5 to 2000.5. The precision criteria of the representation is very satisfactory

2. Basic Formulations

2.1. Harmonic Analysis of a Periodic Function

•Let it be required to find a sum

$$\frac{1}{2}a_0 + \sum_{j=1}^r a_j \cos jx + \sum_{j=1}^r b_j \sin jx , \quad (2.1)$$

which furnishes the best possible representation of a function $u(x)$, given that $u(x)$ takes the values u_0, u_1, \dots, u_{m-1} , when x takes the values x_0, x_1, \dots, x_{m-1} respectively. Finally, m being some number greater than $2r$. The problem is to determine the $(2r+1)$ constants a_0, a_j and $b_j; j = 1, 2, \dots, r$ so as to make the expression (2.1) takes ,as nearly as possible, the m values u_0, u_1, \dots, u_{m-1} when x takes x_0, x_1, \dots, x_{m-1} . To do so we shall make use of the method of least squares[1] and we get

$$\begin{aligned} \frac{1}{2}a_0\eta_{0l} + \sum_{j=1}^r a_j\eta_{lj} + \sum_{j=1}^r b_j\beta_{jl} &= d_l ; l = 0, 1, \dots, r, \\ \frac{1}{2}a_0\beta_{0q} + \sum_{j=1}^r a_j\beta_{qj} + \sum_{j=1}^r b_j\gamma_{qj} &= c_q ; q = 1, 2, \dots, r, \end{aligned} \quad (2.2)$$

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where

$$\begin{aligned}
 \eta_{jl} &= \eta_{jl} = \sum_{k=0}^{m-1} \cos lx_k \cos jx_k; l = 0,1,\dots, r; j = 0,1,\dots, r, \\
 \beta_{ql} &= \sum_{k=0}^{m-1} \cos lx_k \sin qx_k; l = 0,1,\dots, r; q = 1,2,\dots, r, \\
 \gamma_{qt} &= \gamma_{qt} = \sum_{k=0}^{m-1} \sin qx_k \sin tx_k; q = 1,2,\dots, r; t = 1,2,\dots, r, \\
 d_l &= \sum_{k=0}^{m-1} u_k \cos lx_k; l = 0,1,\dots, r, \\
 c_q &= \sum_{k=0}^{m-1} u_k \sin qx_k; q = 1,2,\dots, r.
 \end{aligned} \tag{2.3}$$

Equations (2.3) are called the *normal equations of the least squares method*. These equations represent a set of linear equations in $(2r+1)$ unknowns a 's and b 's coefficients and could be solved by any of the methods adopted for linear systems. However, the coefficient matrix of this set could be reduced to a diagonal one by certain choice of the arguments x_k and in this case the a 's and b 's are determined exactly and the problem is known as *harmonic analysis*.

- In the method of harmonic analysis, the arguments x_k take the special values;

$$0, \frac{2\pi}{m}, 2 \cdot \frac{2\pi}{m}, 3 \cdot \frac{2\pi}{m}, \dots, (m-1) \cdot \frac{2\pi}{m}. \tag{2.4}$$

For these values, the η 's, β 's and γ 's of Equations(2.3) become:

$$\text{For } l = j \neq 0: \eta_{jl} = \gamma_{jl} = \frac{1}{2}m; \beta_{jl} = 0.$$

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For $l \neq j$: $\eta_{lj} = \gamma_{lj} = \beta_{lj} = 0$.

Consequently the a 's and b 's coefficients could then be computed exactly from

$$a_j = \frac{2}{m} \sum_{k=0}^{m-1} u_k \cos j \frac{2\pi}{m} k ; j = 0, 1, \dots, r, \tag{2.5}$$

$$b_q = \frac{2}{m} \sum_{k=0}^{m-1} u_k \sin q \frac{2\pi}{m} k ; q = 1, 2, \dots, r.$$

2.2. Practical Computations of the a 's and b 's Coefficients

The a 's and b 's coefficients of Equations (2.5) could be computed efficiently [4] from

$$a_j = \frac{2}{m} \{u_0 + Q_{1,j} \cos \frac{2\pi}{m} j - Q_{2,j}\}; j = 0, 1, \dots, r, \tag{2.6.1}$$

$$b_q = \frac{2}{m} Q_{1,q} \sin \frac{2\pi}{m} q ; q = 1, 2, \dots, r, \tag{2.6.2}$$

where, for any j the Q 's are computed recursively from

$$Q_{k,j} = u_k + 2 \cos x_j Q_{k+1,j} - Q_{k+2,j}, \tag{2.6.3}$$

by using the initial conditions $Q_{m,j} = Q_{m+1,j} = 0$, starting with $k=m-1$ to compute successively $Q_{m-1,j}, Q_{m-2,j}, \dots, Q_{1,j}$.

2.3. The Sum of the Squares of the Residuals

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The sum of the squares of the residuals is given as [4] :

$$\delta_r^2 = \sum_{i=0}^{m-1} u_i^2 - \frac{m}{2} \left[\frac{a_0^2}{2} + \sum_{j=1}^r (a_j^2 + b_j^2) \right]. \quad (2.7)$$

In practice ,since we do not know r ,we would evaluate a 's and b 's coefficients for $r=1,2,\dots$,then compute δ_r^2 ,and continue as long as δ_r^2 decreases significantly with increasing r .

2.4.Data Smoothing

A series of raw data $\{y_1, y_2, \dots, y_n\}$ is sometimes transformed to a new series of data before it is analyzed. The purpose of this transformation is to smooth out local fluctuations in the raw data, so the transformation is called *data smoothing* or *smoother*[2]. One common type of smoother employs a linear transformation and called a *linear filter*. A linear filter with weights $\{c_0, c_1, \dots, c_{p-1}\}$ transforms the given data to weighted averages $\sum_{j=0}^{p-1} c_j y_{t-j}$ for $t=p, p+1, \dots, n$.

Notice that the new data set has length $n-p+1$. If $\sum_{j=0}^{p-1} c_j = 1$ the linear filter is called an *p-term moving average*. If all weights are equal and they sum to unity ,the linear filter is called a simple moving average.

3.Numerical Applications

3.1.Data

- The used data are listed in the first two columns (and the forth, fifth columns) of Table I as the year and the corresponding annual sunspot's number respectively. These data are obtained from the site :<http://sidc.oma.be>. in the sunspot archive.

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- The third column(and the sixth column) represents the smoothed sunspot's number obtained using a simple 2-term ($p=2$) moving average of Subsection 2.4.

3.2.The a 's and b 's coefficients

- Using the smoothed sunspot's number ,the a 's and b 's coefficients of Equations (2.5) are computed in a recursive manner from Equations (2.6).
- The number of terms r is computed by the artifice mentioned after Equation (2.7) and in this respective we find $r =150$.Table A, gives the values of δ_r^2 (N.S.) for none smoothed and δ_r^2 (S.) for smoothed sunspot's number for different values of r .

Table A: Values of δ_r^2 (N.S.) for None Smoothed and δ_r^2 (S.) for Smoothed sunspot's Number for Different Values of r

r	δ_r^2 (N.S.)	δ_r^2 (S.)
50	34735.1	18895.6
100	8941.85	1514.4
150	4156.73	$4.65661 \cdot 10^{-10}$

- The numerical values of the coefficients a_i ; $i=0,1,2,\dots,150$ and b_j ; $j=1,2,\dots,150$ are listed in Table II.

3.3.Graphical Representations

Graphical representations of the sunspot's number for the years 1700.5 to 2000.5 are displayed in Figure I for both the observed smoothed variations and Fourier smoothed variations

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3.4.Error Analysis

Table III lists the absolute values of the residuals between the observed and the Fourier smoothed variations the of sunspot's number for the years 1700.5 to 2000.5

In concluding the present paper, algorithmic approach for Fourier analysis of smoothed data is developed and applied to the annual sunspot's number from the years 1700.5to 2000.5. The precision criteria of the representation are very satisfactory.

References

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Table I: None Smoothed (N.S.) and Smoothed(S.) Sunspot Number from the Year 1700.5 to 2000.5

Year	N.S.	s.	Year	N.S.	S.
1700.5	5.	8.	1750.5	83.4	65.55
1701.5	11.	13.5	1751.5	47.7	47.75
1702.5	16.	19.5	1752.5	47.8	39.25
1703.5	23.	29.5	1753.5	30.7	21.45
1704.5	36.	47.	1754.5	12.2	10.9
1705.5	58.	43.5	1755.5	9.6	9.9
1706.5	29.	24.5	1756.5	10.2	21.3
1707.5	20.	15.	1757.5	32.4	40.
1708.5	10.	9.	1758.5	47.6	50.8
1709.5	8.	5.5	1759.5	54.	58.45
1710.5	3.	1.5	1760.5	62.9	74.4
1711.5	0.	0.	1761.5	85.9	73.55
1712.5	0.	1.	1762.5	61.2	53.15
1713.5	2.	6.5	1763.5	45.1	40.75
1714.5	11.	19.	1764.5	36.4	28.65
1715.5	27.	37.	1765.5	20.9	16.15
1716.5	47.	55.	1766.5	11.4	24.6
1717.5	63.	61.5	1767.5	37.8	53.8
1718.5	60.	49.5	1768.5	69.8	87.95
1719.5	39.	33.5	1769.5	106.1	103.45
1720.5	28.	27.	1770.5	100.8	91.2
1721.5	26.	24.	1771.5	81.6	74.05
1722.5	22.	16.5	1772.5	66.5	50.65
1723.5	11.	16.	1773.5	34.8	32.7
1724.5	21.	30.5	1774.5	30.6	18.8
1725.5	40.	59.	1775.5	7.	13.4
1726.5	78.	100.	1776.5	19.8	56.15
1727.5	122.	112.5	1777.5	92.5	123.45
1728.5	103.	88.	1778.5	154.4	140.15
1729.5	73.	60.	1779.5	125.9	105.35
1730.5	47.	41.	1780.5	84.8	76.45
1731.5	35.	23.	1781.5	68.1	53.3
1732.5	11.	8.	1782.5	38.5	30.65
1733.5	5.	10.5	1783.5	22.8	16.5
1734.5	16.	25.	1784.5	10.2	17.15
1735.5	34.	52.	1785.5	24.1	53.5
1736.5	70.	75.5	1786.5	82.9	107.45
1737.5	81.	96.	1787.5	132.	131.45
1738.5	111.	106.	1788.5	130.9	124.5
1739.5	101.	87.	1789.5	118.1	104.
1740.5	73.	56.5	1790.5	89.9	78.25
1741.5	40.	30.	1791.5	66.6	63.3
1742.5	20.	18.	1792.5	60.	53.45
1743.5	16.	10.5	1793.5	46.9	43.95
1744.5	5.	8.	1794.5	41.	31.15
1745.5	11.	16.5	1795.5	21.3	18.65
1746.5	22.	31.	1796.5	16.	11.2
1747.5	40.	50.	1797.5	6.4	5.25
1748.5	60.	70.45	1798.5	4.1	5.45
1749.5	80.9	82.15	1799.5	6.8	10.65

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Table I (Continued)

Year	N. S.	S.	Year	N. S.	S.
1800.5	14.5	24.25	1850.5	66.6	65.55
1801.5	34.	39.5	1851.5	64.5	59.3
1802.5	45.	44.05	1852.5	54.1	46.55
1803.5	43.1	45.3	1853.5	39.	29.8
1804.5	47.5	44.85	1854.5	20.6	13.65
1805.5	42.2	35.15	1855.5	6.7	5.5
1806.5	28.1	19.1	1856.5	4.3	13.5
1807.5	10.1	9.1	1857.5	22.7	38.75
1808.5	8.1	5.3	1858.5	54.8	74.3
1809.5	2.5	1.25	1859.5	93.8	94.8
1810.5	0.	0.7	1860.5	95.8	86.5
1811.5	1.4	3.2	1861.5	77.2	68.15
1812.5	5.	8.6	1862.5	59.1	51.55
1813.5	12.2	13.05	1863.5	44.	45.5
1814.5	13.9	24.65	1864.5	47.	38.75
1815.5	35.4	40.6	1865.5	30.5	23.4
1816.5	45.8	43.4	1866.5	16.3	11.8
1817.5	41.	35.55	1867.5	7.3	22.45
1818.5	30.1	27.	1868.5	37.6	55.8
1819.5	23.9	19.75	1869.5	74.	106.5
1820.5	15.6	11.1	1870.5	139.	125.1
1821.5	6.6	5.3	1871.5	111.2	106.4
1822.5	4.	2.9	1872.5	101.6	83.9
1823.5	1.8	5.15	1873.5	66.2	55.45
1824.5	8.5	12.55	1874.5	44.7	30.85
1825.5	16.6	26.45	1875.5	17.	14.15
1826.5	36.3	42.95	1876.5	11.3	11.85
1827.5	49.6	56.9	1877.5	12.4	7.9
1828.5	64.2	65.6	1878.5	3.4	4.7
1829.5	67.	68.95	1879.5	6.	19.15
1830.5	70.9	59.35	1880.5	32.3	43.3
1831.5	47.8	37.65	1881.5	54.3	57.
1832.5	27.5	18.	1882.5	59.7	61.7
1833.5	8.5	10.85	1883.5	63.7	63.6
1834.5	13.2	35.05	1884.5	63.5	57.85
1835.5	56.9	89.2	1885.5	52.2	38.8
1836.5	121.5	129.9	1886.5	25.4	19.25
1837.5	138.3	120.75	1887.5	13.1	9.95
1838.5	103.2	94.45	1888.5	6.8	6.55
1839.5	85.7	75.15	1889.5	6.3	6.7
1840.5	64.6	50.65	1890.5	7.1	21.35
1841.5	36.7	30.45	1891.5	35.6	54.3
1842.5	24.2	17.45	1892.5	73.	79.05
1843.5	10.7	12.85	1893.5	85.1	81.55
1844.5	15.	27.55	1894.5	78.	71.
1845.5	40.1	50.8	1895.5	64.	52.9
1846.5	61.5	80.	1896.5	41.8	34.
1847.5	98.5	111.6	1897.5	26.2	26.45
1848.5	124.7	110.5	1898.5	26.7	19.4
1849.5	96.3	81.45	1899.5	12.1	10.8

Table I (Continued)

Year	N. S.	S.	Year	N. S.	S.
1900.5	9.5	6.1	1950.5	83.9	76.65
1901.5	2.7	3.85	1951.5	69.4	50.45
1902.5	5.	14.7	1952.5	31.5	22.7
1903.5	24.4	33.2	1953.5	13.9	9.15
1904.5	42.	52.75	1954.5	4.4	21.2
1905.5	63.5	58.65	1955.5	38.	89.85
1906.5	53.8	57.9	1956.5	141.7	165.95
1907.5	62.	55.25	1957.5	190.2	187.5
1908.5	48.5	46.2	1958.5	184.8	171.9
1909.5	43.9	31.25	1959.5	159.	135.65
1910.5	18.6	12.15	1960.5	112.3	83.1
1911.5	5.7	4.65	1961.5	53.9	45.75
1912.5	3.6	2.5	1962.5	37.6	32.75
1913.5	1.4	5.5	1963.5	27.9	19.05
1914.5	9.6	28.5	1964.5	10.2	12.65
1915.5	47.4	52.25	1965.5	15.1	31.05
1916.5	57.1	80.5	1966.5	47.	70.35
1917.5	103.9	92.25	1967.5	93.7	99.8
1918.5	80.6	72.1	1968.5	105.9	105.7
1919.5	63.6	50.6	1969.5	105.5	105.
1920.5	37.6	31.85	1970.5	104.5	85.55
1921.5	26.1	20.15	1971.5	66.6	67.75
1922.5	14.2	10.	1972.5	68.9	53.45
1923.5	5.8	11.25	1973.5	38.	36.25
1924.5	16.7	30.5	1974.5	34.5	25.
1925.5	44.3	54.1	1975.5	15.5	14.05
1926.5	63.9	66.45	1976.5	12.6	20.05
1927.5	69.	73.4	1977.5	27.5	60.
1928.5	77.8	71.35	1978.5	92.5	123.95
1929.5	64.9	50.3	1979.5	155.4	155.
1930.5	35.7	28.45	1980.5	154.6	147.55
1931.5	21.2	16.15	1981.5	140.5	128.2
1932.5	11.1	8.4	1982.5	115.9	91.25
1933.5	5.7	7.2	1983.5	66.6	56.25
1934.5	8.7	22.4	1984.5	45.9	31.9
1935.5	36.1	57.9	1985.5	17.9	15.65
1936.5	79.7	97.05	1986.5	13.4	21.3
1937.5	114.4	112.	1987.5	29.2	64.7
1938.5	109.6	99.2	1988.5	100.2	128.9
1939.5	88.8	78.3	1989.5	157.6	150.1
1940.5	67.8	57.65	1990.5	142.6	144.15
1941.5	47.5	39.05	1991.5	145.7	120.
1942.5	30.6	23.45	1992.5	94.3	74.45
1943.5	16.3	12.95	1993.5	54.6	42.25
1944.5	9.6	21.4	1994.5	29.9	23.7
1945.5	33.2	62.9	1995.5	17.5	13.05
1946.5	92.6	122.1	1996.5	8.6	15.05
1947.5	151.6	143.95	1997.5	21.5	42.9
1948.5	136.3	135.5	1998.5	64.3	78.8
1949.5	134.7	109.3	1999.5	93.3	106.45
1950.5	83.9	76.65	2000.5	119.6	115.3

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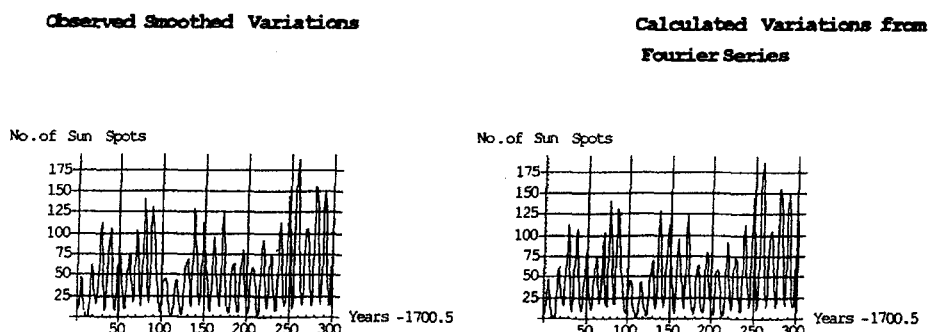
Table II: Coefficients of the Fourier Series Representation of the Smoothed Sunspot Number from the Year 1700.5 to 2000.5

n	a_n	b_n	n'	$a_{n'}$	$b_{n'}$
0	100.0106				
1	7.48504	-5.09572	26	3.03333	-2.0218
2	0.141041	-8.37604	27	-1.77886	22.5352
3	-11.8307	-11.6447	28	12.9163	-3.73933
4	4.24623	-0.342449	29	1.08868	-14.5961
5	-5.99333	3.60816	30	1.93651	-24.0084
6	-2.74139	-9.41246	31	0.18164	-3.11958
7	3.63621	-6.3119	32	-2.74288	2.7361
8	-2.67046	-5.34028	33	1.01539	-3.98494
9	-1.46849	-2.8647	34	1.31715	-7.18193
10	-3.2864	-1.72527	35	4.01142	-2.47013
11	2.73267	2.76585	36	-6.13162	1.56146
12	-2.15892	-0.182019	37	-6.67972	-6.23785
13	1.30293	0.855962	38	-3.08758	0.641266
14	5.19794	0.412113	39	0.452891	1.23301
15	-1.09012	-0.177693	40	-3.08727	1.89994
16	1.45199	-1.85105	41	-0.958273	-2.20152
17	-1.00159	-0.771233	42	1.78376	-1.73505
18	1.22283	4.0895	43	-1.74631	-0.320171
19	-2.29063	0.847369	44	0.198448	-1.73074
20	2.77777	-1.76843	45	-1.94586	-0.375096
21	0.195412	3.96707	46	0.0548009	-0.472735
22	-1.5889	-1.97327	47	0.276094	-1.3278
23	4.01576	-3.63373	48	-1.02186	-1.34861
24	0.745488	3.7603	49	0.01278	-0.920713
25	-1.15399	13.9832	50	-1.83692	0.319189
51	-0.231955	-2.56794	76	-1.53489	0.0867802
52	-2.75147	1.13042	77	-0.576647	-0.526484
53	1.90604	-2.13917	78	0.539956	0.0739499
54	-0.880962	-1.20651	79	-0.248733	-0.216868
55	3.98057	-2.11396	80	0.370881	-0.380862
56	-0.0565528	-1.98952	81	-0.785722	-0.34081
57	-0.165898	-3.6656	82	-0.48068	0.654861
58	-1.55515	-1.08663	83	-1.3054	-0.854005
59	-1.66723	1.07233	84	-0.824684	0.00764465
60	-1.3085	1.18217	85	-0.574565	-0.235255
61	-0.779782	-1.21277	86	-0.0438118	0.396044
62	0.656397	-2.12885	87	0.0350113	0.365818
63	-1.98753	0.730908	88	-0.46429	-0.728841
64	-1.92552	0.108135	89	-0.251644	-0.187594
65	-0.994134	-0.833992	90	-0.25182	0.167727
66	0.878087	-0.247464	91	-0.367253	0.313905
67	0.118184	0.195128	92	-0.274774	0.201094
68	-0.59458	-1.10805	93	0.29002	-0.235588
69	0.457669	-1.4732	94	0.411862	-0.230459
70	0.607732	0.31453	95	-0.578228	-0.532476
71	0.171706	0.493522	96	-0.853074	-0.427287
72	-1.16003	-0.284855	97	-0.83384	-0.99296
73	-0.473471	-1.37747	98	-0.341983	-0.0984207
74	-0.223121	0.461983	99	-0.579552	-0.37311
75	-0.0883077	-0.469658	100	-0.444639	-0.336274

Table II(Continued)

101	-0.508281	-0.349089	126	-0.35433	-0.36044
102	-0.401912	-0.00136354	127	-0.457765	-0.187245
103	-0.834038	-0.0969005	128	-0.291335	-0.104189
104	-0.154135	-0.443866	129	-0.547753	-0.0181787
105	-0.349221	0.0413715	130	-0.556252	0.215365
106	-0.520665	0.119467	131	-0.322019	0.0834856
107	-0.709318	0.10458	132	-0.303247	-0.0230149
108	-0.450959	-0.763333	133	-0.364192	-0.163719
109	-0.577919	-0.267945	134	-0.389196	-0.183305
110	-0.265949	-0.0917259	135	-0.266341	-0.0113574
111	-0.597018	0.0206636	136	-0.370835	0.0156732
112	-0.396892	-0.178887	137	-0.310693	0.0692979
113	-0.450467	0.0737386	138	-0.367719	-0.069298
114	-0.176808	-0.042315	139	-0.263107	0.0355537
115	-0.244805	-0.294876	140	-0.302629	-0.0295513
116	-0.395102	-0.194543	141	-0.444911	0.0294662
117	-0.480492	-0.151253	142	-0.284169	0.0386647
118	-0.407908	-0.0678901	143	-0.322551	-0.0393769
119	-0.229361	-0.140918	144	-0.287019	-0.048477
120	-0.403882	0.000691037	145	-0.360409	-0.0371023
121	-0.122404	-0.166315	146	-0.341793	0.00275942
122	-0.270846	-0.457441	147	-0.365547	-0.0324296
123	-0.394778	-0.170506	148	-0.372811	0.00803271
124	-0.441075	-0.101473	149	-0.342705	0.00563458
125	-0.381469	-0.322042	150	-0.352393	-0.00225757

Figure I : Graphical Representations of the Sun Spot Number for the Years 1700.5 to2000 .5



Algorithmic Approach for the Fourier ...

Table III : Residuals Between the Observed Smoothed (O) and the Fourier Calculated Smoothed (C) Variations in the Sun Spots Number for the Years 1700 .5 (15) 2000.5

Year	O	C	O - C
1700.5	8.	8.	1.13687×10^{-13}
1715.5	37.	37.	1.42109×10^{-14}
1730.5	41.	41.	4.9738×10^{-14}
1745.5	16.5	16.5	7.10543×10^{-14}
1760.5	74.4	74.4	1.27898×10^{-13}
1775.5	13.4	13.4	7.63833×10^{-14}
1790.5	78.25	78.25	2.55795×10^{-13}
1805.5	35.15	35.15	7.10543×10^{-15}
1820.5	11.1	11.1	2.66454×10^{-14}
1835.5	89.2	89.2	5.82645×10^{-13}
1850.5	65.55	65.55	1.56319×10^{-13}
1865.5	23.4	23.4	2.94875×10^{-13}
1880.5	43.3	43.3	4.26326×10^{-14}
1895.5	52.9	52.9	2.06057×10^{-13}
1910.5	12.15	12.15	1.08358×10^{-13}
1925.5	54.1	54.1	4.9738×10^{-13}
1940.5	57.65	57.65	7.95808×10^{-13}
1955.5	89.85	89.85	6.39488×10^{-13}
1970.5	85.55	85.55	4.12115×10^{-13}
1985.5	15.65	15.65	4.47642×10^{-13}
2000.5	115.3	115.3	4.68958×10^{-13}

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**خوارزمية لتحليل فورير مع تطبيق على العدد السنوي
للبيع الشمسية من عام
1700.5 إلى 2000.5**

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**٢- قسم الرياضيات كلية التربية للبنات بجدة
المملكة العربية السعودية**

ملخص البحث :

تم في هذا البحث تشيد خوارزمية لتحليل فورير لبيانات ممهدة وطبق ذلك على العدد السنوي
المسجل للبيع الشمسية من عام 1700.5 إلى 2000.5 وكان مقياس الدقة في التمثيل
مُرضي إلى درجة بالغة .