MATHEMATICAL SIMULATION OF MACHINE TOOL STRUCTURES

BY

PROF.DR. A.A. NASSER[1], PROF.DR. H.R. EL-SAYED[2],
DR. S.M. SERAG[3] AND ENG. S.M. SAMAK[4]

1 - SUMMARY:

Machine tool structure is the principal part of the machine which contributes to its dynamic behaviour. For this reason, the study of the different alternative design configurations of the structure will be necessary, and the mathematical simulation of such structure becomes an effective tool.

In this work, the dynamic characteristics of the structure of a horizontal knee-type milling machine are investigated. Basing upon the classical beam theory, a mathematical model for the structure is introduced. With the aid of this model, fourteen suggested design solution are studied and compared with each other. The suggested fourteen structures are of the same height, main dimensions, wall thickness and mass.

The study results in predicting the most suitable structure. Thereby a considerable improvement of the dynamic behaviour of the considered machine is achieved.

2 - INTRODUCTION:

Due to the development of digital computers, several analytical techniques have been adapted for predicting the static and dynamic behaviour of machine tool structures. In fact, these techniques are based on simulating the structure of the machine by a mathematical mode. Thereby

(1) Abdel-Hady Abdel-Bary Nasser, Prof. Dr., Head of Production Engineering & Machine Design Department, Faculty of Engineering & Technology, Menoufia University.
(2) Hassan Ragab El-Sayed, Prof. Dr. Faculty of Engineering, Alexandria University.
(3) Soad Mohamed Serag, B.Sc., Ph.D., Lecturer, Production Engineering & Machine Design Department, Faculty of Engineering & Technology, Menoufia University.
(4) Salem M. Samak, B.Sc., Demonstrator, Faculty of Engineering & Technology, Menoufia University.

the different influencing parameters could be inter-related and their effects could be investigated. Applications of such techniques to some types of machine tools are found in the literature[1-9].

In this work, one of these techniques, namely the classical beam theory, is used to investigate the effects of the structural configuration on its dynamic behaviour. The structure of the horizontal knee-type milling machine is chosen, and the different possible variations in its shape are studied.

3 - THE MATHEMATICAL MODEL:

Fig. (1) shows the general shape structural main parts and the main dimensions of the considered horizontal knee-type milling machine. The basic model of the machine is shown in Fig. (2). To adapt such structure to beam equations the structure should be replaced by an idealized model Fig. (3), which is in turn transformed to the mathematical model shown in Fig. (4).

4 - DERIVATION OF THE FREQUENCY EQUATION:

As shown in Fig. (4), the mathematical model of the machine comprises five beams. Accordingly the general solution of the beam equation (10) could be written in the following general form:

$$y_r = C_{1r} \sin\frac{Z_r}{L_r} x_r + C_{2r} \cos\frac{Z_r}{L_r} x_r + C_{3r} \sinh\frac{Z_r}{L_r} x_r + C_{4r} \cosh\frac{Z_r}{L_r} x_r \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Substituting the end conditions of the different beams into equation (1) yields:-

$$C_{21} + C_{41} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$C_{11} + C_{31} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\sin Z_4 C_{14} + \cos Z_4 C_{24} - \sinh Z_4 C_{34} - \cosh Z_4 C_{44} = 0 \quad \ldots\ldots\ldots (4)$$

$$\cos Z_4 C_{14} - \sin Z_4 C_{24} - \cosh Z_4 C_{34} - \sinh Z_4 C_{44} = 0 \quad \ldots\ldots\ldots (5)$$

$$\sin Z_5C_{15}+\cos Z_5C_{25}-\sinh Z_5C_{35}-\cosh Z_5C_{45} = 0 \quad \ldots\ldots\ldots(6)$$

$$\cos Z_5C_{15}-\sin Z_5C_{25}-\cosh Z_5C_{35}-\sinh Z_5C_{45} = 0 \quad \ldots\ldots\ldots(7)$$

$$\sin Z_3C_{13}+\cos Z_3C_{23}+\sinh Z_3C_{33}+\cosh Z_3C_{43}=C_{24}+C_{44} \quad \ldots\ldots(8)$$

$$a_3(\cos Z_3C_{13}-\sin Z_3C_{23}+\cosh Z_3C_{33}+\sinh Z_3C_{43})=a_4(C_{14}+C_{34}) \quad \ldots\ldots\ldots\ldots(9)$$

$$E\ I_3a_3^2\ (-\sin Z_3C_{13}-\cos Z_3C_{23}+\sinh Z_3C_{33} + \cosh Z_3C_{43}) = E\ I_4\ a_4^2\ (-C_{24}+C_{44}) \quad \ldots\ldots\ldots(10)$$

$$E\ I_3a_3^3(-\cos Z_3C_{13}+\sin Z_3C_{23}+\cosh Z_3C_{33} + \sinh Z_3C_{43}) = E\ I_4\ a_4^3\ (-C_{14}+C_{34}) \quad \ldots\ldots\ldots(11)$$

$$C_{25} + C_{45} = 0 \quad \ldots\ldots\ldots(12)$$

$$a_2(\cos Z_2C_{12} - \sin Z_2C_{22} + \cosh Z_2C_{32} + \sinh Z_2C_{42}) = a_5\ (C_{15} + C_{35})$$

which could be written in the form:

$$\cos Z_2C_{12} - \sin Z_2C_{22} + \cosh Z_2C_{32} + \sinh Z_2C_{42} - B_{25}C_{15} - B_{25}C_{35} = 0 \quad \ldots\ldots\ldots(13)$$

$$E\ I_2\ a_2^2\ (-\sin Z_2\ C_{12} - \cos Z_2C_{22} + E\ \sinh Z_2C_{32} + \cosh Z_2C_{42}) = E\ I_5\ a_5^2\ (-C_{25} + C_{45})$$

or

$$\sin Z_2C_{12} + \cos Z_2C_{22} - \sinh Z_2C_{32} - \cosh Z_2C_{42} - D_{25}\ C_{25} + D_{25}\ C_{45} = 0 \quad \ldots\ldots\ldots(14)$$

$$E\ I_2a_2^3(-\cos Z_2C_{12} + \sin Z_2C_{22} + \cosh Z_2C_{32}+\sinh Z_2C_{42})$$
$$= m_5w^2(\sin Z_2C_{12}+\cos Z_2C_{22} + \sinh Z_2C_{32}+\cosh Z_2C_{42}) \quad \ldots\ldots(15)$$

$$\sin Z_1C_{11}+ \cos Z_1C_{21} + \sinh Z_1C_{31}+ \cosh Z_1C_{41}=C_{22}+C_{42} \ldots(16)$$

$$C_{23} + C_{43} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

$$a_1(\cos Z_1C_{11} - \sin Z_1C_{21} + \cosh Z_1C_{31} + \sinh Z_1C_{41})$$
$$= a_2(C_{12} + C_{32}) \qquad \ldots\ldots\ldots (18)$$

$$a_2(C_{12} + C_{32}) = a_3 (C_{13} + C_{33})$$

which could be written in the following form:

$$C_{12} + C_{32} - B_{23} C_{13} - B_{23} C_{33} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

$$E I_1 a_1^2 (-\sin Z_1C_{11} - \cos Z_1 C_{21} + \sinh Z_1 C_{31} + \cosh Z_1C_{41})$$
$$-E I_2 a_2^2 (-C_{22} + C_{42}) = E I_3 a_3^2 (-C_{23} + C_{43}) \quad \ldots\ldots \quad (20)$$

$$E I_2 a_1^3 (-\cos Z_1C_{11} + \sin Z_1 C_{21} + \cosh Z_1 C_{31} + \sinh Z_1C_{41})$$
$$-E I_2 a_2^3 (-C_{12} + C_{32}) = -M_1 w^2 (C_{22} + C_{42}) \qquad \ldots\ldots\ldots (21)$$

Equations (8), (10), (9), (11), (16), (20), (18) and (21) could be replaced by equations $(\bar{8})$, $(\overline{10})$ ..... by combining each two successive equations. This gives:-

$$\sin Z_3C_{13} + \cos Z_3C_{23} + \beta_{34}C_{24} + \alpha_{34}C_{34} = 0 \qquad (\bar{8})$$

$$\sinh Z_3C_{33} + \cosh Z_3 C_{43} + \alpha_{34} C_{24} + \beta_{34}C_{44} = 0 \qquad (\overline{10})$$

$$\cos Z_3 C_{13} - \sin Z_3 C_{23} + \Delta_{34} C_{14} + \gamma_{34}C_{34} = 0 \qquad (\bar{9})$$

$$\cosh Z_3 C_{33} + \sinh Z_3C_{43} + \gamma_{34}C_{14} + \Delta_{34}C_{34} = 0 \qquad (\overline{11})$$

$$\sin Z_1 C_{11} + \cos. Z_1 C_{21} + \beta_{12} C_{22} + \alpha_{12}C_{42}$$
$$- D_{12}D_{23}C_{23} = 0 \qquad (\overline{16})$$

$$\sinh Z_1C_{31} + \cosh Z_1C_{41} + \alpha_{12}C_{22} + \beta_{12}C_{42}$$
$$+ D_{12}D_{23}C_{23} = 0 \qquad (\overline{20})$$

$$\cos Z_1C_{11} - \sin Z_1C_{21} + \Delta_{12}C_{12} - R_1Z_1C_{22}$$
$$+ \gamma_{12}C_{32} - R_1Z_1C_{42} = 0 \qquad (\overline{18})$$

$$\cosh Z_1C_{31} + \sinh Z_1C_{41} - \gamma_{12}C_{12} + R_1Z_1C_{22}$$
$$+ \Delta_{12}C_{32} + R_1Z_1C_{42} = 0 \qquad (\overline{21})$$

where;

$$a_r = \frac{Z_r}{L_r}$$

$$B_{r(r+1)} = \frac{a_{r+1}}{a_r}$$

$$D_{r(r+1)} = B^2_{r(r+1)} \cdot \frac{I_{r+1}}{I_r}$$

$$\alpha_{r(r+1)} = 0.5\ (D_{r(r+1)} - 1)$$

$$\beta_{r(r+1)} = -\ (\alpha_{r(r+1)} + 1) = -0.5(D_{r(r+1)}+1)$$

$$\gamma_{r(r+1)} = \alpha_{r(r+1)} \cdot B_{r(r+1)}$$

$$\Delta_{r(r+1)} = \beta_{r(r+1)} \cdot B_{r(r+1)}$$

$$R_1 = \frac{M_1}{2\ m_1}$$

Simplifying equation (15) by the following substitution:

$$U = \cos\ Z_2 + R_2\ Z_2\ \sin\ Z_2$$
$$V = -\sin\ Z_2 + R_2\ Z_2\ \cos\ Z_2$$
$$W = -\cosh\ Z_2 + R_2\ Z_2\ \sinh\ Z_2$$
$$X = -\sinh\ Z_2 + R_2\ Z_2\ \cosh\ Z_2$$

where

$$\frac{m_5\ w^2}{E\ I_2\ a_2^3} = \frac{m_5}{m_2}\ Z_2 = R_2\ Z_2$$

gives

$$U\ C_{12} + V\ C_{22} + W\ C_{32} + X\ C_{42} = 0 \qquad \text{........(15)}$$

Equations from (2) to (7), from (8) to (11), from (12) to (14), (15), (16), (17), (18), (19), (20) and (21) form a set of equations containing the unknown constants C_{11}, C_{12}, ...,C_{45} and their unknown coefficients. Since the values of these constants are not zero, hence the set of equations yield to the frequency equation in the form of 20th order deterimant, after reduction to 17th order and rearrangement the determinant equation (22) is established.

$\sin Z_1$	$\cos Z_1$		β_{12}		α_{12}		$-D_{12}D_{23}$								
$\cos Z_1$	$-\sin Z_1$	Δ_{12}	$-R_1 Z_1$	γ_{12}	$-R_1 Z_1$										
$\sinh Z_1$	$\cosh Z_1$		$-\alpha_{12}$		$-\beta_{12}$		$-D_{12}D_{23}$								
$\cosh Z_1$	$\sinh Z_1$	$-\gamma_{12}$	$-R_1 Z_1$	$-\Delta_{12}$	$-R_1 Z_1$										
		$\sin Z_2$	$\cos Z_2$	$-\sinh Z_2$	$-\cosh Z_2$									$-2\ D_{25}$	
		$\cos Z_2$	$-\sin Z_2$	$\cosh Z_2$	$\sinh Z_2$								$-B_{25}$		$-B_{25}$
		U	V	W	X										
		1		1		$-B_{23}$		$-B_{23}$							
						$\sin Z_3$	$\cos Z_3$			β_{34}		α_{34}			
						$\cos Z_3$	$-\sin Z_3$		Δ_{34}		γ_{34}				
							$-\cosh Z_3$	$\sinh Z_3$		α_{34}		β_{34}			
							$-\sinh Z_3$	$\cosh Z_3$	γ_{34}		Δ_{34}				
									$\sin Z_4$	$\cos Z_4$	$-\sinh Z_4$	$-\cosh Z_4$			
									$\cos Z_4$	$-\sin Z_4$	$-\cosh Z_4$	$-\sinh Z_4$			
													$\sin Z_5$	$\cos Z_5 + \cosh Z_5$	$-\sinh Z_5$
													$\cos Z_5$	$-\sin Z_5 + \sinh Z_5$	$-\cosh Z_5$

The values of Z_2, Z_3, Z_4 and Z_5 could be determined in terms of Z_1 from the equation (2):

$$Z_{r+1} = Z_r \cdot \frac{L_{r+1}}{L_r} \quad B_{r(r+1)}.$$

Substituting these values into equation (22) yields the frequency equation in terms of Z_1 alone.

5 - THE NATURAL FREQUENCIES AND MODERFORMS OF THE CONSIDERED MACHINE:

The different natural frequencies of the machine could be determined from the following equation[2]:

$$f = \frac{Z_1^2}{2\pi L_1^2} \sqrt{\frac{E\ I_1\ g}{\rho A_1}} \quad \text{CPS}$$

by substituting the different values of (Z_1). The corresponding modeforms of the machine could be determined by calculating the deflection of each beam using equation (1). For this purpose the values of C_{11}, C_{21}, ..., C_{45} should be evaluated. This could be done by solving the set of equations given in section (4). This set of equations could be first reduced to nineteen equations by omitting the last equation and substituting $C_{11} = 1$. The moderform is only found on a relative basis, so that all the modeform is only found on a relative basis, so that all the modeform equation (1) can be multiplied on the right-hand side by the same arbitrary constant $\neq 0$.

6 - THE BASIC MODEL OF THE MACHINE AND THE SUGGESTED MODIFICATIONS:

The basic model of the machine is shown in Fig. (2). Four modifications are suggested and shown in Figs (5), (6), (7) and (8). All modifications are done in such a way that the column of the machine has the same mass, height and wall thickness, whilst the dimensions and weights of the table and overarm are kept constant. In the first modification the column has vertical sides and tapered back, whilst in the second the column assumes squared cross-section. In the third and fourth modifications the back side wall is given a circular shape. The idealized models of the four suggested modifications are shown in Figs. (9), (10), (11) and (12).

The configuration of the overarm of the machine is also investigated. Five different cross-sectional alternative shapes of the overarm are suggested and shown in Fig. (13). The five suggested overarms have the same mass and length.

The investigation of the dynamic behaviour of the basic and suggested models will be done in the following way:-

1. The natural frequencies and modeforms of the basic model will be first determined and considered as basic data.
2. The natural frequencies and moderforms of each of the suggested modifications will be measured and compared with the basic data in order to evaluate their effects. In this way the suggested model which will have the superior properties will be detected.
3. The dynamic behaviour of the superior model will be further investigated by changing its overarm in the sequence shown in Fig. (13).

7 - RESULTS, DISCUSSION AND CONCLUSIONS:

The results of this investigation are tabluated and plotted in the form shown hereafter.

Table (1) shows the characteristics of the basic model of the machine, while tables (2) to (5) show the characteristics of each suggested modification. Tables (6) to (9) give the results of comparison between the characteristics of the suggested modifications and the basic model, which show that the third and fourth suggested modified models are superior than the basic.

The characteristics of the third modified model when equipped with the different suggested shapes of overarms, shown in Fig. (13), are included in tables (10) to (14). Tables (15) to (19) show the results of comparison between the characteristics of this model, when equipped with the different overarm shapes, with its basic model. The basic model in this case will be the third modified model equipped with the original overarm as it is shown in Fig. (7). Hence the results given in table (4) will be the basic data. Tables (15) to (19) show that the third modification equipped with overarm possessing the second and/or the fifth shapes gives the best results. These two combinations will be termed (III - 2) and (III - 5).

Tables (20) to (24) are concerned with the characteristics of the fourth modification when equipped with the different suggested shapes of overarms, whilst tables (25) to (29) give the results of comparison between its characteristics and that of its basic model. The basic model in this case will be the fourth modification equipped with the original overarm as it is shown in Fig. (8), and its data given in table (5) will be considered as basic data. The results of comparison show that the fourth modification equipped with overarm possessing the second and/or the fifth shapes gives the best results. These two combinations will be termed (IV - 2) and (IV - 5).

The main results of this investigation are summarized in Figs (14) to (18). These figures compare the characteristics of the basic model of the machine with those of the four best models, namely, (III - 2), (III - 5), (IV - 2) and (IV - 5). Regarding the moderforms, the best four models behave approximately in the same manner, as it is shown in Fig. (14), and from Figs (15) to (17) which show the deflections of the main parts of each model at the different modes. However, the modified model (III - 2) may be the best one, in the sense that, it ensures, the minimum deflection and hence the maximum rigidity. The percentage reduction in the deflection of the main parts of that model is given in Fig. (18), which shows how far the deflection of each part of the suggested model is reduced in comparison with those of the basic model.

The percentage reduction reachs 21% for the column, 23% for the table and 71% for the overarm. Further investigation on prototype scale seems to be necessary in order to indentify securely the suggested model which could be recommended.

Regarding the magnitude of the natural frequencies, it could be seen that, the first natural frequencies of the best model range from 203 to 221 C/S compared to 181 C/S for the basic model of the machine. This gives a mean increase of about 31 C/S corresponding to 1860 r.p.m., which makes the first natural frequencies of the suggested models far away, with an example space, from the working range of the machine.

In order to visualize the individual effects of the column and hence the overarm, the set of results illustrated in Figs (19) to (22) is presented. Figs (19) to (20) compare the deflections of the different main parts of the basic, third and fourth suggested models when equipped with the original overarm. On the other hand, Fig. (22) indicates the percentage reduction reachs 4% for the column, 9% for the table and 28% for the overarm.

From the presented results and discussion the following main conclusions could be drawn:-

1. When the model mass and main dimensions are kept constant, the rigidity of a machine tool structure could be increased by altering its configuration. The optimum configuration could be detected by checking the various alternative possible shapes of its main parts.
2. For the purpose of checking the different possible configurations of a machine tool structure, the classical beam theory may be one of the useful tools.
3. The four suggested models for the considered milling machine exhibit reduced deformation and increased natural frequency.
4. The percentage reduction in deformation of the suggested models amounts to 21% for the column, 23% for the table and 71% for the overarm. The natural frequency increased by about 31 C/S.
5. Rounding the back-side of the column and/or the overarm yield considerable improvement of the dynamic characteristics of the milling machine.
6. Rounding the back-side of the column of the horizontal milling machine while keeping the other parts having the original shapes results in reducing the deflection by about 4% for the column, 9% for the table and 28% for the overarm.

NOMENCLATURE

A_r	Cross-sectional area.
a_r	Frequency constant.
$B_{r(r+1)}$	Area constant.
$C_{1r}, C_{2r}, \ldots, C_{4r}$	Boundary constants.
E	Modulus of elasticity.
$E\ I$	Flexural rigidity.
f	Frequency.
g	Acceleration of gravity.
I_r	Moment of inertia of area.
L_r	Length.
m_r	Mass.
R_1, R_2	Masses ratio.
r	Subscript taking the values 1,2, ...,5 and indicates the number of beam.
w	Circular frequency.
X_r	Abscissa.
y_r	Ordinate.
Z_r	frequency constant.
	Density of beam material.
ρA	Mass per unit length.

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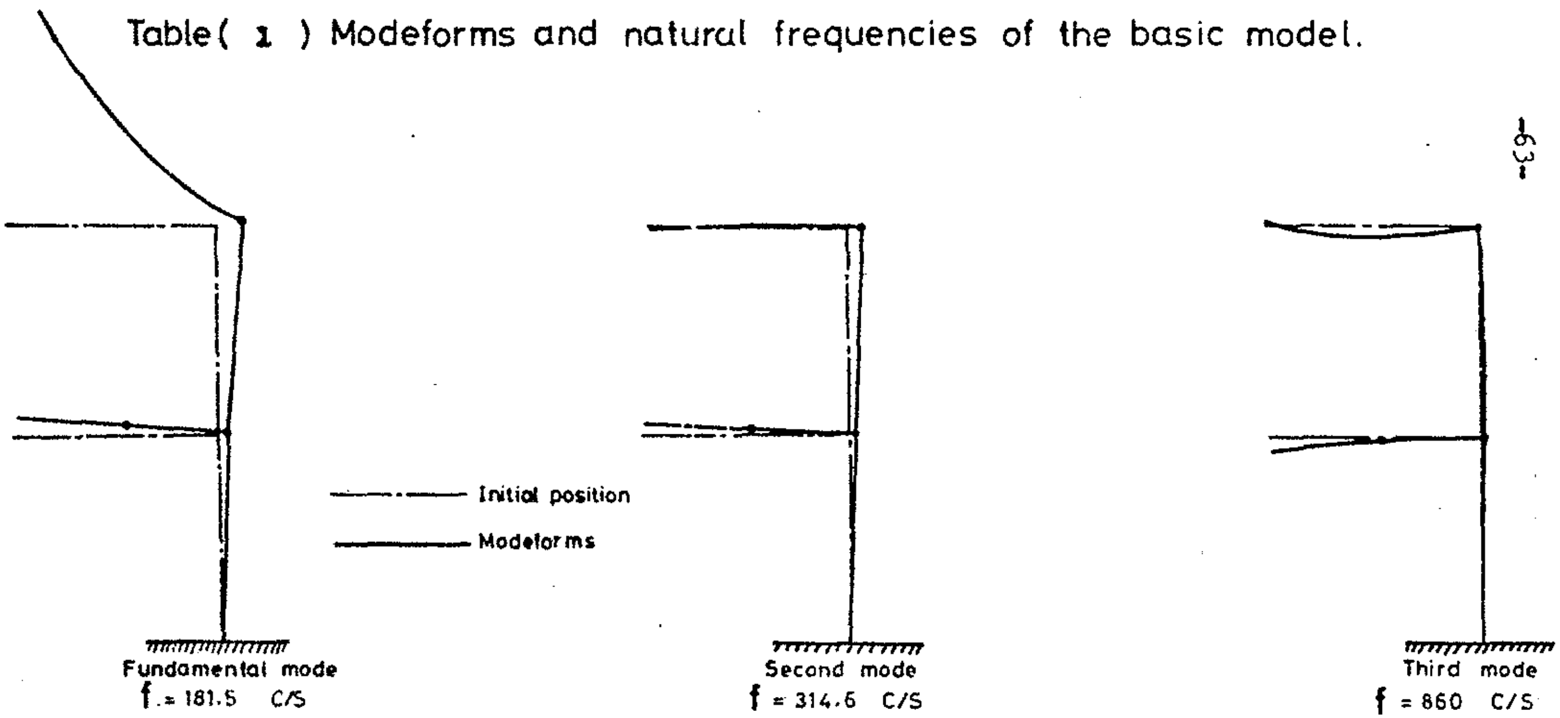
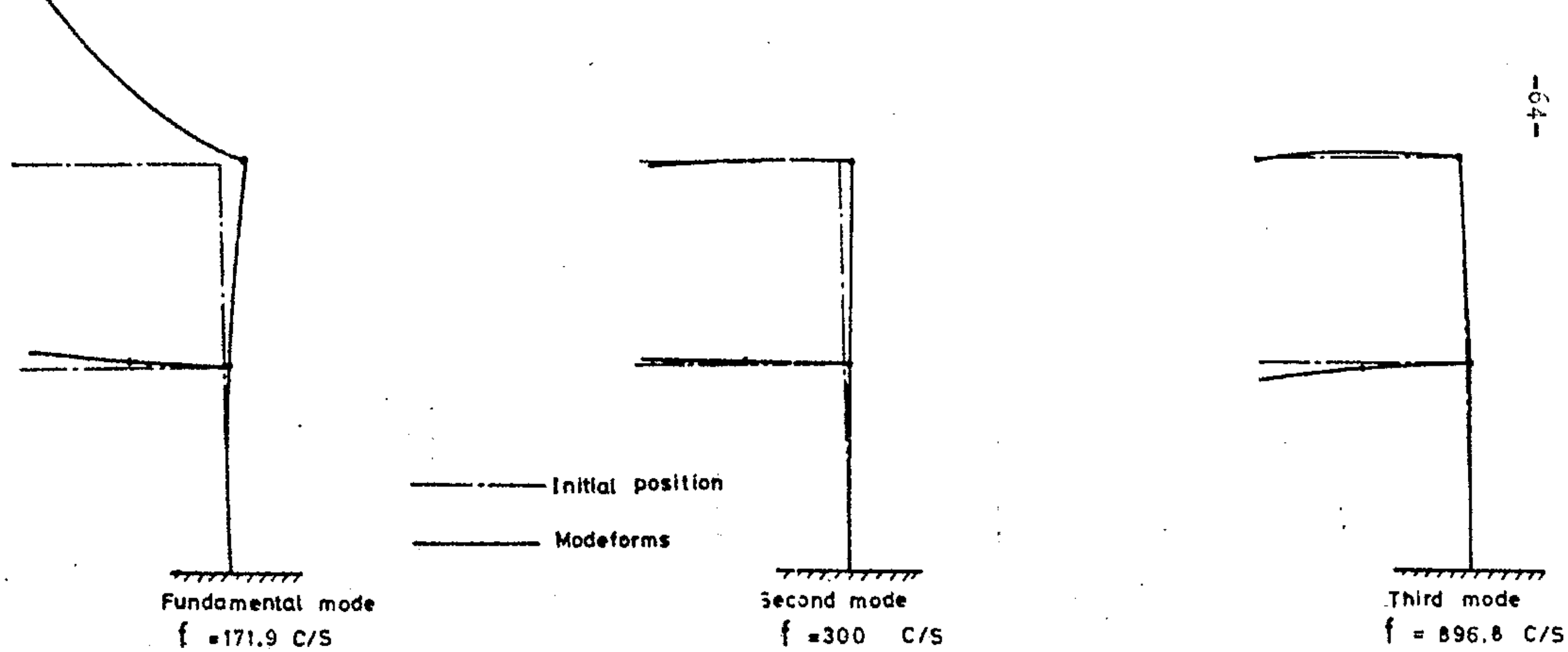
Mode no.	z_1	f c/s	Deflections																								
			Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
			X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
			0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1^{st}	0.79	181.5	0	0.09	0.34	0.74	1.28	1.28	1.93	2.68	3.35	4.48	0	0.3	0.61	0.92	1.23	1.23	1.54	1.87	2.18	2.5	0	4.51	14.22	26.67	39.98
2^{nd}	1.04	314.6	0	0.08	0.28	0.59	0.96	0.96	1.34	1.72	2.08	2.42	0	0.2	0.41	0.63	0.85	0.85	1.08	1.31	1.55	1.79	0	0.18	0.09	-0.17	-0.51
3^{rd}	1.72	860.5	0	0.11	0.35	0.56	0.59	0.59	0.51	0.41	0.28	0.12	0	-0.1	-0.31	-0.59	-0.94	-0.94	-1.34	-1.8	-2.29	-2.79	0	-1.02	-2.08	-1.56	0.38

Table(1) Modeforms and natural frequencies of the basic model.



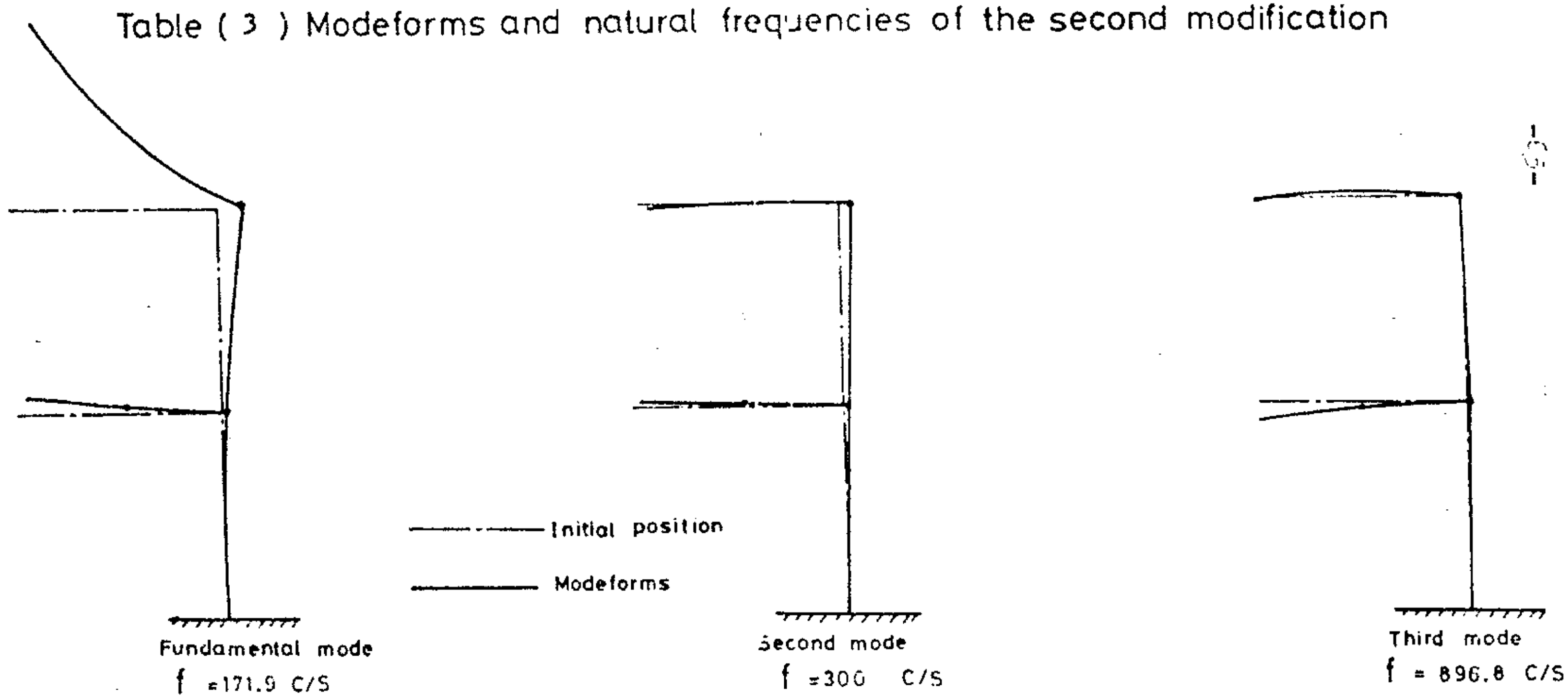
Mode no.	z_1	f c/s	Deflections																								
			Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
			X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
			0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st	0.81	171.9	0	0.09	0.35	0.76	1.31	1.31	1.98	2.75	3.62	4.59	0	0.3	0.6	0.91	1.21	1.21	1.52	1.83	2.14	2.46	0	4.19	13.03	24.27	36.25
2nd	1.07	300	0	0.08	0.29	0.6	0.97	0.97	1.36	1.73	2.08	2.41	0	0.19	0.39	0.59	0.8	0.8	1.02	1.23	1.45	1.67	0	0.16	0.04	-0.25	-0.36
3rd	1.85	896.8	0	0.13	0.4	0.61	0.59	0.59	0.44	0.33	0.25	0.19	0	-0.15	-0.41	-0.76	-1.17	-1.17	-1.65	-2.2	-2.76	-3.36	0	0.58	1.24	0.84	-0.51

Table (2) Modeforms and natural frequencies of the second modification



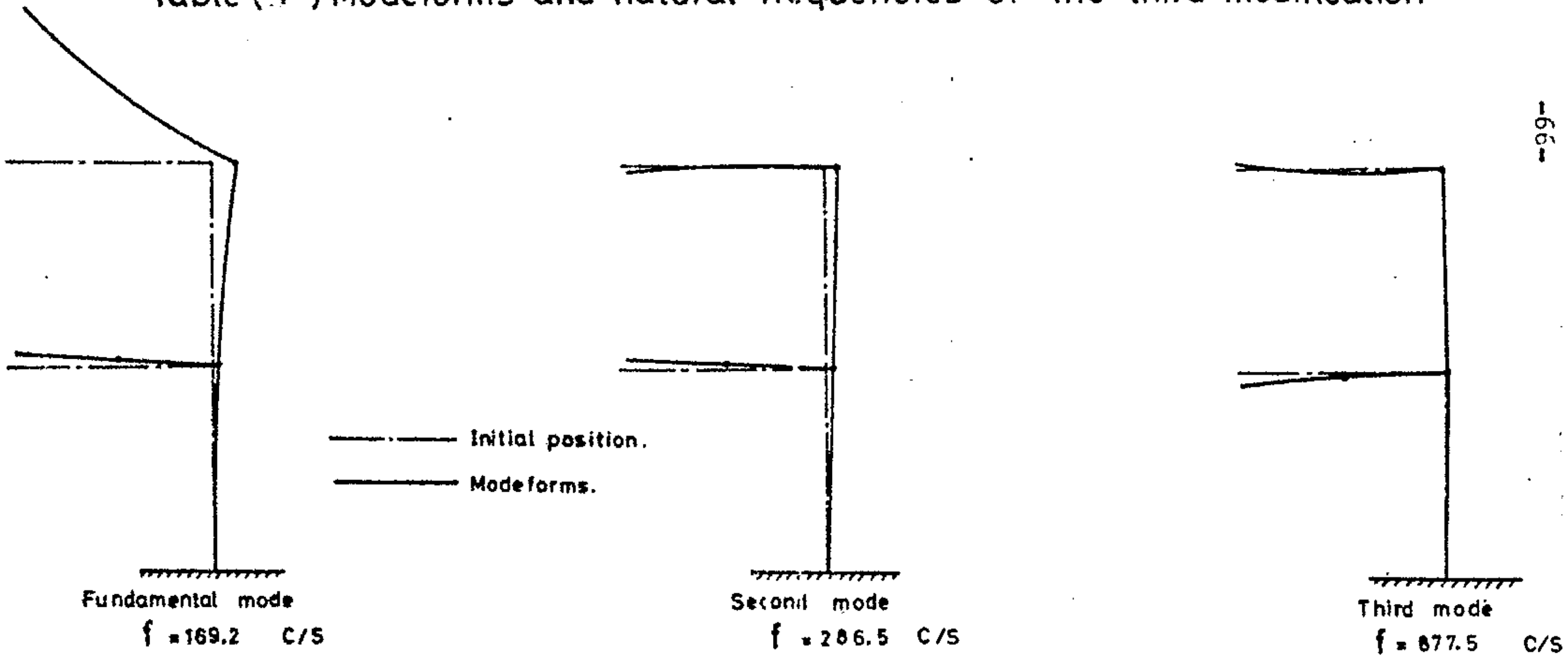
Mode no.	z_1	f c/s	Deflections																								
			Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
			X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
			0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1^{st}	0.81	171.9	0	0.09	0.35	0.76	1.31	1.31	1.98	2.75	3.62	4.59	0	0.3	0.6	0.91	1.21	1.21	1.52	1.83	2.14	2.46	0	4.19	13.03	24.27	36.25
2^{nd}	1.07	300	0	0.08	0.29	0.6	0.97	0.97	1.36	1.73	2.08	2.41	0	0.19	0.39	0.59	0.8	0.8	1.02	1.23	1.45	1.67	0	0.16	0.04	-0.25	-0.36
3^{rd}	1.85	896.8	0	0.13	0.4	0.61	0.59	0.59	0.44	0.33	0.25	0.19	0	-0.35	-0.41	-0.76	-1.17	-1.17	-1.65	-2.2	-2.76	-3.36	0	0.58	1.24	0.84	-0.51

Table (3) Modeforms and natural frequencies of the second modification



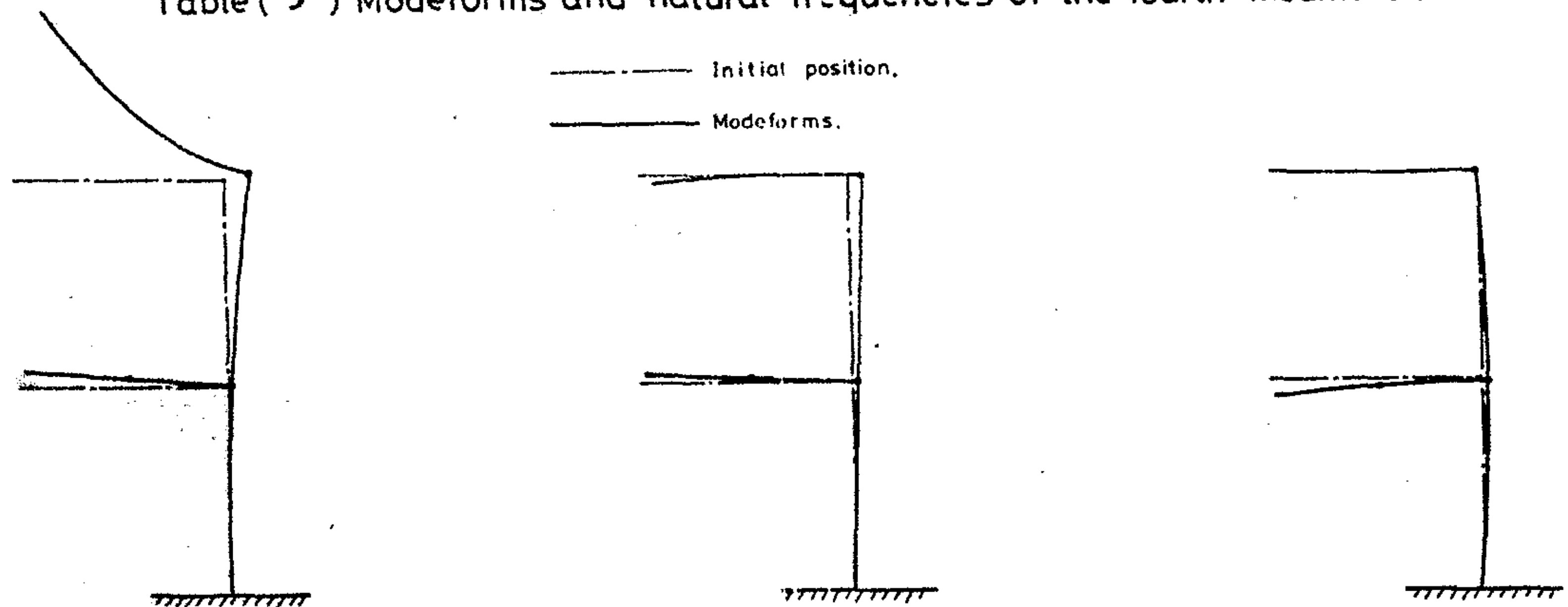
Mode no.	z_1	f c/s	Deflections																								
			Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
			X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
			0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st	0.83	169.2	0	0.09	0.34	0.74	1.27	1.27	1.91	2.64	3.46	4.37	0	0.28	0.56	0.84	1.12	1.12	1.41	1.7	1.99	2.28	0	3.46	10.62	19.7	29.4
2nd	1.08	286.5	0	0.08	0.29	0.59	0.96	0.96	1.33	1.68	2.01	2.31	0	0.18	0.37	0.56	0.75	0.75	0.95	1.15	1.35	1.55	0	0.03	-0.34	-0.93	-1.57
3rd	1.89	877.5	0	0.14	0.41	0.61	0.57	0.57	0.37	0.2	0.04	-0.13	0	-0.16	-0.41	-0.73	-1.11	-1.11	-1.55	-2.04	-2.56	-3.06	0	-0.39	-0.66	-0.33	0.45

Table (4) Modeforms and natural frequencies of the third modification



Mode no.	z	f c/s	Deflections																								
			Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
			X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
			0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st	0.82	183.5	0	0.08	0.33	0.73	1.25	1.25	1.87	2.59	3.4	4.3	0	0.28	0.57	0.86	1.15	1.15	1.44	1.74	2.03	2.33	0	3.77	11.75	21.89	32.7
2nd	1.06	306.7	0	0.08	0.29	0.59	0.95	0.95	1.33	1.69	2.03	2.35	0	0.19	0.38	0.59	0.79	0.79	1	1.22	1.43	1.65	0	0.05	-0.27	-0.81	-1.41
3rd	1.8	884.3	0	0.12	0.38	0.58	0.59	0.59	0.46	0.35	0.25	0.15	0	-0.13	-0.37	-0.69	-1.07	-1.07	-1.52	-2.03	-2.55	-3.1	0	-0.04	0.01	0.09	0.15

Table (5) Modeforms and natural frequencies of the fourth modification



Fundamental mode
$f = 183.5$ C/S

Second mode
$f = 306.7$ C/S

Third mode
$f = 884.3$ C/S

Mode number	Frequency ratios.	Deflection ratios																								
		Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
		X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
		0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st mode	1.02	0	1	1.03	1.03	1.02	1.02	1.01	1.01	1.01	1.01	0	0.93	0.93	0.93	0.94	0.94	0.94	0.95	0.95	0.93	0	0.79	0.76	0.75	0.74
2nd mode	0.86	0	1	1.03	1.01	1.01	1.01	1	0.98	0.97	0.95	0	0.9	0.9	0.89	0.88	0.88	0.88	0.88	0.87	0.86	0	0.33	-2.79	4.47	2.63
3rd mode	0.99	0	1.27	1.2	1.12	0.96	0.96	0.68	0.46	0.21	-0.25	0	1.7	1.29	1.22	1.22	1.22	1.18	1.15	1.13	1.11	0	-0.33	-0.37	-0.3	-0.73

Table (6) Comparison between the first modification and basic model .

Mode number	Frequency ratios.	Deflection ratios																								
		Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
		X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
		0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st mode	0.95	0	1	1.03	1.03	1.02	1.02	1.02	1.02	1.02	1.02	0	1	0.98	0.98	0.98	0.98	0.99	0.98	0.98	0.98	0	0.93	0.91	0.91	0.9
2nd mode	0.95	0	1	1.03	1.01	1.01	1.01	1.01	1	1	0.99	0	0.95	0.95	1.07	0.94	0.94	1.06	0.93	0.93	0.93	0	0.89	0.44	1.47	1.23
3rd mode	1.04	0	1.18	1.14	1.09	1	1	0.86	0.8	0.89	1.58	0	1.5	1.32	1.29	1.24	1.24	1.23	1.22	1.21	1.2	0	-0.57	-0.59	-0.54	-1.34

Table(7) Comparison between the second modification and basic model.

Mode number	Frequency ratios.	Deflection ratios																								
		Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
		X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
		0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st mode	0.93	0	1	1	1	0.99	0.99	0.99	0.98	0.98	0.97	0	0.93	0.92	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0	0.77	0.74	0.74	0.73
2nd mode	0.91	0	1	1.03	1	1	1	0.99	0.97	0.96	0.95	0	4.05	0.9	0.89	0.89	0.89	0.88	0.88	0.87	0.85	0	0.16	-3.78	5.47	3.08
3rd mode	1.02	0	1.27	1.17	1.09	0.96	0.96	0.72	0.49	0.14	1.08	0	1.6	1.3	1.24	1.19	1.19	1.15	1.13	1.12	1.1	0	0.38	0.32	0.21	1.18

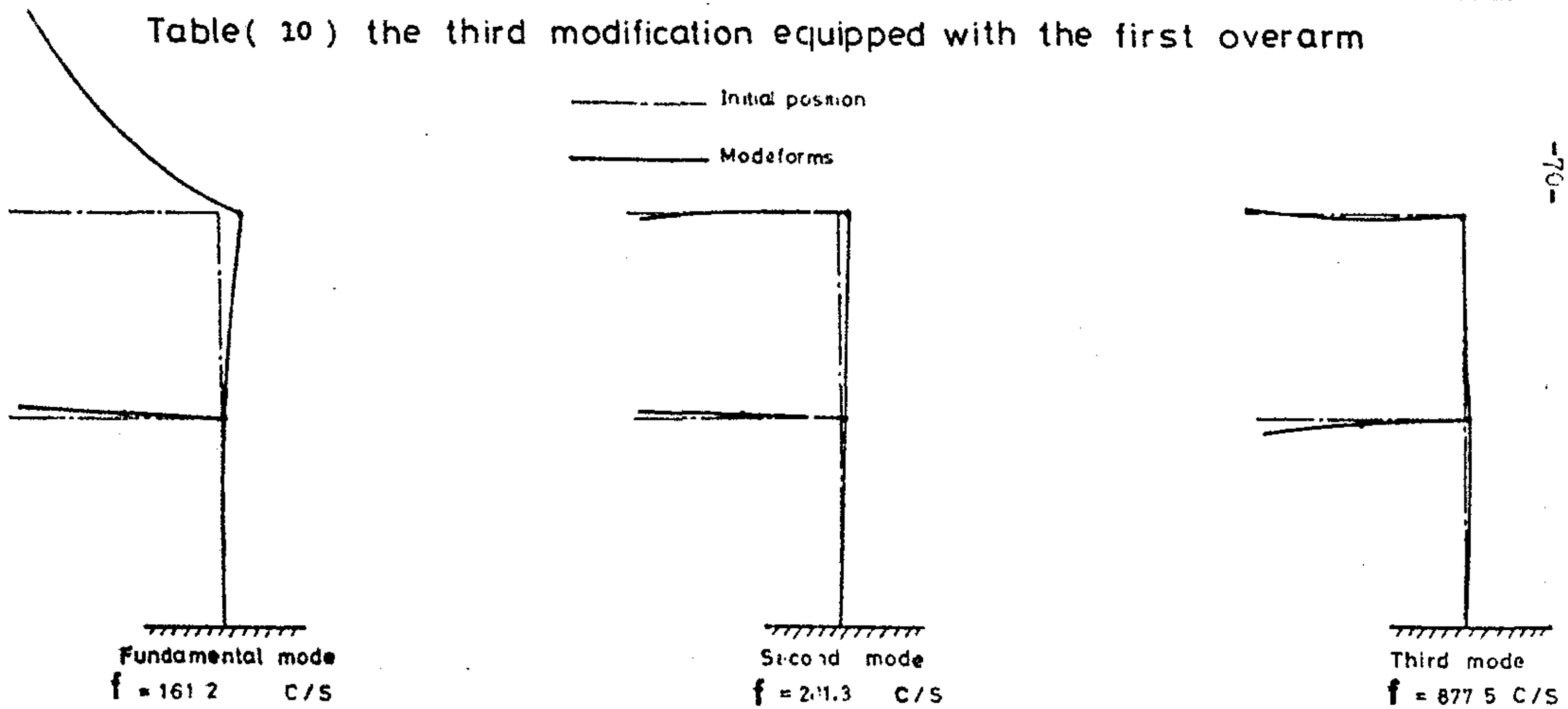
Table(8) Comparison between the third modification and basic model.

Mode number	Frequency ratios.	Deflection ratios																								
		Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
		X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
		0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st mode	1.01	0	0.89	0.97	0.98	0.97	0.97	0.97	0.96	0.96	0.96	0	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0	0.83	0.82	0.82	0.82
2nd mode	0.97	0	1	1.03	1	0.99	0.99	0.99	0.98	0.97	0.97	0	0.95	0.92	0.93	0.93	0.93	0.92	0.93	0.92	0.92	0	0.27	-3	4.76	2.76
3rd mode	1.03	0	1.09	1.08	1.03	1	1	0.9	0.85	0.89	1.25	0	1.3	1.19	1.17	1.14	1.14	1.13	1.12	1.12	1.11	0	0.04	0	-0.06	0.39

Table (9) Comparison between the fourth modification and basic model.

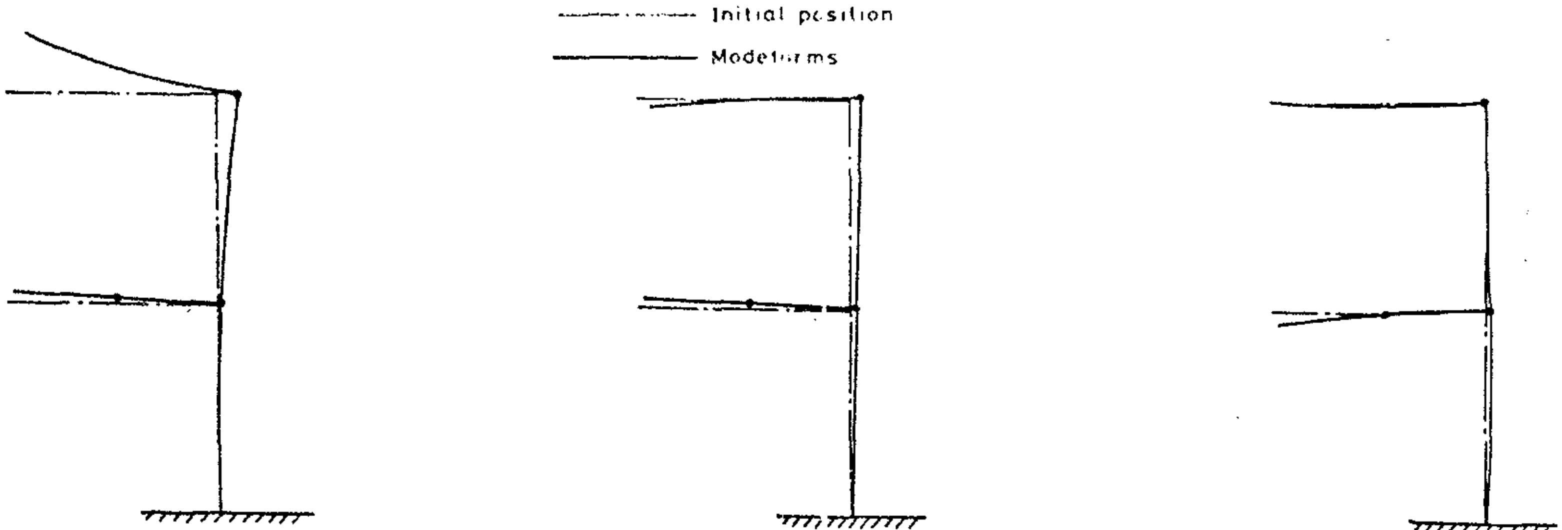
Mode no.	z_1	f (c/s)	Beam 1 X_1/L_1 = 0	0.25	0.5	0.75	1	Beam 2 X_2/L_2 = 0	0.25	0.5	0.75	1	Beam 3 X_3/L_3 = 0	0.25	0.5	0.75	1	Beam 4 X_4/L_4 = 0	0.25	0.5	0.75	1	Beam 5 X_5/L_5 = 0	0.25	0.5	0.75	1
1st	0.81	161.2	0	0.09	0.35	0.75	1.3	1.3	1.96	2.73	3.59	4.55	0	0.28	0.57	0.87	1.16	1.16	1.45	1.75	2.05	2.34	0	4.16	12.96	24.16	36.11
2nd	1.07	281.3	0	0.08	0.29	0.6	0.97	0.97	1.35	1.72	2.06	2.39	0	0.18	0.37	0.57	0.76	0.76	0.96	1.17	1.37	1.58	0	0.15	0.02	-0.28	-0.67
3rd	1.89	877.5	0	0.14	0.41	0.61	0.57	0.57	0.37	0.2	0.04	0.13	0	-0.16	-0.41	-0.73	-1.12	-1.12	-1.55	-2.44	-2.56	-3.08	0	-0.43	-0.63	-0.27	0.65

Table(10) the third modification equipped with the first overarm



Mode no.	z_1	f c/s	Deflections																								
			Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
			X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
			0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st	0.29	207.9	0	0.08	0.31	0.66	1.11	1.11	1.64	2.23	2.68	3.55	0	0.73	0.47	0.71	0.95	0.95	1.19	1.44	1.69	1.93	0	1.56	4.38	7.84	11.51
2nd	1.09	291.9	0	0.08	0.78	0.58	0.92	0.92	1.28	1.61	1.91	2.17	0	0.7	0.35	0.53	0.71	0.71	0.9	1.09	1.29	1.48	0	-0.05	-0.55	-1.29	-2.12
3rd	1.89	877.5	0	0.14	0.41	0.61	0.57	0.57	0.37	0.2	0.03	-0.14	0	-0.16	0.41	-0.74	-1.12	-1.12	-1.56	-2.05	-2.57	-3.0[illegible]	0	-0.32	-0.51	-0.53	-0.13

Table (11) The third modification equipped with the second overarm



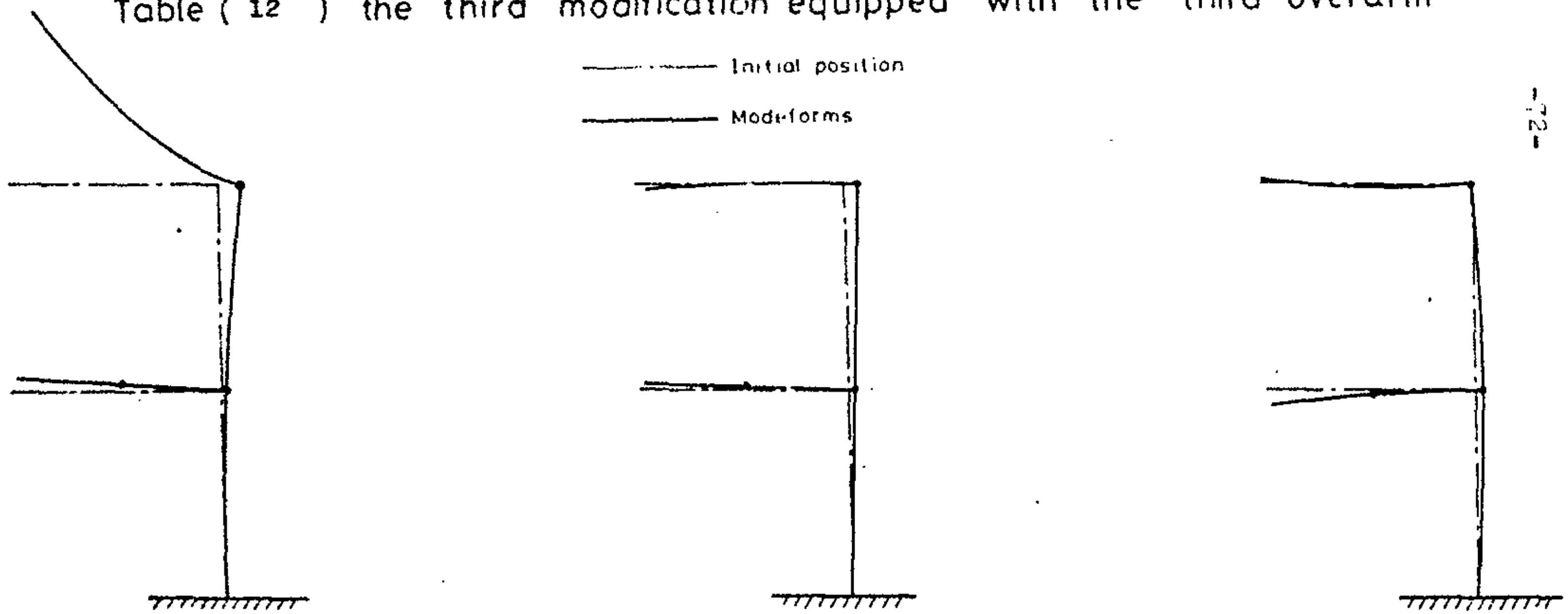
Fundamental mode f = 207.9 C/S

Second mode f = 291.9 C/S

Third mode f = 877.5 C/S

Mode no.	z_1	f c/s	Deflections																								
			Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
			X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
			0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st	0.82	165.2	0	0.09	0.34	0.75	1.29	1.29	1.94	2.69	3.53	4.47	0	0.28	0.57	0.86	1.15	1.15	1.44	1.73	2.03	2.37	0	3.87	11.97	22.27	33.25
2nd	1.08	286.5	0	0.08	0.29	0.6	0.96	0.96	1.33	1.64	2.02	2.32	0	0.18	0.37	0.56	0.75	0.75	0.95	1.15	1.35	1.56	0	0.02	-0.37	-0.98	-1.64
3rd	1.89	877.5	0	0.14	0.41	0.61	0.57	0.57	0.37	0.2	0.04	-0.13	0	-1.10	-0.41	-0.73	-1.11	-1.11	-1.55	-2.04	-2.56	-3.05	0	-0.41	-0.68	-0.29	0.57

Table (12) the third modification equipped with the third overarm



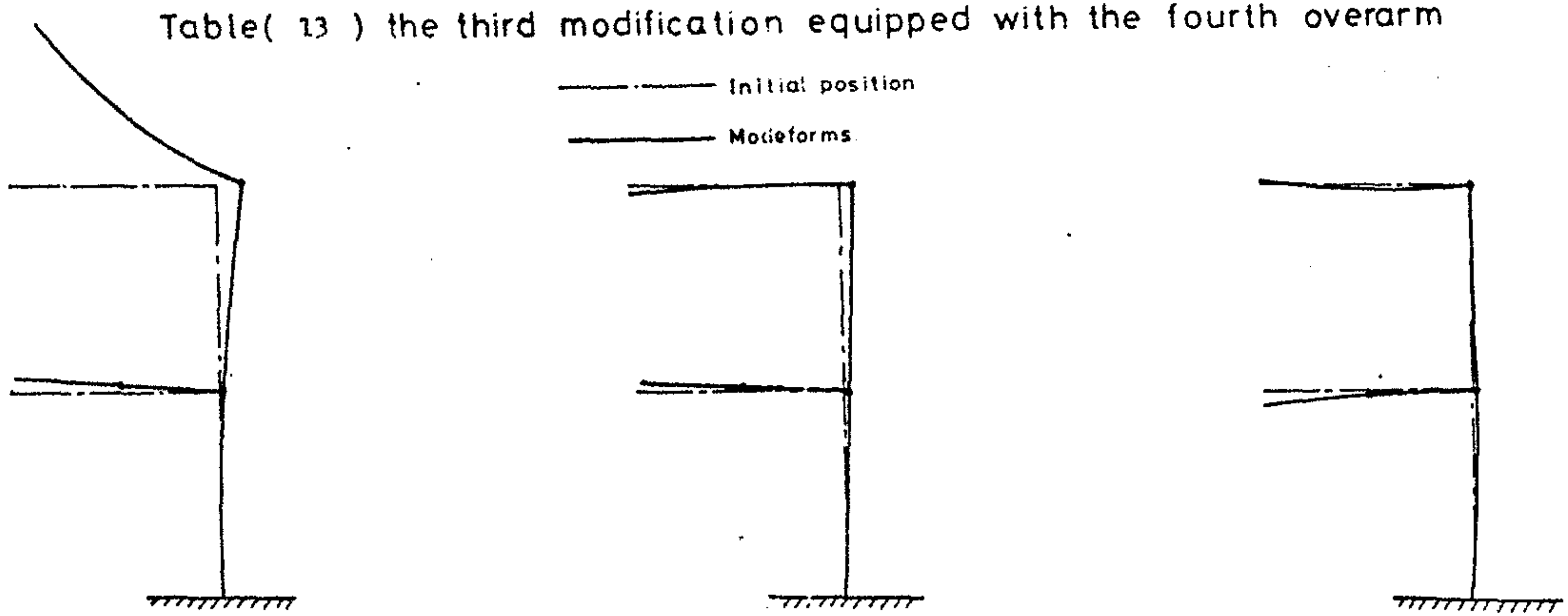
Fundamental mode
f = 165.2 C/S

Second mode
f = 286.5 C/S

Third mode
f = 877.5 C/S

Mode no.	z_1	f c/s	Deflections																								
			Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
			X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
			0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st	0.83	196.2	0	0.09	0.34	0.74	1.27	1.27	1.91	2.64	3.46	4.37	0	0.28	0.56	0.84	1.12	1.12	1.41	1.7	1.99	2.28	0	3.51	10.77	19.99	29.82
2nd	1.08	286.5	0	0.08	0.29	0.59	0.96	0.96	1.33	1.68	2.01	2.31	0	0. 8	0.37	0.56	0.75	0.75	0.95	1.15	1.35	1.59	0	0.03	-0.36	-0.95	-1.61
3rd	1.89	877.5	0	0.14	0.41	0.61	0.57	0.57	0.37	0.2	0.04	-0.13	0	-0.16	-0.41	-0.73	-1.12	-1.12	-1.55	-2.04	-2.56	-3.08	0	-0.4	-0.67	-0.33	0.47

Table(13) the third modification equipped with the fourth overarm



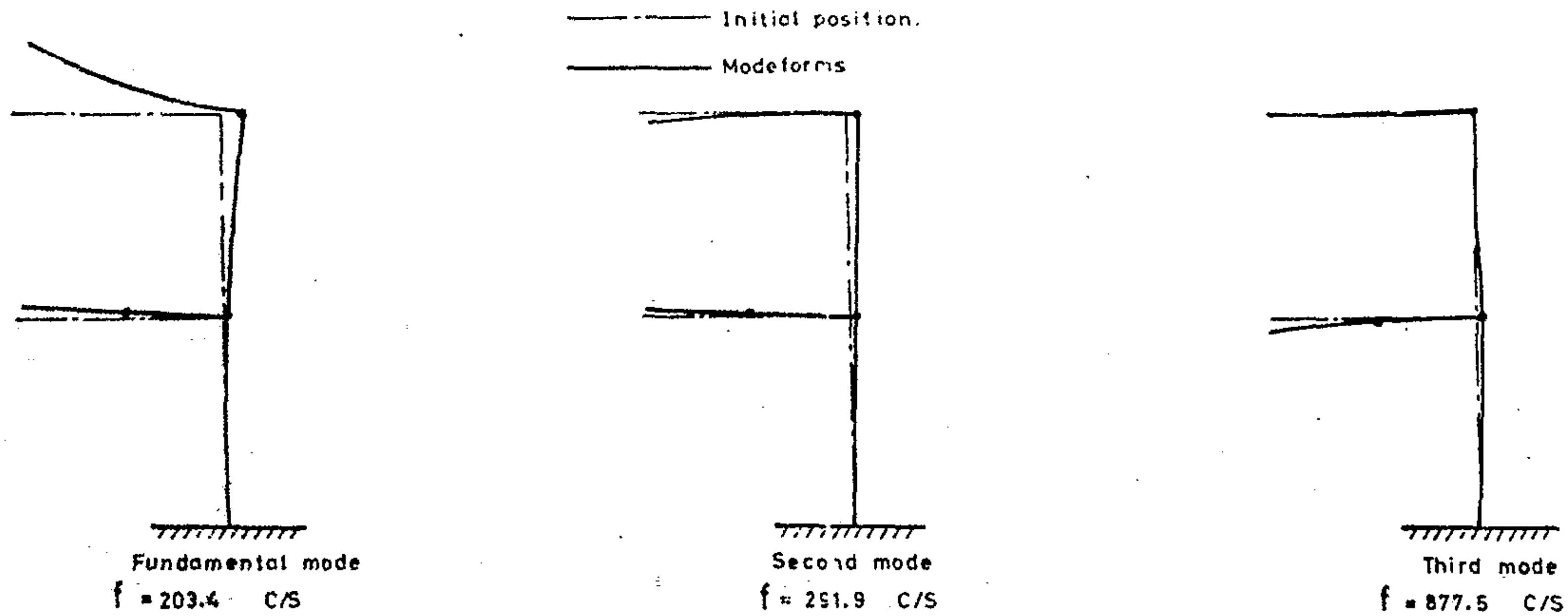
Fundamental mode $f = 196.2$ C/S

Second mode $f = 286.5$ C/S

Third mode $f = 877.5$ C/S

Mode no.	z_1	f c/s	Deflections																								
			Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
			X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
			0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st	0.91	203.4	0	0.08	0.31	0.67	1.13	1.13	1.67	2.27	2.93	3.64	0	0.24	0.48	0.72	0.97	0.97	1.22	1.47	1.72	1.97	0	1.69	4.79	8.63	12.69
2nd	1.09	291.9	0	0.08	0.28	0.58	0.93	0.93	1.29	1.62	1.91	2.18	0	0.17	0.35	0.53	0.72	0.72	0.91	1.1	1.29	1.49	0	-0.04	-0.54	-1.28	-2.1
3rd	1.89	877.5	0	0.14	0.41	0.61	0.57	0.57	0.37	0.2	0.03	-0.14	0	-0.13	-0.41	-0.74	-1.12	-1.12	-1.56	-2.05	-2.56	-3.09	0	-0.33	-0.61	-0.52	-0.1

Table (14) the third modification equipped with the fifth overarm



Mode number	Frequency ratios.	Deflection ratios																								
		Beam					Beam					Beam					Beam					Beam				
		X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
		0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st mode	0.95	0	1	1.03	1.01	1.02	1.02	1.02	1.03	1.04	1.04	0	1	1.02	1.03	1.03	1.03	1.03	1.03	1.03	1.02	0	1.2	1.22	1.22	1.23
2nd mode	0.98	0	1	1	1.01	1.01	1.01	1.01	1.02	1.02	1.03	0	1	1	1.02	1.01	1.01	1.01	1.02	1.01	1.02	0	5	-0.06	0.3	0.4
3rd mode	1	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1.1	1.04	0.82	1.44

Table (15) Comparison between the third modification equipped with the first overarm with its basic model.

Mode number	Frequency ratios.	Deflection ratios																								
		Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
		X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
		0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st mode	1.23	0	0.89	0.91	0.89	0.87	0.87	0.86	0.84	0.82	0.81	0	0.82	0.84	0.84	0.85	0.85	0.84	0.85	0.85	0.84	0	0.45	0.41	0.4	0.34
2nd mode	1.02	0	1	0.96	0.98	0.96	0.96	0.96	0.96	0.95	0.94	0	0.94	0.94	0.94	0.94	0.94	0.95	0.95	0.95	0.95	0	-1.66	0.15	1.39	1.35
3rd mode	1	0	1	1	1	1	1	1	1	0.75	1.07	0	1	1	1.01	1	1	1	1	1	1	0	0.82	0.92	1.6	-0.29

Table (16) Comparison between the third modification equipped with the second overarm with its basic model.

Mode number	Frequency ratios.	Deflection ratios																								
		Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
		X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
		0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st mode	0.97	0	1	1	1.01	1.01	1.01	1.01	1.02	1.02	1.02	0	1	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	0	1.12	1.13	1.13	1.13
2nd mode	1	0	1	1	1.01	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0.66	1.09	1.05	1.04
3rd mode	1	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1.05	1.03	0.88	1.26

Table (17) Comparison between the third modification equipped with the third overarm with its basic model.

Mode number	Frequency ratios.	Deflection ratios																								
		Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
		X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
		0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st mode	1	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1.01	1.01	1.01	0.98
2nd mode	1	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1	1.06	1.02	1.02
3rd mode	1	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1.02	1.01	1	1.04

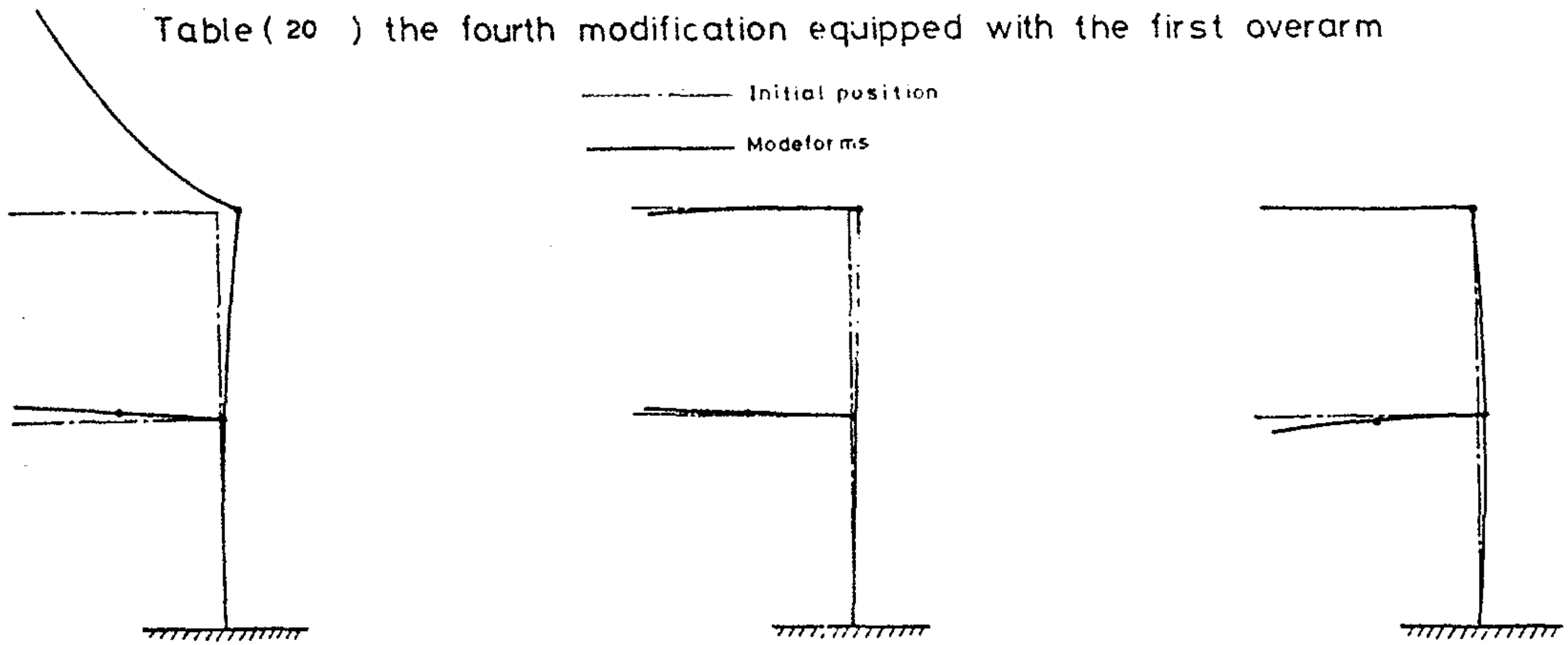
Table (18) Comparison between the third modification equipped with the fourth overarm with its basic model.

Mode number	Frequency ratios.	Deflections ratios																								
		Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
		X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
		0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st mode	1.2	0	0.89	0.91	0.9	0.89	0.89	0.87	0.86	0.84	0.83	0	0.86	0.85	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0	0.49	0.45	0.44	0.43
2nd mode	1.02	0	1	0.96	0.98	0.97	0.97	0.97	0.96	0.95	0.94	0	0.94	0.94	0.94	0.96	0.96	0.96	0.95	0.95	0.96	0	-1.33	1.59	1.37	1.34
3rd mode	1	0	1	1	1	1	1	1	1	0.75	1.07	0	1	1	1.01	1	1	1	1	1	1	0	0.84	0.92	1.57	0.22

Table (19)Comparison between the third modification equipped with the fifth overarm with its basic model.

Mode no.	Z_1	f c/s	Deflections																								
			Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
			X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
			0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st	0.79	170.3	0	0.09	0.34	0.75	1.29	1.29	1.69	2.72	3.59	4.57	0	0.23	0.59	0.9	1.2	1.2	1.51	1.82	2.12	2.43	0	4.74	14.99	28.11	42.14
2nd	1.05	300.9	0	0.08	0.29	0.59	0.96	0.96	1.35	1.72	2.08	2.43	0	0.13	0.39	0.6	0.81	0.81	1.02	1.24	1.46	1.68	0	0.2	0.17	0.04	-0.33
3rd	1.8	884.3	0	0.12	0.38	0.58	0.59	0.59	0.46	0.35	0.25	0.15	0	-0.13	-0.37	-0.69	-1.07	-1.07	-1.52	-2.02	-2.56	-3.1	0	-0.03	0.02	0.09	0.13

Table (20) the fourth modification equipped with the first overarm



Fundamental mode f = 170.3 C/S

Second mode f = 300.9 C/S

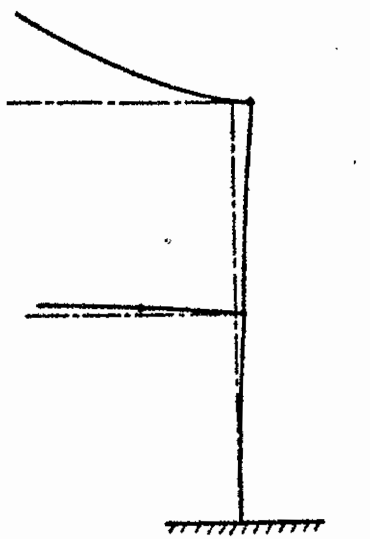
Third mode f = 884.3 C/S

Mode no.	z_1	f c/s	Deflections																								
			Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
			X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
			0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st	0.9	221.1	0	0.08	0.3	0.65	1.1	1.1	1.64	2.23	2.87	3.57	0	0.24	0.49	0.74	0.99	0.99	1.24	1.5	1.76	2.02	0	1.73	4.99	9.04	13.34
2nd	1.07	312.5	0	0.07	0.28	0.57	0.92	0.92	1.28	1.62	1.93	2.2	0	0.18	0.37	0.56	0.76	0.76	0.96	1.16	1.37	1.55	0	-0.04	-0.55	-1.31	-2.15
3rd	1.8	884.3	0	0.12	0.38	0.58	0.58	0.58	0.45	0.35	0.25	0.15	0	-0.13	-0.37	-0.7	-1.08	-1.08	-1.45	-2.05	-2.59	-3.13	0	-0.02	0.05	0.14	0.2

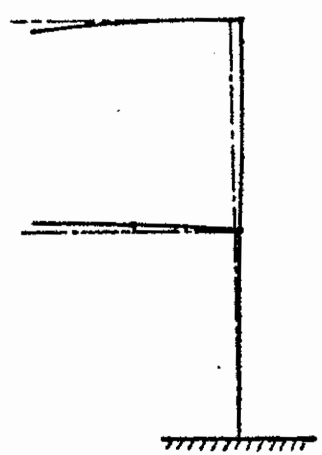
Table (21) the fourth modification equipped with the second overarm

—·—·— Initial position

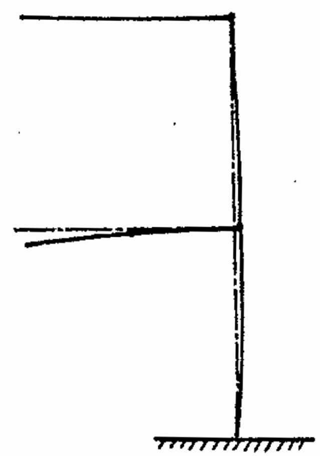
——— Modeforms.



Fundamental mode
$f = 221.1$ C/S



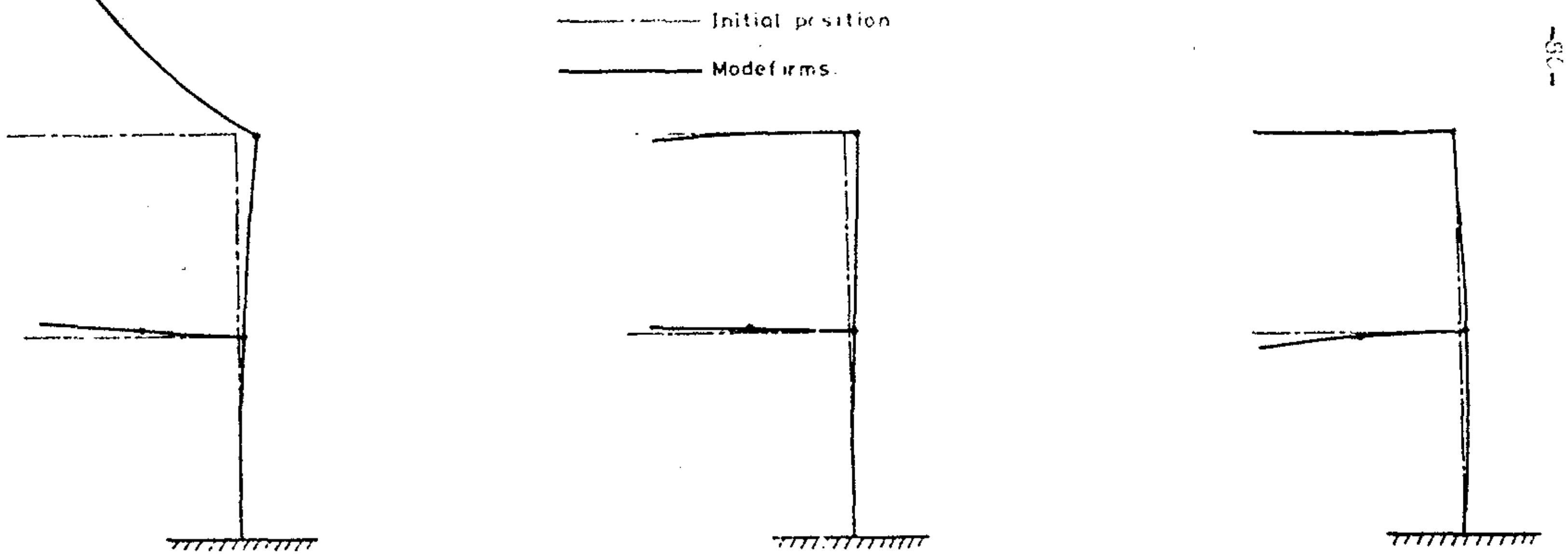
Second mode
$f = 312.5$ C/S



Third mode
$f = 884.3$ C/S

Mode no.	z_1	f c/s	Deflections																								
			Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
			X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
			0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st	0.8	174.7	0	0.09	0.34	0.74	1.28	1.28	1.93	2.69	3.53	4.48	0	0.29	0.59	0.89	1.19	1.19	1.49	1.79	2.1	2.41	0	4.39	13.82	25.84	38.75
2nd	1.06	306.7	0	0.08	0.29	0.59	0.95	0.95	1.33	1.69	2.04	2.30	0	0.19	0.38	0.59	0.76	0.76	1.01	1.22	1.44	1.6[illegible]	0	0.05	-0.3	-0.85	-1.40
3rd	1.8	884.3	0	0.12	0.38	0.58	0.59	0.59	0.46	0.35	0.25	0.15	0	-0.13	-0.37	-0.69	-1.07	-1.07	-1.52	-2.07	-2.5[illegible]	-3.[illegible]	0	-0.04	0	0.08	0.14

Table(22) the fourth modification equipped with the third overarm



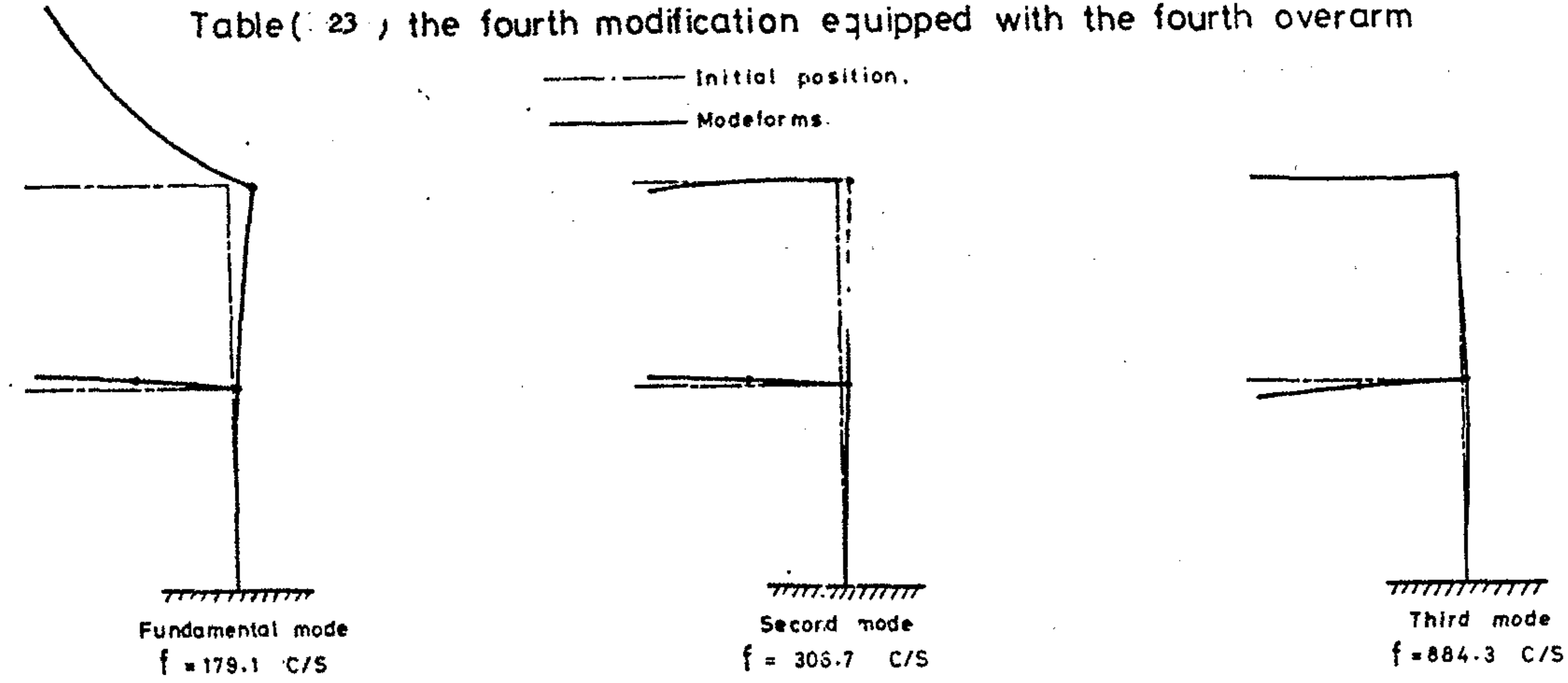
Fundamental mode f = 174.7 C/S

Second mode f = 306.7 C/S

Third mode f = 884.3 C/S

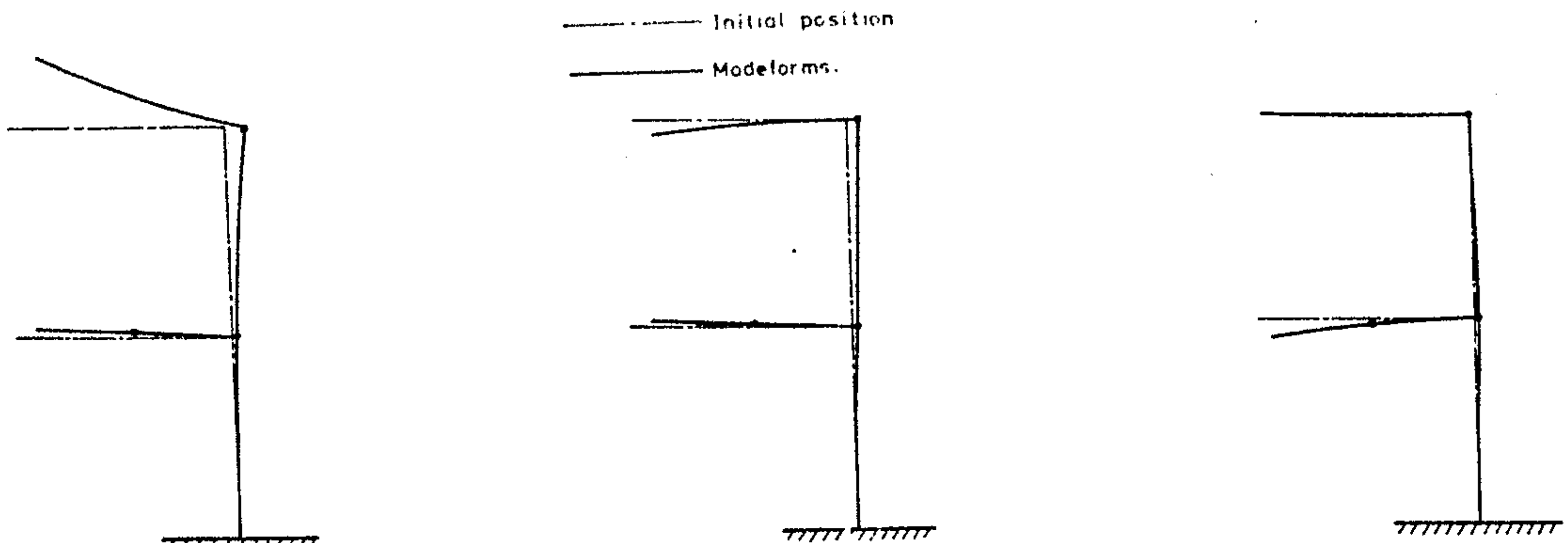
Mode no.	z_1	f c/s	Deflections																								
			Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
			X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
			0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st	0.81	179.1	0	0.09	0.34	0.73	1.26	1.26	1.9	2.64	3.47	4.39	0	0.28	0.58	0.87	1.16	1.16	1.46	1.76	2.06	2.36	0	3.98	12.43	23.22	34.76
2nd	1.06	306.7	0	0.08	0.29	0.59	0.95	0.95	1.33	1.69	2.03	2.35	0	0.19	0.38	0.59	0.79	0.79	1	1.22	1.34	1.65	0	0.05	-0.29	-0.84	-1.44
3rd	1.8	884.3	0	0.12	0.38	0.58	0.59	0.59	0.46	0.35	0.25	0.15	0	-0.13	-0.37	-0.69	-1.07	-1.07	-1.52	-2.02	-2.56	-3.1	0	-0.04	0.01	0.09	0.14

Table(23) the fourth modification equipped with the fourth overarm



Mode no.	Z_1	f C/S	Deflections																								
			Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
			X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
			0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st	0.9	221.1	0	0.08	0.3	0.65	1.11	1.11	1.64	2.24	2.88	3.58	0	0.24	0.49	0.74	0.99	0.99	1.25	1.51	1.77	2.03	0	1.78	5.14	9.32	13.76
2nd	1.07	312.5	0	0.07	0.28	0.57	0.92	0.92	1.29	1.63	1.94	2.22	0	0.18	0.37	0.56	0.76	0.76	0.96	1.17	1.39	1.59	0	0.04	-0.54	-1.3	-2.13
3rd	1.8	884.3	0	0.12	0.38	0.58	0.58	0.58	0.45	0.35	0.25	0.15	0	-0.13	-0.37	-0.7	-1.08	-1.08	-1.53	-2.04	-2.58	-3.13	0	-0.03	0.04	0.13	0.2

Table (24) the fourth modification equipped with the fifth overarm



Fundamental mode $f = 221.1$ C/S

Second mode $f = 312.5$ C/S

Third mode $f = 884.3$ C/S

Mode number	Frequency ratios	Deflection ratios																								
		Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
		X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
		0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st mode	0.93	0	1.12	1.03	1.03	1.03	1.03	1.05	1.05	1.05	1.06	0	1.03	1.03	1.04	1.04	1.04	1.04	1.04	1.04	1.04	0	1.26	1.27	1.28	1.29
2nd mode	0.98	0	1	1	1	1.01	1.01	1.01	1.02	1.02	1.03	0	1	1.02	1.01	1.02	1.02	1.02	1.01	1.02	1.02	0	4	-0.63	-0.05	0.23
3rd mode	1	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0.75	2	1	0.86

Table (25)Comparison between the fourth modification equipped with the first overarm with its basic model.

Mode number	Frequency ratios	Deflection ratios																								
		Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
		X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
		0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st mode	1.2	0	1	0.91	0.89	0.88	0.88	0.88	0.86	0.84	0.83	0	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0	0.46	0.42	0.41	0.41
2nd mode	1.02	0	0.87	0.96	0.96	0.97	0.97	0.96	0.96	0.95	0.93	0	0.95	0.97	0.95	0.96	0.96	0.96	0.95	0.96	0.94	0	-0.8	2.04	1.62	1.52
3rd mode	1	0	1	1	1	1	1	0.98	1	1	1	0	1	1	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0	0.5	5	1.55	1.33

Table(26)Comparison between the fourth modification equipped with the second overarm with its basic model.

Mode number	Frequency ratios.	Deflection ratios																								
		Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
		X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
		0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st mode	0.95	0	1.12	1.03	1.01	1.02	1.02	1.03	1.04	1.04	1.04	0	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	0	1.16	1.17	1.18	1.18
2nd mode	1	0	1	1	1	1	1	1	1	1	1	0	1	1	1	0.96	0.96	1.01	1	1	1	0	1	1.11	1.05	1.03
3rd mode	1	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0.99	0.99	0	1	0	0.89	0.93

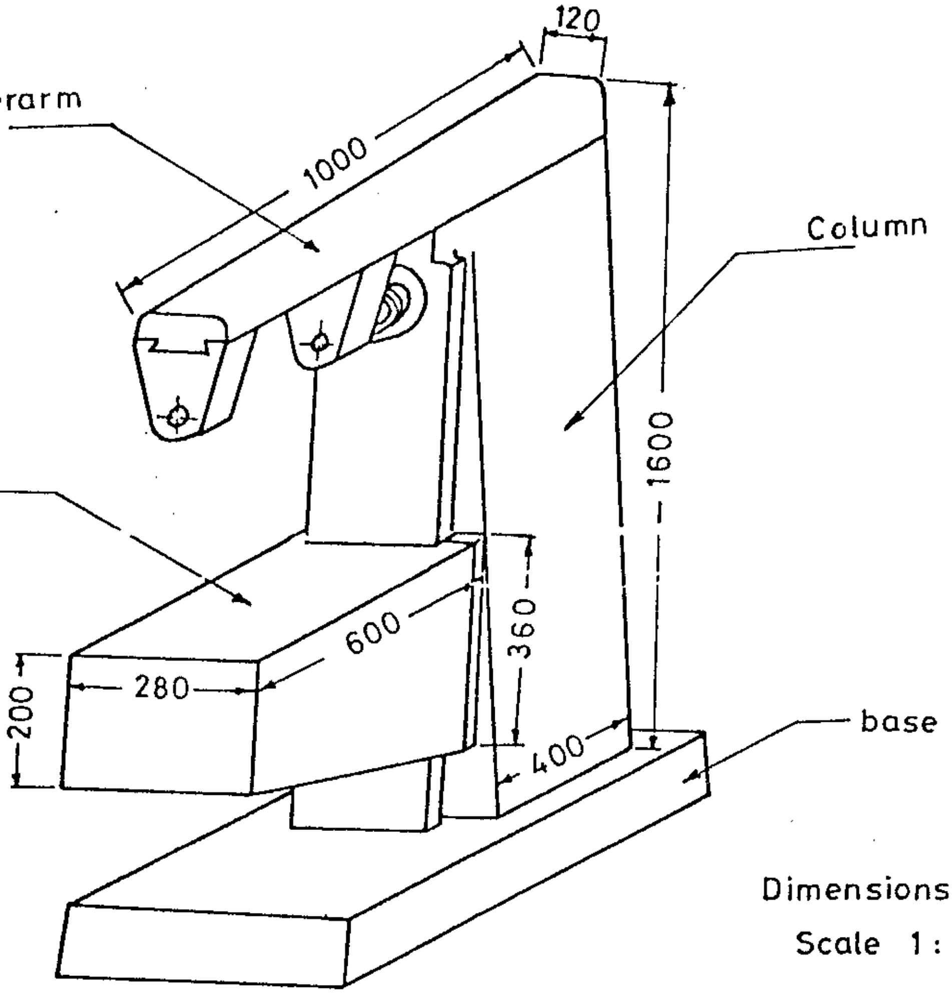
Table(27)Comparison between the fourth modification equipped with the third overarm with its basic model.

Mode number	Frequency ratios.	Deflection ratios																								
		Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
		X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
		0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st mode	0.97	0	1.12	1.03	1	1.01	1.01	1.02	1.02	1.02	1.02	0	1	1.02	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0	1.05	1.06	1.06	1.06
2nd mode	1	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1	1.07	1.04	1.02
3rd mode	1	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1	1	1	0.93

Table(28) Comparison between the fourth modification equipped with the fourth overarm with its basic model.

Mode number	Frequency ratios.	Deflection ratios																								
		Beam 1					Beam 2					Beam 3					Beam 4					Beam 5				
		X_1/L_1					X_2/L_2					X_3/L_3					X_4/L_4					X_5/L_5				
		0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1	0	0.25	0.5	0.75	1
1st mode	1.2	0	1	0.91	0.89	0.89	0.89	0.88	0.86	0.85	0.83	0	0.86	0.86	0.86	0.86	0.86	0.87	0.87	0.87	0.87	0	0.47	0.44	0.42	0.42
2nd mode	1.02	0	0.87	0.96	0.96	0.97	0.97	0.97	0.96	0.95	0.94	0	0.95	0.97	0.95	0.96	0.96	0.96	0.96	0.96	0.96	0	0.8	2	1.6	1.51
3rd mode	1	0	1	1	1	0.98	0.98	0.98	1	1	1	0	1	1	1.01	1.01	1.01	1	1.01	1.01	1.01	0	0.75	4	1.44	1.33

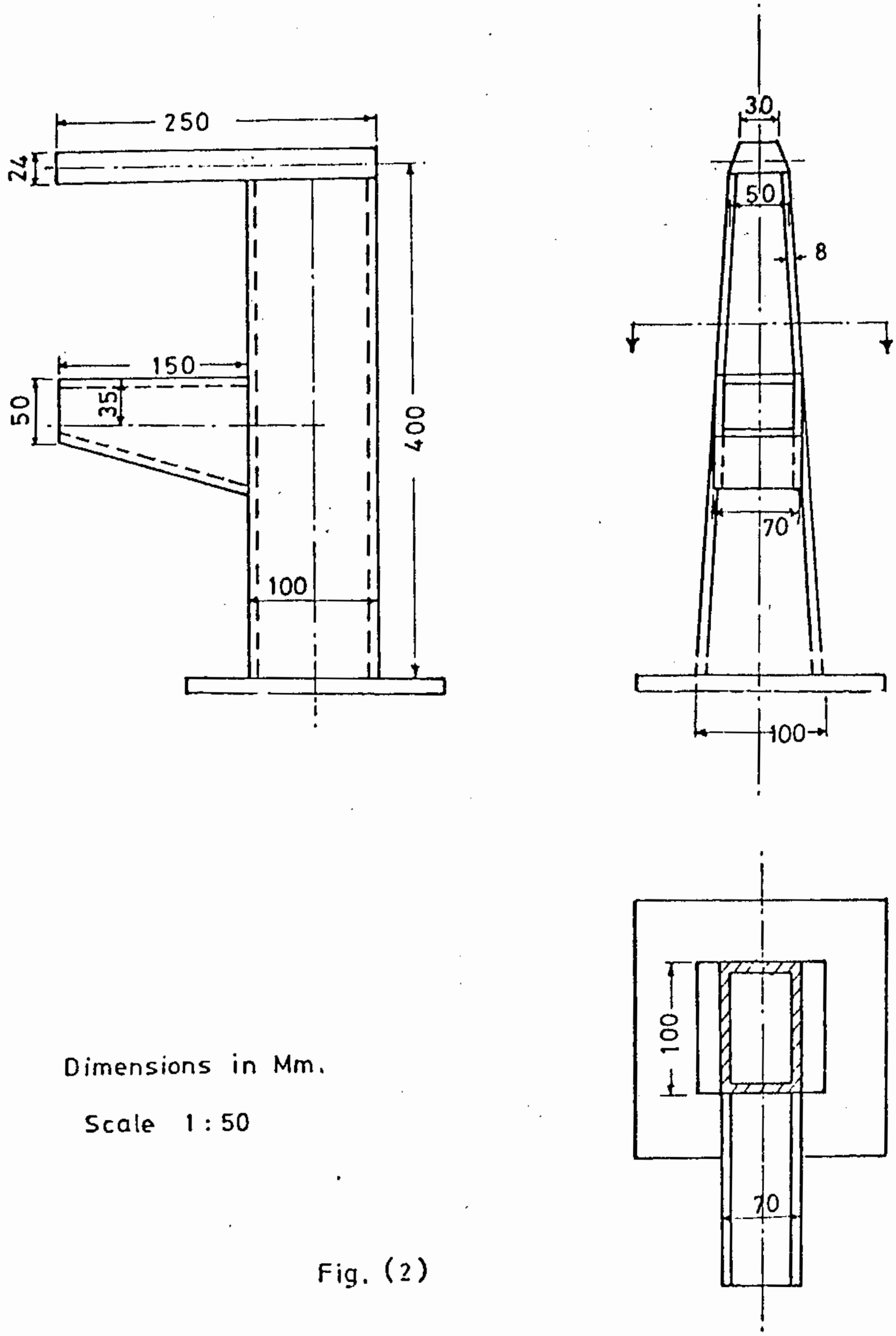
Table (29)Comparison between the fourth modification equipped with the fifth overarm with its basic model.



Dimensions in Mm.
Scale 1 : 200

Fig. (1)

Main parts and dimensions of the horizontal knee_type milling machine.



Dimensions in Mm.

Scale 1 : 50

Fig. (2)

The basic model of the machine.

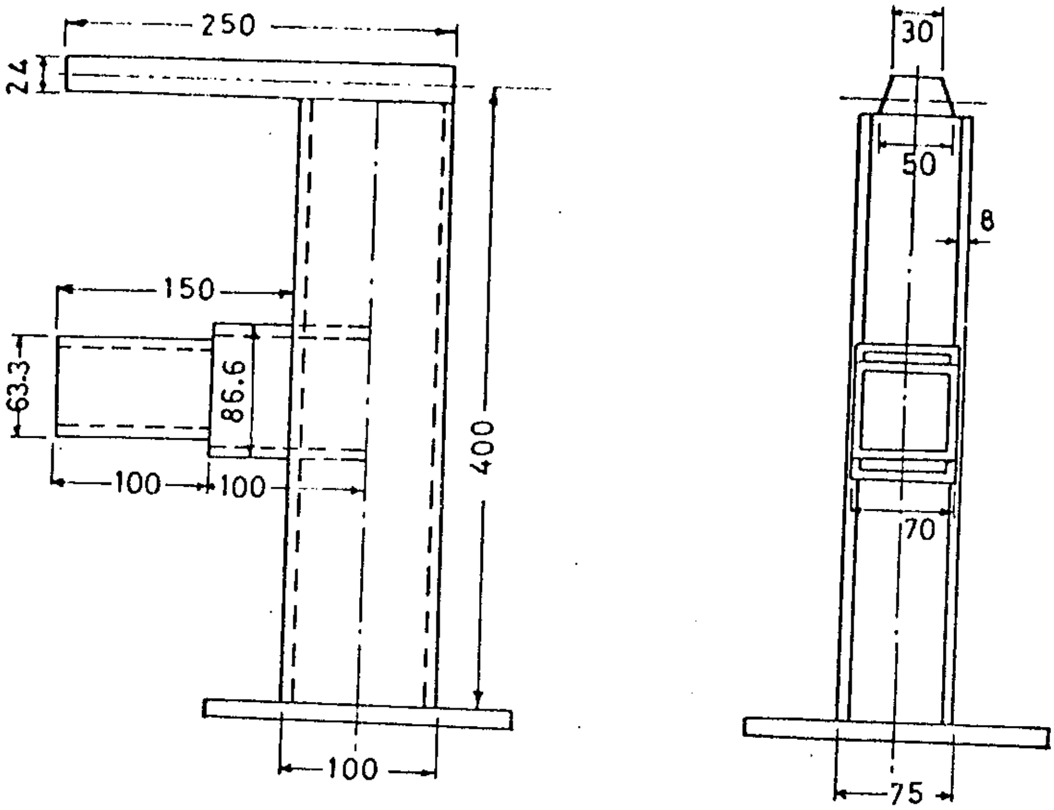


Fig.(3)

Idealized basic model

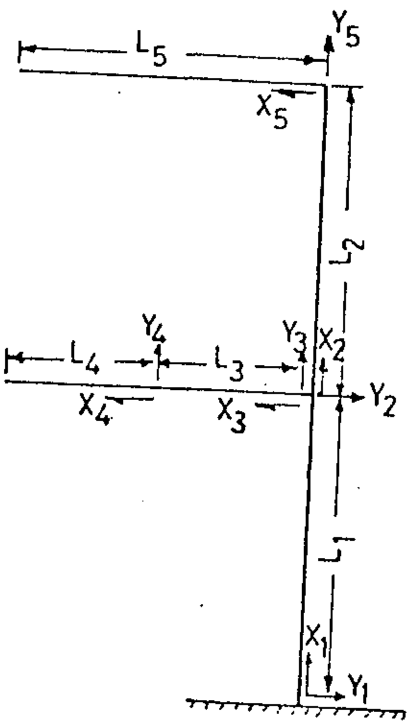


Fig. (4)

Mathematical model

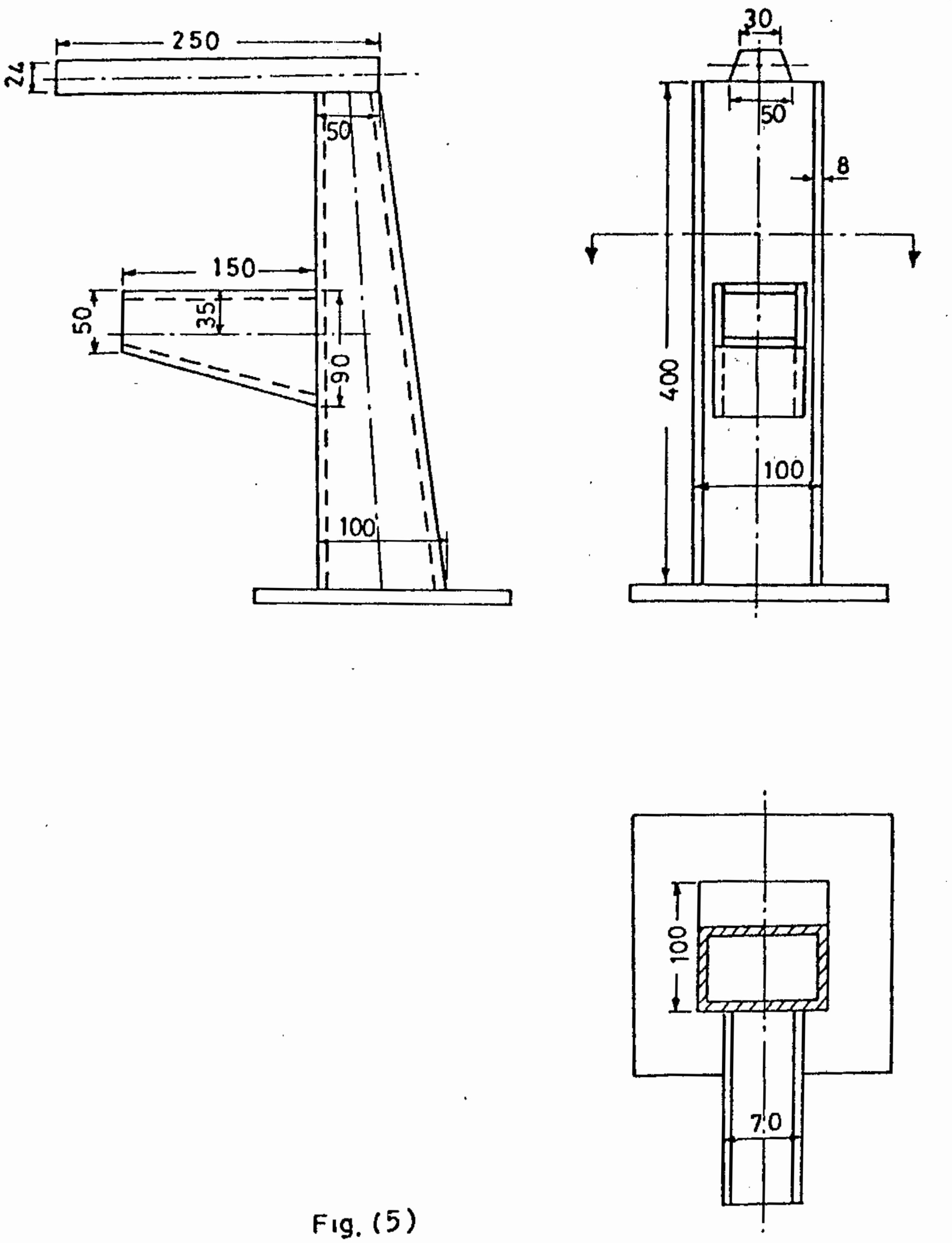


Fig. (5)

Model of the first modification.

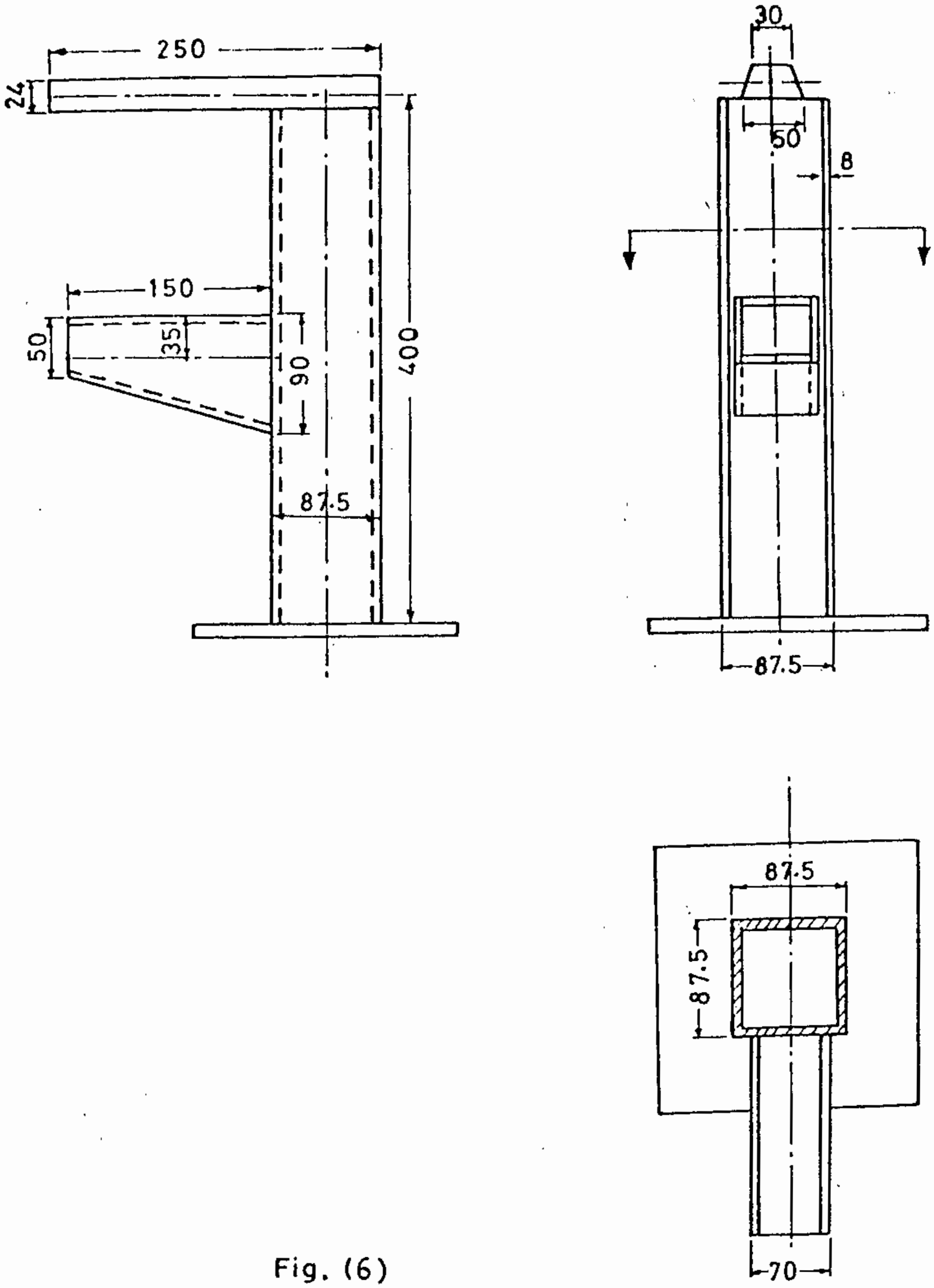


Fig. (6)

Model of the second modification

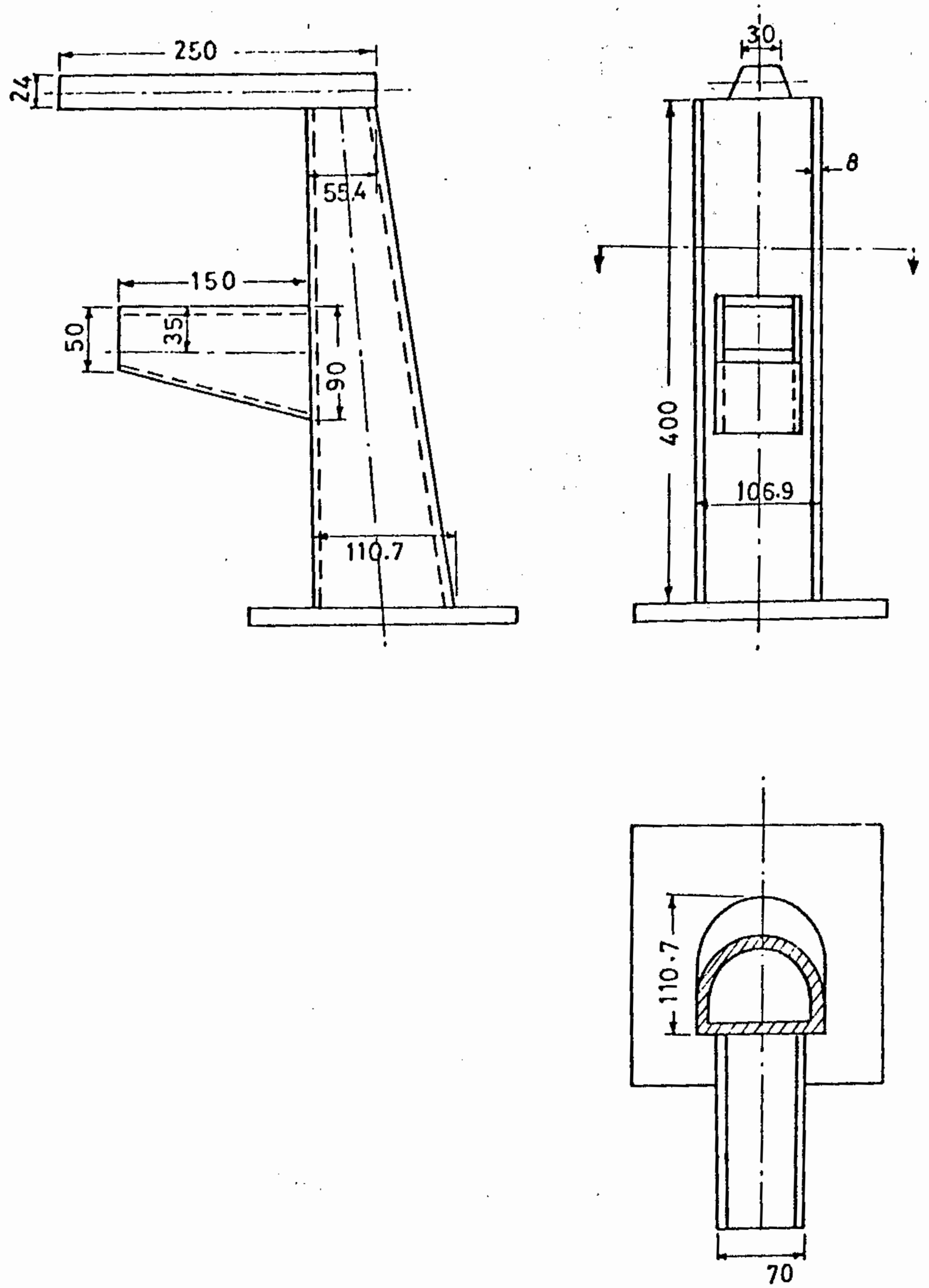


Fig. (7) .

Model of the third modification.

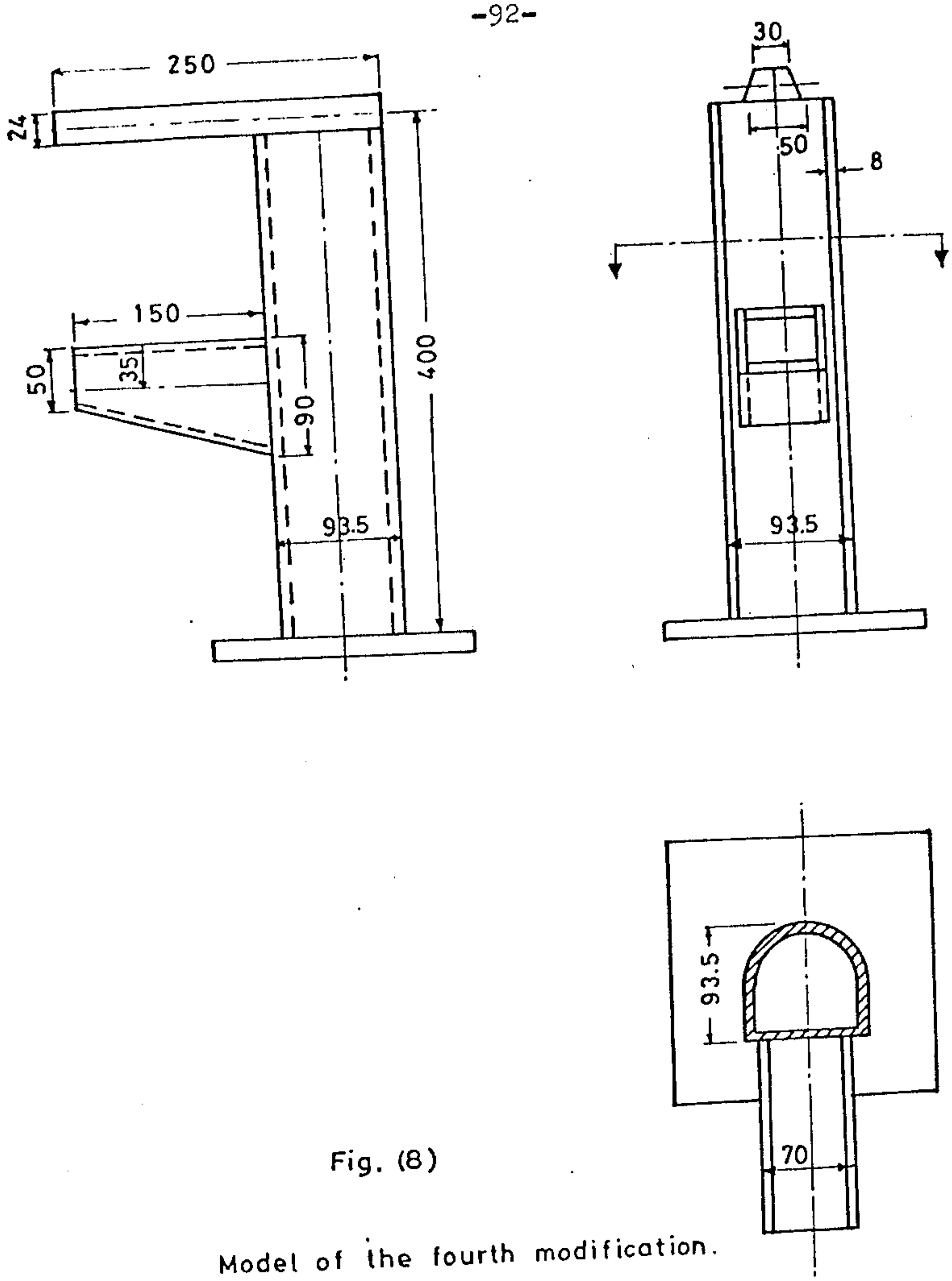


Fig. (8)

Model of the fourth modification.

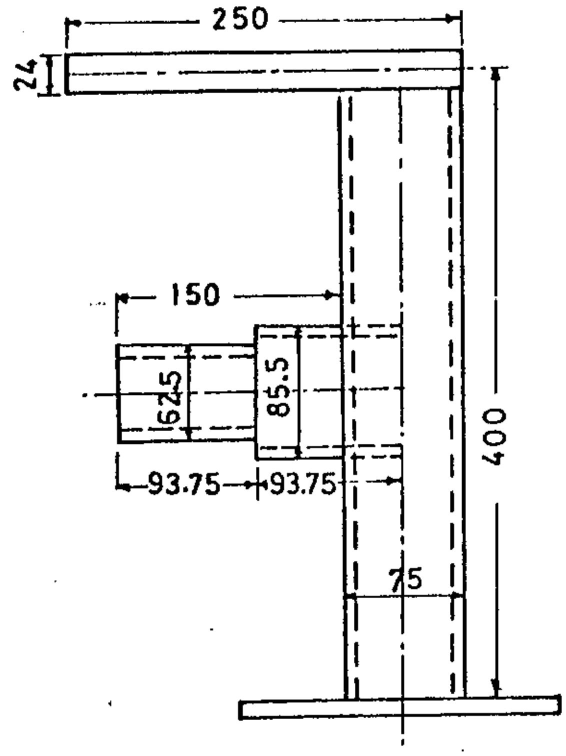


Fig.(9)

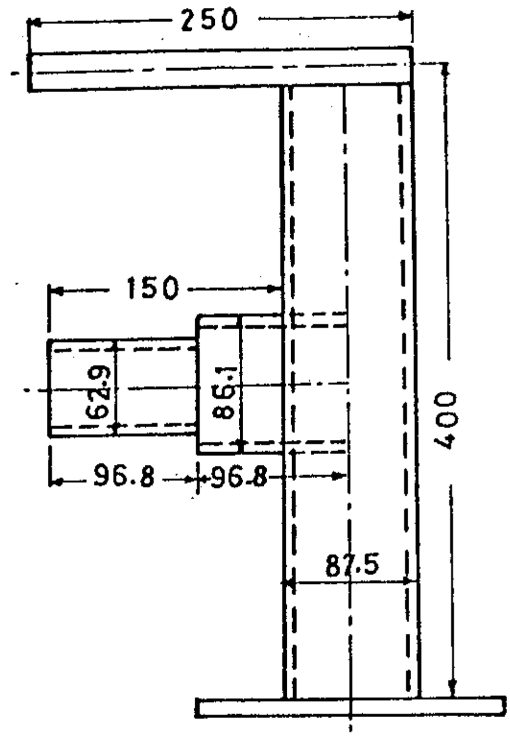


Fig. (10)

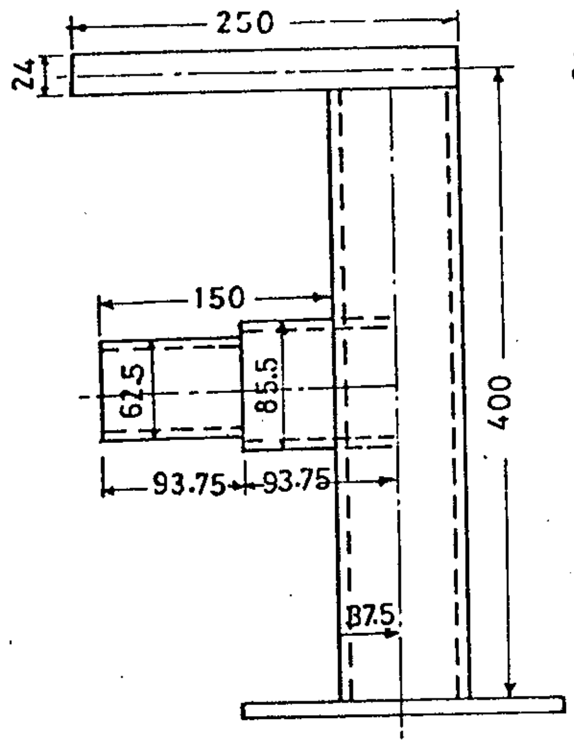


Fig.(11)

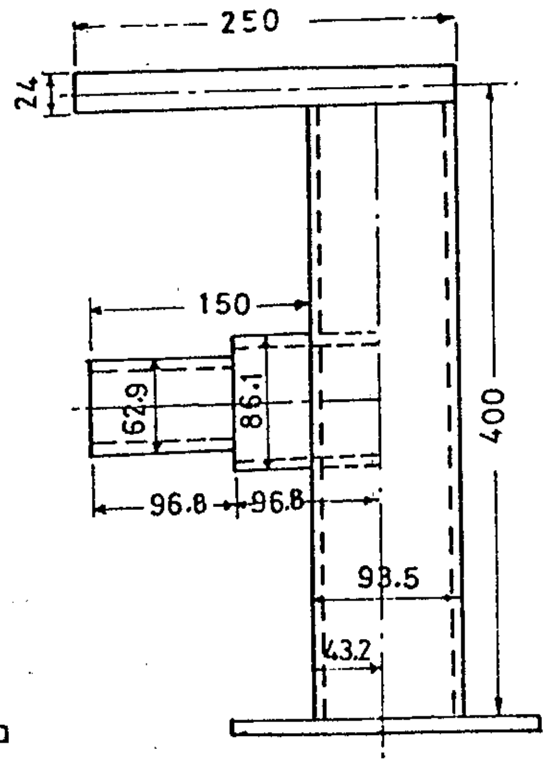
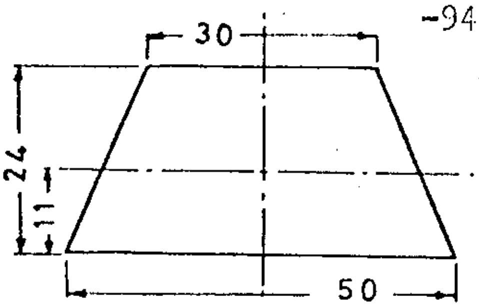


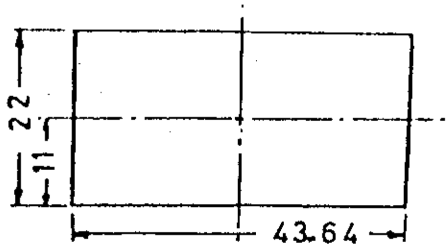
Fig. (12)

The idealized models of the four suggested modification .



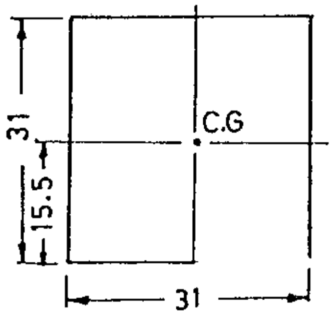
The basic cross-section

(Trapejoid)



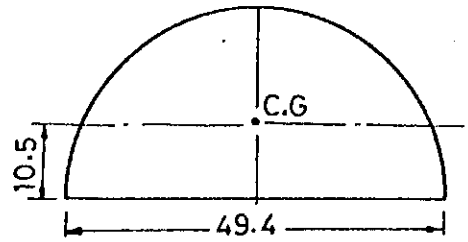
The first modification

(Rectangle)



The second modification

(Square)

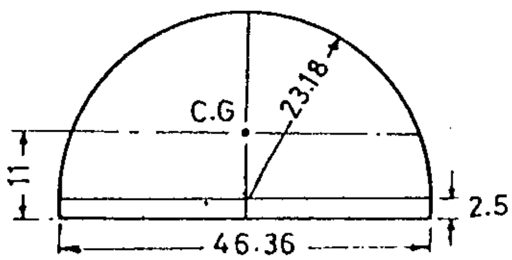


The third modification

(Semi – Circle)

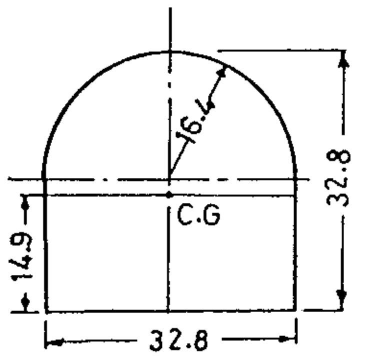
Dimensions in Mm.

Scale 1:1



The fourth modification

(Semi – Circular on rectanglar base)

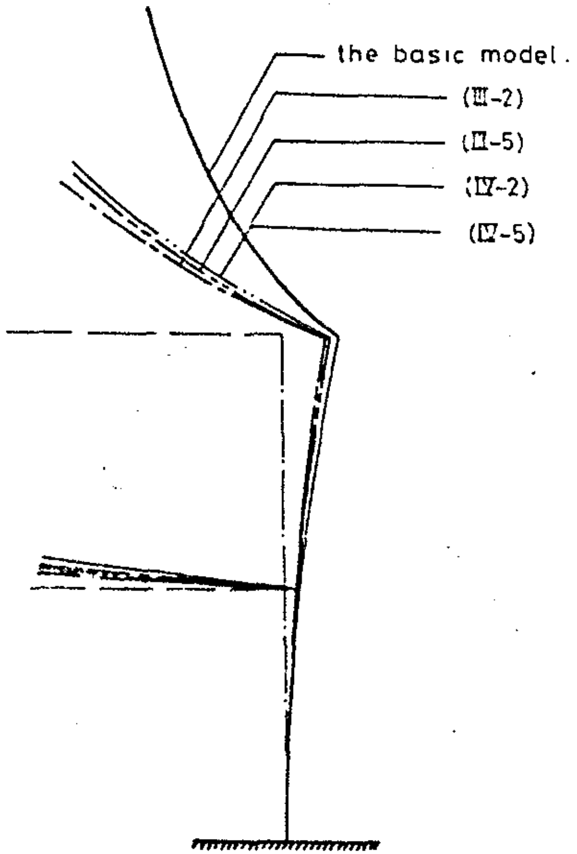


The fifth modification

(Semi – Circular tail)

Fig. (13)

Different Cross–Sectional alternative shapes of the overarm.

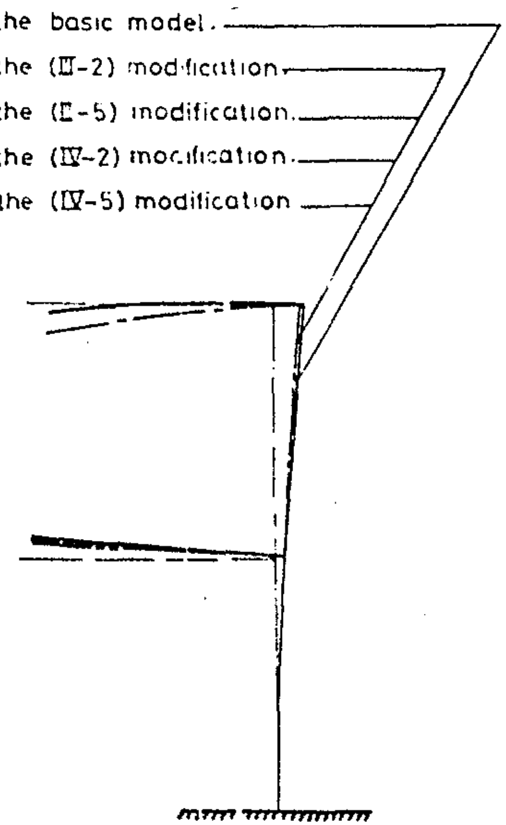


a) Fundamental modeforms

f_{basic} = 181.5

$f_{(III-2)}$ = 207.9 $f_{(III-5)}$ = 203.4

$f_{(IV-2)}$ = 221.1 $f_{(IV-5)}$ = 221.1

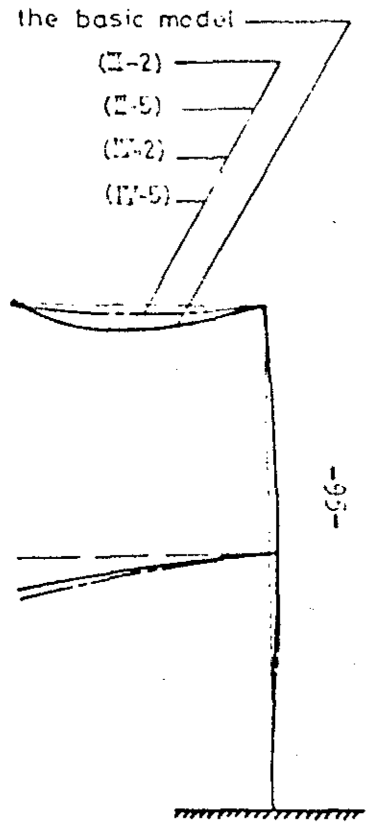


b) Second modeforms

f_{basic} = 314.6

$f_{(III-2)}$ = 291.9 $f_{(III-5)}$ = 291.9

$f_{(IV-2)}$ = 312.5 $f_{(IV-5)}$ = 312.5



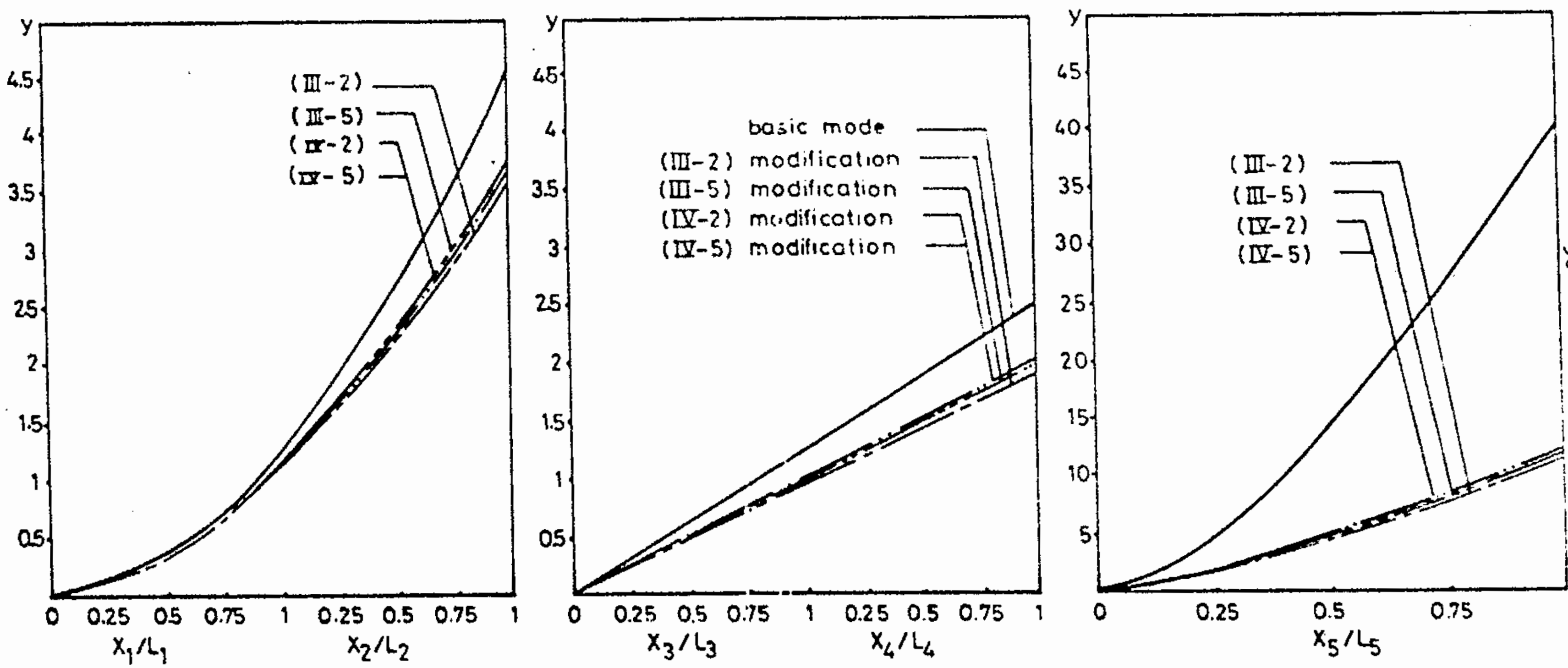
c) Third modeforms

f_{basic} = 860.5

$f_{(III-2)}$ = 877.5 $f_{(III-5)}$ = 877.5

$f_{(IV-2)}$ = 884.3 $f_{(IV-5)}$ = 884.3

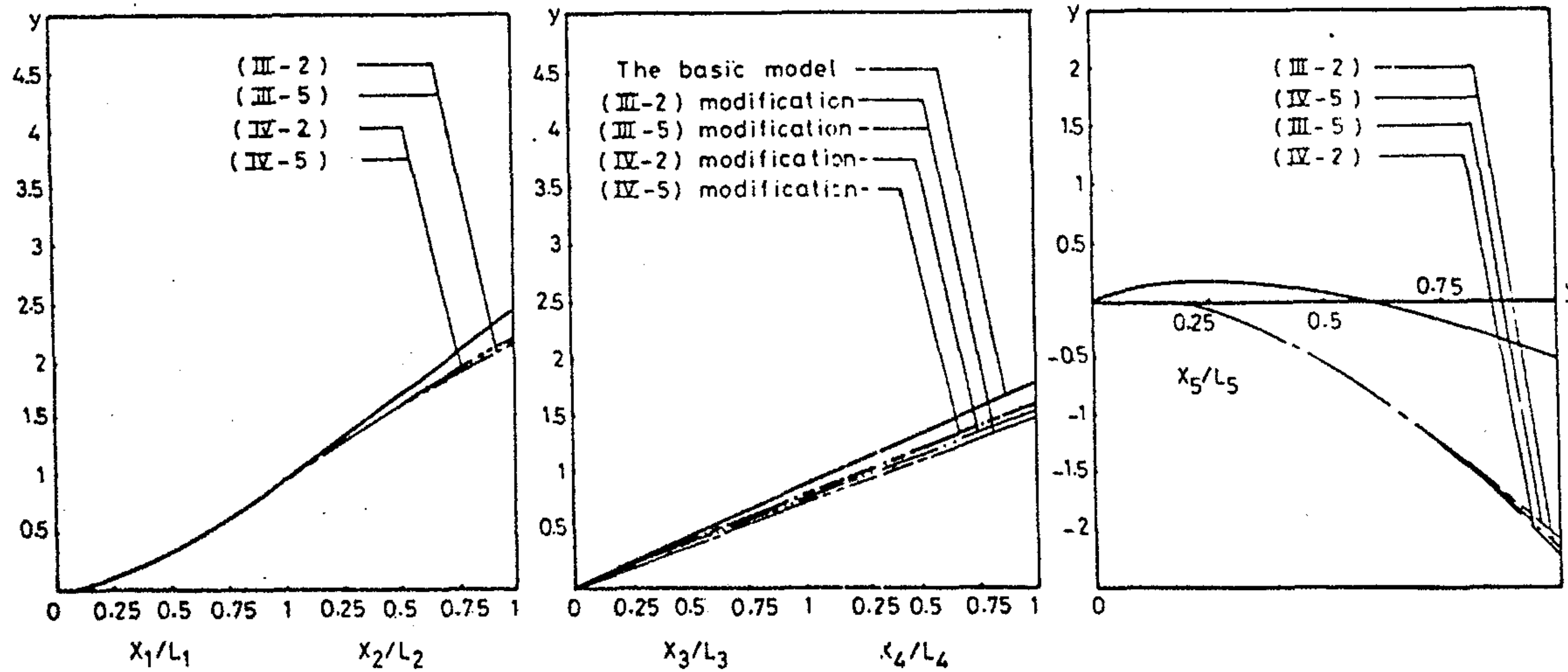
Fig.(14) Modeforms of the basic model and the best modifications



a) The column b) The table c) The overarm

Fig.(15) Deflections of the main parts of the basic model and best modifications

(First mode)



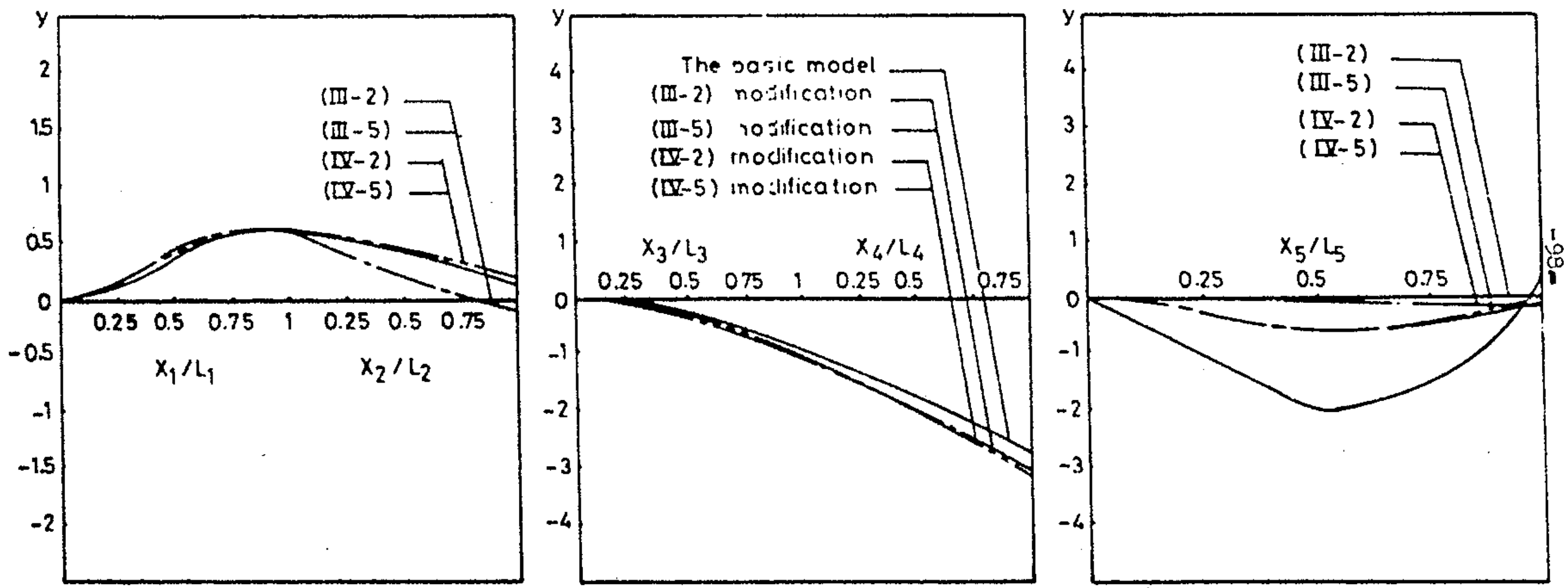
a) The column.

b) The table.

c) The overarm.

Fig. (16) Deflections of the main parts of the basic model and best modifications.

(Second model).



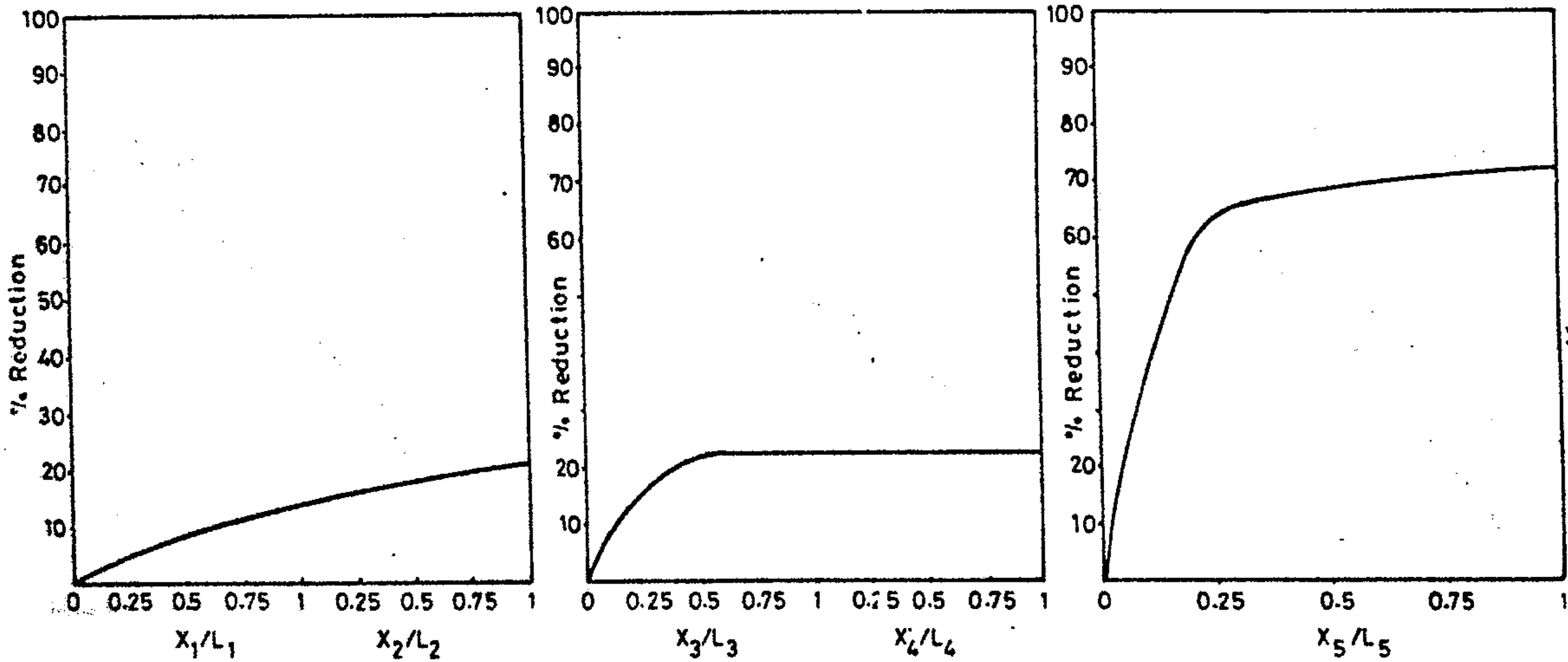
a) The column

b) The table

c) The overarm

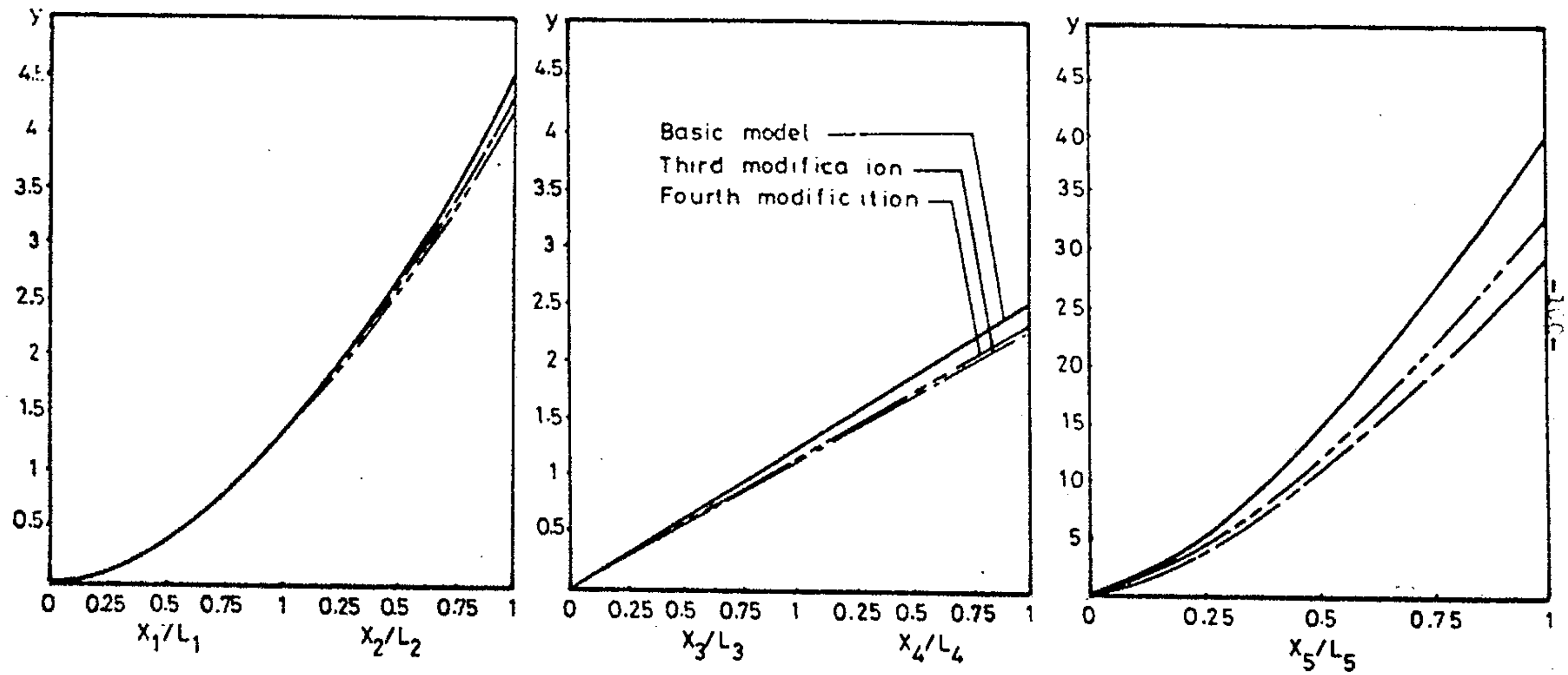
Fig.(17) Deflections of the main parst of the basic model and best modifications

(Third mode)



a) The column b) The table c) The overarm

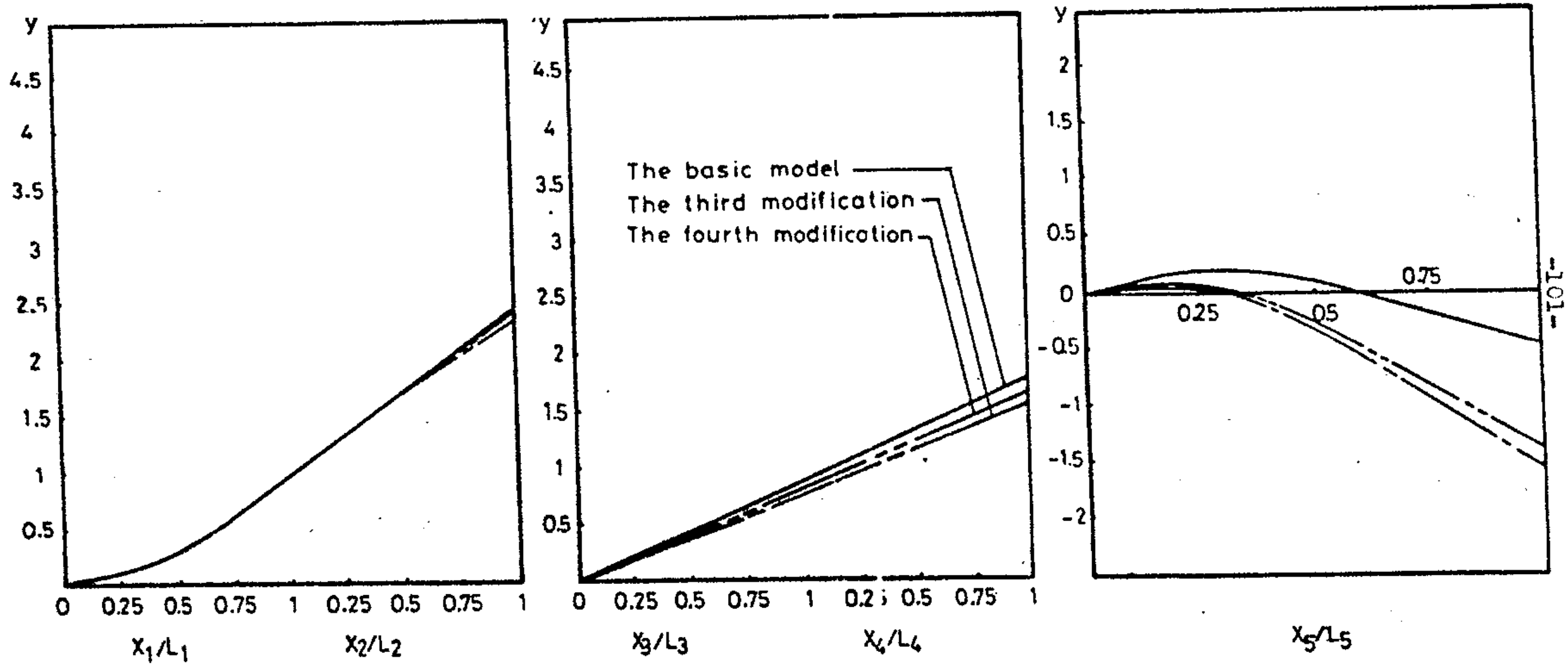
Fig (18) The percentage reduction in the deflection of the main parts of the model (III - 2) as compared with the basic model



a) The column b) The table c) The overarm

Fig. (19) Deflections of the main parts of the basic, third and fourth modifications

(First mode)



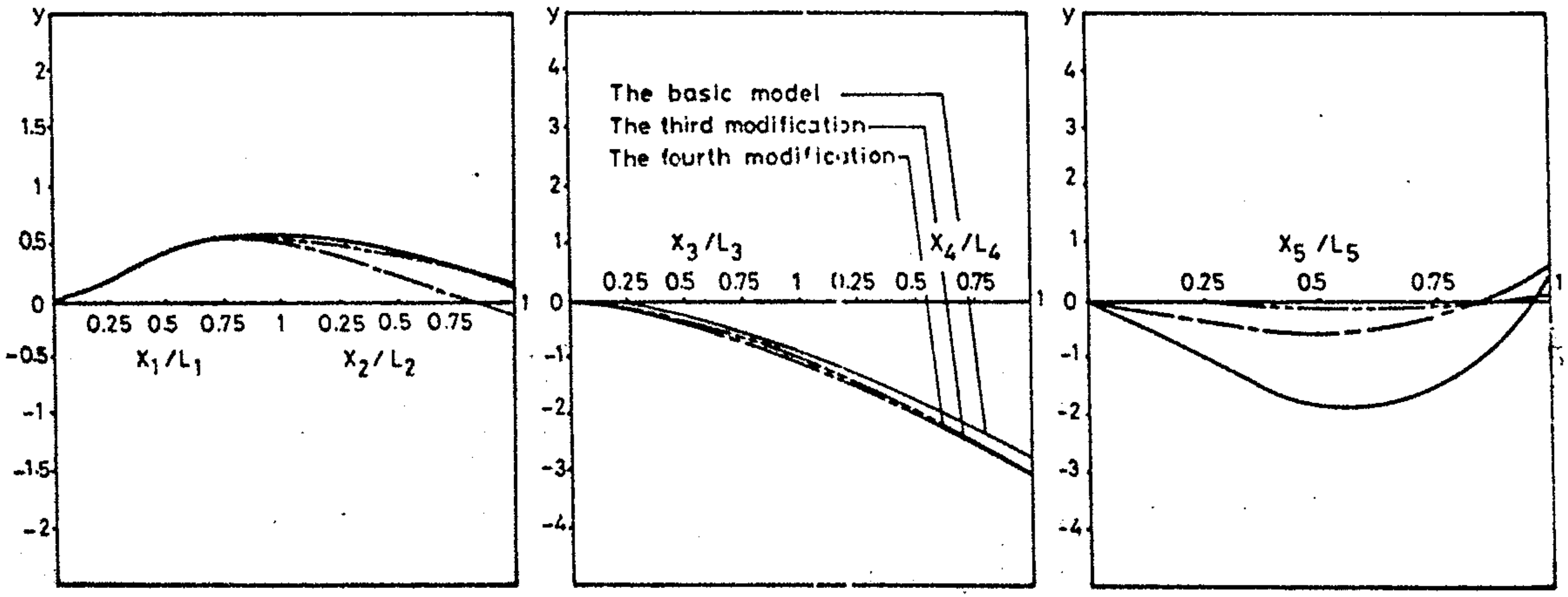
a) The column

b) The table

c) The overarm

Fig. (20) Deflections of the main parts of the basic third and fourth modifications

(Second mode)



a) The column. b) The table. c) The overarm.

Fig. (21) Deflection of the main parts of basic, third and fourth modifications.
(third mode).

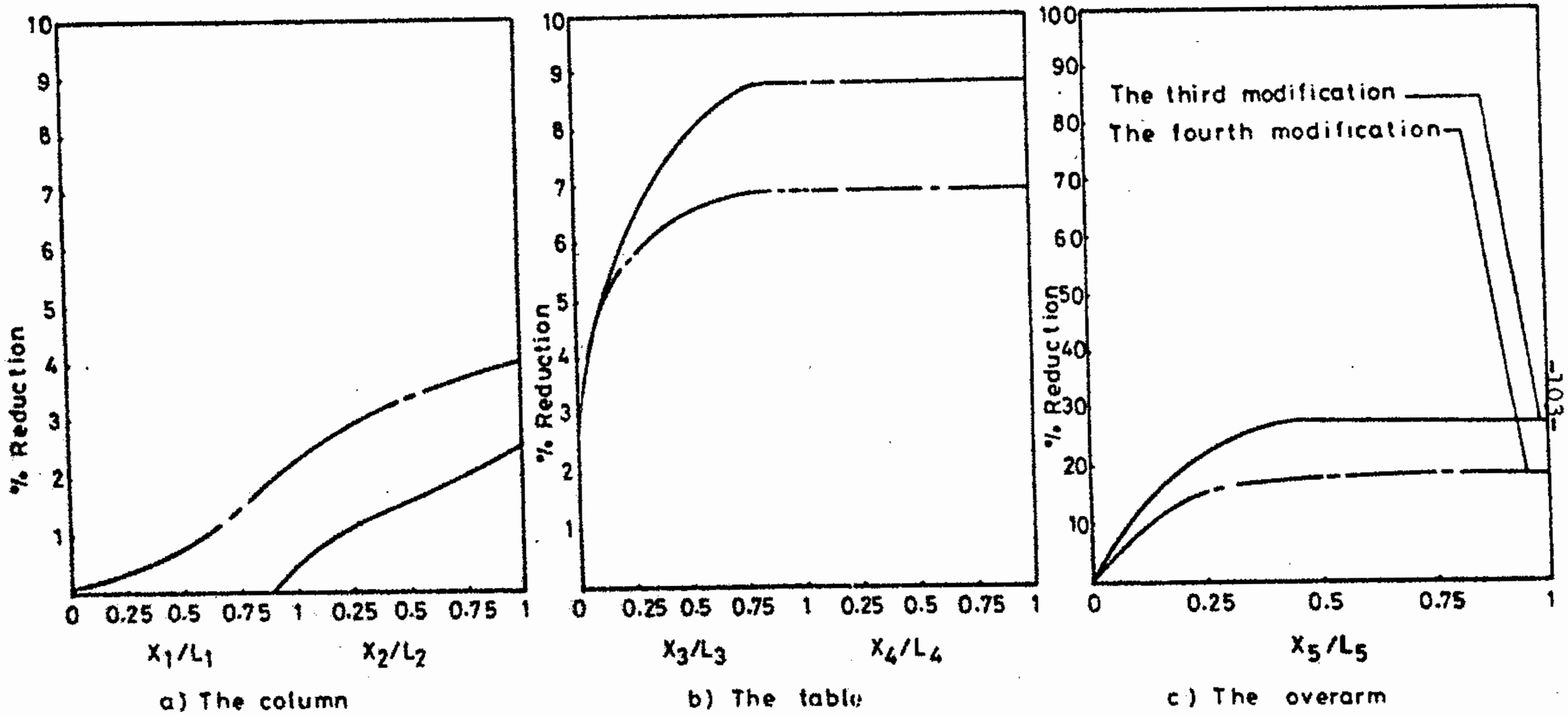


Fig. (22.) The percentage reduction in the deflection of the main parts of the third and modifications as compared with the basic model.

MATHEMATICAL SIMULATION OF MACHINE TOOL STRUCTURES

التوصيف الرياضى لهياكل آلات الانتاج

أستاذ دكتور / عبدالهادى ناصر • أستاذ دكتور / حسن رجب السيد
دكتوره / سعاد محمد سراج • مهندس / سالم ســـــك

هيكل آلة الانتاج هو الجزء الرئيسى لها حيث يساهم فى تحديد حالتها الديناميكية • لهذا السبب كان من الضرورى دراسة أشكال تصميمية مختلفة للهيكل بمساعدة التوصيف الرياضى للهيكل •

فى هذا العمل فحصت الخصائص الديناميكية لهيكل فريزة أفقية على أساس طريقة نظرية العتب مشتملا على النموذج الرياضى للهيكل • وبمساعدة النموذج الرياضى المقدم درس وقورن ١٣ نموذج مقترح يمثلوا حلولا تصميمية لها نفس الوزن ، الأبعاد ، السمك والكتلة •

الدراسة المقدمة بينت امكان توقع الحالة الديناميكية لمعظم الهياكل الملائمة وبالتالى امكان عمل التحسينات الممكنة لها •