الفرقة الثانية

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University	: Menoufia		Date	:	10/6/2019
Faculty	: Electronic Engineering		Time	:	3.0 Hours (10 Am-1 PM)
Department	: Physics and Eng. Math.		No. of pages	:	2
Academic year	: Second Year		Full Mark	:	100 Marks
Course Name	: Eng. Mathematics (6)		Exam	:	Final (Part I)

Part (I): Answer all of the following questions

Question No 1: {10 Marks}

1. Compute the limits a. $\lim_{z\to i}\dfrac{z-i}{z^2+1}$ b. $\lim_{z\to 0}\dfrac{x^2y^2}{x^4+iy^4}$ c. $\lim_{z\to 0}\dfrac{x^2+iy^2}{|z|}$

2. Using Cauchy-Riemann equation, show if the function $f(z)=2x^2+y+i(y^2-x)$ is differentiable, analytic, or entire function in an appropriate domain D. Then, find its first order derivative if it exists.

Question No 2: {20 Marks}

1. Verify that the function $u(x,y)=-e^{-x}\sin y$ is harmonic in an appropriate domain D. Then, find its harmonic conjugate $v(x,y)$ and form the corresponding analytic function $f(z)=u(x,y)+iv(x,y)$.

2. Evaluate the integral $\int_C (3z^2+2z+2)dz$, where C is:

   a. The circle $|z|=R$, R is any real constant.

   b. The straight-line segments connecting the points $\{-1\to -i\to 1\to 1+i\}$.

   c. The straight-line segments connecting the points $\{-1\to -1-i\to 1-i\to 1+i\}$.

3. Using Cauchy integral formula, evaluate the integral $\int_C \dfrac{z^3+3}{z(z-i)^2}dz$, where C is the contour shown in Fig. 1.

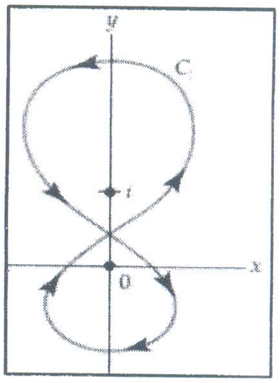

Fig.1

Question No 3: {20 Marks}

1. Expand the complex function $f(z)=\dfrac{1}{z(z-1)}$ in a Laurent series to be valid for the following annular domains:

   a. $0<|z|<1$ b. $0<|z-1|<1$ c. $1<|z-2|<2$

2. Using Cauchy's-Residue theorem evaluate the following integrals:

   a. $\oint_C \dfrac{e^z}{(z-\frac{1}{2}-\frac{\sqrt{15}}{2}i)(z+3)^2}dz$, where C is the circle $|z|=\dfrac{7}{2}$.

   b. $\oint_C \dfrac{z+2}{z^3+z^2+16z+16}dz$, where C is the straight-line segments connecting the points $\{-2i\to 1\to 5i\to -2\to -2i\}$.

Best wishes

Dr. Nasser A. Saeed