EL-MANSOURA UNIVERSITY
FACULTY OF ENGINEERING
DEPT. OF PROD. & MECH. DESIGN ENG.
TIME ALLOWED: 3 HOURS

FINAL EXAM., JAN. 2009
THEORY OF STRESS ANALYSIS
2nd YEAR STUDENTS
EQUATION SHEET IS PROVIDED

SOLVE THE FOLLOWING PROBLEMS; NEAT SKETCHES ARE REQUIRED
ALL PROBLEMS HAVE SAME POINTS

PROBLEM # 1:

(a) REWRITE THE FOLLOWING STATEMENTS AND MARK EACH EITHER (√) or (×)

1. The principal stresses are independent of the Cartesian coordinate system. ( )
2. Addition of a hydrostatic pressure to a given stress state does not affect the magnitude of the maximum shear stress. ( )
3. Rigid-body motion produces linear strain. ( )
4. The five elastic constants, E, G, K, λ & υ are all independent constants. ( )
5. If : $\sigma_{11} = \sigma_{xx}$, $\sigma_{22} = \sigma_{yy}$, $\sigma_{33} = \sigma_{zz}$, the third stress invariant $I_3 = \sigma_{11} + \sigma_{22} + \sigma_{33}$ ( )
6. At a given point on a photo-elastic model, if N=10, f=60 KN/m, h=6 mm, the maximum shear stress at this point = 100 MPa ( )
7. In the case of $\sigma_{xx} = -\sigma_{yy} = 100$ MPa, $\sigma_{zz} = \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$, the maximum shear stress = 0 ( )
8. Moiré fringe technique is an all-field stress analysis technique ( )
9. Mechanical strain gages have high sensitivity ( )
10. The Potentiometer electrical circuit is suitable for static strain measurements( )

(b) Show how a single-element strain gage can be used and how it should be oriented to determine σ_1, σ_2 & τ_{max} in the following plane stress states:

1. A round bar subjected to a torque T,
2. A thin-walled cylindrical pressure vessel,
3. A thin-walled spherical pressure vessel.

Represent each case on a sketch.

PROBLEM # 2:

A set of Cartesian strain components are:

$\varepsilon_{xx} = 400 \times 10^{-6}$, $\varepsilon_{yy} = 300 \times 10^{-6}$, $\varepsilon_{zz} = 200 \times 10^{-6}$,
$\gamma_{xy} = 100 \times 10^{-6}$, $\gamma_{yz} = 200 \times 10^{-6}$, $\gamma_{xz} = 0$

(a) It is required to transform this given set into a new set of strain components relative to the new set of axes: O x'y'z' where $\theta(x, x') = 90°$, $\theta(y, y') = 90°$, and $\theta(z, z') = 0$.

(b) If E = 200 GPA & $\upsilon = 0.30$,
  1) calculate the Cartesian set of stresses: σ_{xx}, σ_{yy}, σ_{zz}, τ_{xy}, τ_{zy}, τ_{zx} in MPa.
  2) calculate the magnitudes of the principal stresses and the maximum shear stress.

PROBLEM # 3:

At a particular point in a body manufactured from steel :
(E= 200 GPa and $\upsilon = 0.30$), the three principal stresses are:

$\sigma_1 = 120$ MPa, $\sigma_2 = 60$ MPa, $\sigma_3 = -40$ MPa,

(a) Sketch the three-dimensional Mohr's circle for stress,
(b) Determine the maximum shear stress and the minimum shear stress in MPa,

(c) On a plane through the point, the shearing stress τ = 70 MPa. What normal stress must exist on this plane?

(d) On another plane, the shearing stress is τ = 50 MPa. What range of normal stress which can exist on this plane?

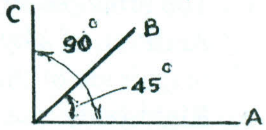
PROBLEM # 4:

The following readings were taken from 3-element rectangular strain rosette mounted on a steel specimen (E=205 GPa & $\upsilon = 0.3$).

Determine the principal strains ε_1 and ε_2 and the maximum shear strain γ_{max}.

Also calculate the corresponding stresses σ_1, σ_2, τ_{max}:

$\varepsilon_A = 1000 \times 10^{-6}$, $\varepsilon_B = -500 \times 10^{-6}$, $\varepsilon_C = 0$



PROBLEM # 5:

Explain the difference between the following as related to stress analysis:

a. Brittle coating & photoelastic coating.
b. Isochromatic fringes & Isoclinic fringes.
c. Circular polariscope & plane polariscope.
d. Electrical resistance strain gages & Moiré fringe method

BEST WISHES

Prof. Dr. M. Sh., Jan., 2009

EQUATION SHEET

$$(\sigma_1 - \sigma_2) = Nf/h \tag{1}$$

$$\tau_{\max} = \tfrac{1}{2}(\sigma_{\max} - \sigma_{\min}) = \tfrac{1}{2}(\sigma_1 - \sigma_3) \tag{2}$$

$$\begin{aligned}\epsilon_{x'x'} = {} & \epsilon_{xx} \cos^2(x', x) + \epsilon_{yy} \cos^2(x', y) \\ & + \epsilon_{zz} \cos^2(x', z) + \gamma_{xy} \cos(x', x) \cos(x', y) \\ & + \gamma_{yz} \cos(x', y) \cos(x', z) + \gamma_{zx} \cos(x', z) \cos(x', x)\end{aligned} \tag{3}$$

$$\begin{aligned}\epsilon_{y'y'} = {} & \epsilon_{yy} \cos^2(y', y) + \epsilon_{zz} \cos^2(y', z) \\ & + \epsilon_{xx} \cos^2(y', x) + \gamma_{yz} \cos(y', y) \cos(y', z) \\ & + \gamma_{zx} \cos(y', z) \cos(y', x) + \gamma_{xy} \cos(y', x) \cos(y', y)\end{aligned} \tag{4}$$

$$\begin{aligned}\epsilon_{z'z'} = {} & \epsilon_{zz} \cos^2(z', z) + \epsilon_{xx} \cos^2(z', x) \\ & + \epsilon_{yy} \cos^2(z', y) + \gamma_{zx} \cos(z', z) \cos(z', x) \\ & + \gamma_{xy} \cos(z', x) \cos(z', y) + \gamma_{yz} \cos(z', y) \cos(z', z)\end{aligned} \tag{5}$$

$$\begin{aligned}\gamma_{x'y'} = {} & 2\epsilon_{xx} \cos(x', x) \cos(y', x) \\ & + 2\epsilon_{yy} \cos(x', y) \cos(y', y) + 2\epsilon_{zz} \cos(x', z) \cos(y', z) \\ & + \gamma_{xy}[\cos(x', x) \cos(y', y) + \cos(x', y) \cos(y', x)] \\ & + \gamma_{yz}[\cos(x', y) \cos(y', z) + \cos(x', z) \cos(y', y)] \\ & + \gamma_{zx}[\cos(x', z) \cos(y', x) + \cos(x', x) \cos(y', z)]\end{aligned} \tag{6}$$

$$\begin{aligned}\gamma_{y'z'} = {} & 2\epsilon_{yy} \cos(y', y) \cos(z', y) \\ & + 2\epsilon_{zz} \cos(y', z) \cos(z', z) + 2\epsilon_{xx} \cos(y', x) \cos(z', x) \\ & + \gamma_{yz}[\cos(y', y) \cos(z', z) + \cos(y', z) \cos(z', y)] \\ & + \gamma_{zx}[\cos(y', z) \cos(z', x) + \cos(y', x) \cos(z', z)] \\ & + \gamma_{xy}[\cos(y', x) \cos(z', y) + \cos(y', y) \cos(z', x)]\end{aligned} \tag{7}$$

$$\begin{aligned}\gamma_{z'x'} = {} & 2\epsilon_{zz} \cos(z', z) \cos(x', z) \\ & + 2\epsilon_{xx} \cos(z', x) \cos(x', x) + 2\epsilon_{yy} \cos(z', y) \cos(x', y) \\ & + \gamma_{zx}[\cos(z', z) \cos(x' x) + \cos(z', x) \cos(x', z)] \\ & + \gamma_{xy}[\cos(z', x) \cos(x', y) + \cos(z', y) \cos(x', x)] \\ & + \gamma_{yz}[\cos(z', y) \cos(x', z) + \cos(z', z) \cos(x', y)]\end{aligned} \tag{8}$$

$$\sigma_{xx} = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\epsilon_{xx} + \nu(\epsilon_{yy} + \epsilon_{zz})] \tag{9}$$

$$\begin{aligned}\sigma_{yy} &= \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\epsilon_{yy} + \nu(\epsilon_{xx} + \epsilon_{zz})] \\ \sigma_{zz} &= \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\epsilon_{zz} + \nu(\epsilon_{xx} + \epsilon_{yy})]\end{aligned} \tag{10}$$

$$\tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}, \quad \tau_{yz} = \frac{E}{2(1+\nu)}\gamma_{yz}, \quad \tau_{zx} = \frac{E}{2(1+\nu)}\gamma_{zx} \tag{11}$$

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)} \tag{12}$$

$$\begin{aligned}& \sigma_n^3 - (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})\sigma_n^2 \\ & + (\sigma_{xx}\sigma_{yy} + \sigma_{yy}\sigma_{zz} + \sigma_{zz}\sigma_{xx} - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2)\sigma_n \\ & - (\sigma_{xx}\sigma_{yy}\sigma_{zz} - \sigma_{xx}\tau_{yz}^2 - \sigma_{yy}\tau_{zx}^2 - \sigma_{zz}\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{zx}) = 0\end{aligned} \tag{13}$$

$$\epsilon_1 = \tfrac{1}{2}(\epsilon_A + \epsilon_C) + \tfrac{1}{2}\sqrt{(\epsilon_A - \epsilon_C)^2 + (2\epsilon_B - \epsilon_A - \epsilon_C)^2} \tag{14}$$

$$\epsilon_2 = \tfrac{1}{2}(\epsilon_A + \epsilon_C) - \tfrac{1}{2}\sqrt{(\epsilon_A - \epsilon_C)^2 + (2\epsilon_B - \epsilon_A - \epsilon_C)^2} \tag{15}$$