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TWO-LINK ROBOTIC ARM. PART 1: GEOMETRICAL AND ANALYTICAL REPRESENTATIONS OF THE ACCESSIBLE REGION.

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ABSTRACT

The present paper deals with the analytical and geometrical representations of accessible region for two link robotic arm in planar case. The first part presents the derivation and representation of the loci-curves traced the accessible region. These are basic requirements for the determination of the characteristics as well as the shape of the accessible region and for developing design charts for general accessible region. The rest of the paper concerns the investigation of the design parameters and their affect on the area, boundary contour and shape of the accessible region. The quantitative evaluation of boundary counter of the accessible region and its area are also presented.

This study, as we believed, provides one of the basic tools necessary for evaluation of robotic performance.

INTRODUCTION

Manipulators have been used extensively in hostile environments,such as in the nuclear industries, deep undersea exploration and maintenance operations, and in space. Manipulators have also been used increasingly in industrial automation applications without the involvement of human operators [1 and 2].

One of the most important accomplishments of the industrial robots is the capability with a given mechanical structure that they are able to perform a series of different jobs. This will be defined as technological flexibility. The technological flexibility of the robots is described ainly by the structral and kinematic characteristics of such systems.

The working space, accessible region, of a robot is one of the most important specifications for both robot designer as well as the user. The accessible region can be used to measure the efficiency of a designed robot mechanism. Presentation of workspace of robot not only helps to evaluate the different performance characteristics of the robot, but, in itself also represents an important criterion for the evaluation of robot geometries. The reachable working space of a given robot is defined as the region within which every point can be reached by a reference point on the robot hand.

One of the basic questions encountered in robot design is the determination of the shape of workspace. Given the structure of a robot, can one specify the shape of its workspace and study its characteristics? The problem is a difficult one, primarily because of the large number of degrees of freedom involved in the robot system and inherent complexity on spatial geometery.

There exists a large number of investigations dedicated to the workspace of manipulators. A few references which, according to the author's opinion , are fairly representative of these papers are [3-12]. Roth [3] was the first to present work related to the workspace of robots, Shimano [4]has attempted to solve the problem of describing the reachable workspace. Fichter and Hunt [5] presented the torus surface of the general 2R open linkage. Tsai and Soni [6-7]solved the accessible region of 2R robot arms for planar case in closed-form. They presented a new algorithm which is based on optimizing the co-ordinate transformation equations, the optimization is carried out using small increments of the joint displacements. Kumar and Waldron [8] presented a numerical method to trace the bounded surface of the working space of robot with ideal R pairs. Gupta and Roth [9]presented the primary and secondary workspace of robots. Yang and Lee[10-11] derived a set of recursive equation to represent the workspace analytically and formulated a set of criteria to investigate certain characteristics of this work-space. They developed an algorithm to outline the boundary of the workspace and evaluate its volume quantatively. They presented a manipulator performance index which is based on workspace and can be evaluated efficently. Sugimoto and Duffy [12]proved the theories for determining the extreme distance of manipulator hand.

This paper represents an attempt to treat the foregoing question. The prime interest of author of this paper is to present an engineering apporach to the accessible region of two-link robot arm which should overcome problems arising at the selection of the kinematic dimensions of a robot which can reach a set of specified planar working positions or trace some specified planar path.

The present paper is restricted to robots which have only limited revolute joints. "Limited" here means that each joint is not capable of complete rotation but it rotates between two extreme positions. The robot hand was treated as a point in planar case. The accessible region is mapped on the sagittal plane of the robot, i.e. The(X*Y*)plane, Fig. (2). This figure illsutrates 3a 3R robot arm, two-link robot arm, in planar cases, the geometry of which is typical of a large class of manipulator. The origin 0 of the (X*Y*Z*) corrdinate system is at the center of the second joint.

ACCESSIBLE REGION OF TWO-LINK ROBOT ARM

Figure(3).Shows the two-link robot arm considered in this study. Its configuration is typical of a large class of manipulator. Referring to Fig. (3), it is obvious that the distance h, the length of stand, only affects the relative height of the accessible region, the origin 0 of the (X*Y*) coordinates system, with respect to the base point of robot. Therefore, h is considered to be zero. The coordinates of point P, end of robot arm in planar case Fig. (2) can be written as

$$X = L_2 \cos \theta_2 - L_3 \cos (\theta_2 + \theta_3) \qquad (1)$$

$$y = L_2 \sin \theta_2 - L_3 \sin (\theta_2 + \theta_3) \qquad (2)$$

From Equ.(1) and Equ.(2), the following equations can be obtained :

$$(x - L_2 \cos \theta_2)^2 - (y - L_2 \sin \theta_2)^2 = L_3^2 \qquad (3)$$

$$x^2 + y^2 = (L_2^2 + L_3^2) - (2 L_2 L_3 \cos \theta_3) \qquad (4)$$

From Equations(3)and(4), θ_2 and θ_3 can be obtained as :

$$\theta_2 = \cos^{-1} \frac{(L_2^2 - L_3^2) + (x^2 + y^2)}{2 L_3 \cdot \sqrt{x^2 + y^2}} + \tan^{-1} \frac{y}{x} \qquad (5)$$

$$\theta_3 = \cos^{-1} \frac{(L_2^2 + L_3^2) - (x^2 + y^2)}{2L_2 L_3} \qquad (6)$$

Obviously, both Equ. (3) and Equ. (4) represent loci of circles. More important, θ_2 and θ_3 are not interrelated, i.e. θ_2 only appears in Equ.(3) and θ_3 appears only in Equ.(4). It is important to note that, the value of θ_3 and their limiting values (θ_{3max} and θ_{3min}) are a function of the angles of the revolute joints 2 and 3, i.e.

$$\theta_3 = f\ (\theta_2,\ \gamma^+ \text{ and } \gamma^-\ ).$$

From Fig. (3), it can be seen that Equ. (3) describes the circular ares ab ($\theta_2 = \theta_{2min}$) and de ($\theta_2 = \theta_{2max}$), wheras Equ.(4)describes another circular arcs : $\widehat{bc}$, $\widehat{ef}$, $\widehat{ed}$ ($\theta_3 = 90 - \gamma^+_{max}$) and fa ($\theta_3 = 180 - \theta_{2min}$). Extending these results, the above equations can be used to get the accessible region of two-link robot arm with different arm ratio L_3/L_2 and with θ_2 varying from some θ_{2min} to θ_{2max}, γ^+ for some γ_{max} and γ^- for some γ_{max}. Figures 4(a-d) show the area covered by the manipulator having specific joints angular displacement (θ_{2min}, θ_{2max}, γ^-_{max}. and γ^+_{min})..

Figure 4-a shows the accessible region for the link ratio L_3/L_2 = 0.66. Figures 4 (b, c and d) show the accessible regions for the link ratios L_3/L_2 =1, 1.33 and 1.66 respectively. From Figures 4 (a-d) it can be seen that an increase in the ratio L_3/L_2 leads to an increase in the area of the accessible region. This is because an increase in L_3/L_2 causes an increase in the length of arcs $\widehat{ab}$, $\widehat{ed}$, $\widehat{de}$ and $\widehat{fa}$ while arcs $\widehat{bc}$ and $\widehat{ef}$ are not affected. Aslo, the accessible region gets farer away from the axis O^*Y^* as the ratio L_3/L_2 increase,i.e. the value of L_3/L_2 controls the position of the accessible region.

The effect of γ^-_{max}. on both the shape and area of the accessible region is obviously clear in Fig.(5).It is found that γ^-_{max} has two effects on the length of the circular arcs. An increase in γ^-_{max}. causes an increase in arcs $\widehat{de}$ and $\widehat{fa}$ and a decrease in arc $\widehat{ef}$ while the another arcs are not affected. Consequently, the area of the accessible region will increase in a specific direction as shown in Fig.(5). Any change in the value of γ^+_{max}. will affect only the length of arcs $\widehat{ab}$ and $\widehat{cd}$ while other arcs will not change as shown in Fig.(6). Therefore, the length of arcs $\widehat{ab}$ and $\widehat{cd}$ will increase as a result of an increase in the value of γ^+_{max}. which, in turn, leads to an increase in the area of the accessible region. The rate of change of the area covered by the accessible region has a direction oppisite to that when γ^-_{max} increases. This can be seen by a simple comparison between Fig.(5) and Fig.(6). On the contrary, θ_{2min} has an effect different from both γ^-_{max} and γ^+_{max}..It can be seen from Fig. (7) that an increase in the value of θ_{2min} results in a decrease in the area of the accessible region because the length of arcs $\widehat{ef}$ and $\widehat{cb}$ decrease. In other words, as θ_{2min} decreases, the area of accessible region increases. On the other hand, an increase in θ_{2max} leads to an increase in the area of the accessible region due to the increase in the length of arc $\widehat{ef}$ only Fig. (8). This increasing takes a direction oppisite to that when θ_{2min} decreases (compare between Fig.(7), and Fig. (8)).

DESIGN CHARTS

Extending the previous representations one can construct charts for general accessible region, and we can easily get the accessible negion of two-link robotic arm for a certain θ_2, γ^- and γ^+ at a specific arm ratio. To clarify how to use the general design charts, the following is an example for determining the accessible region for a given two link robot arm having the values of $L_3/L_2 = 1.33$, $\theta_{2min} = 50°$, $\theta_{2max} = 130°$, $\gamma^+_{max} = 40$ and $\gamma^-_{max} = 20°$. The area covered by the accessible region can be determined using the design charts as shown in Fig. (9) (doted region). Point(a)must lie on arc 00 and can be located by the intersection with arc which represents θ_{2min}, i.e. (θ_{2min} - arc). Point(b) lies on the latter arc and it can located by the intersection with the(γ^+_{max} - arc). Point (c)lies on arc ($\theta_{2min} = 90$) and it can be positioned by the intersection with (γ^+_{max} - arc). Again, arc 00 which involves point(a)should be intersected with (θ_{2min} - arc) to determine point(d). Then point(e) is the result of intersecting (γ^-_{max} - arc) with (θ_{2max} - arc). Finally, point(f) is determined by the intersection of (γ^-_{max} - arc) with (θ_{2min} - arc).

BOUNDARY CONTOUR AND AREA OF THE ACCESSIBLE REGION.

Having an analytical and a geometrical representations of the accessible region, it is now possible to investigate the boundary contour and the area of robot accessible region quantitatively. The basic approach involves the construction of the accessible region in sagittal plane Fig. (2), and from which numberical methods are utilized to determine its boundary contour length and subsequently its area.

Boundary Contour

Figure (3) shows that the boundary contour of the accessible region consists of six circul arcs $\widehat{ab}$, $\widehat{bc}$, $\widehat{cd}$ $\widehat{de}$, $\widehat{ef}$ and $\widehat{fa}$. The length of all these arcs, i.e. the

$$L_b = \frac{\pi L_2}{180} [\gamma^-_{max} (\delta - 1 + \sqrt{\delta^2 + 2\delta \cos \theta_{2min} + 1}) + \gamma^+_{max} + 2 (\theta_{2max} - \theta_{2min})] \qquad (7)$$

where : (δ) is the link proportion L_3/L_2.
Examining Equ. (7) and figures (3-8), it can be noted that :

1] The length of the boundary contour and the area of the accessible region are a function of the kinematic parameters of the robot, i.e., (L_3/L_2, γ^+_{max}, γ^-_{max}, θ_{2max}, θ_{2min} and the difference between θ_{2max} and θ_{2min}).

2] The constraints which make the accessible region consists of six circular arcs are: $\gamma^-_{max}, \gamma^+_{min} >$ zero), $\theta_{2max} > 90°$, $\theta_{2min} < 90°$ and $\theta_{2max} > (\theta_{2min} + \gamma^-_{max})$. Other than that, the number of arcs will be less than six and the shape of the accessible region will be changed and the area will be decreased.

3] An increase in L_3/L_2, θ_{2max}, γ^-_{max} and γ^+_{max} leads to an increase in the length of boundary contour and the area of the accessible region. On the countrary, θ_{2min} has a different effect.

4] The rate of change of the area covered by the accessible region and its boundary contour length has no optimum value.

5] The value of L_3/L_2 controls the position and area of the accessible region, however, it has no effect on the shape of that region.

Area of Accessible Region

Knowing the boundary contour or the accessible region, the area can then be calculated by

$$A = \sum_{X_{min}}^{X_{max}} [Y_{max} (x) - Y_{min} (x)] \quad \Delta x \qquad (8)$$

referring to Fig. (10), where X_{max} and X_{min} denote the maximum and minimum of the x value of the boundary profile; $Y_{max}(x)$ and $Y_{min}(x)$ are extreme values of Y and x and Δx represents the width of the dividing rectangle.

where

$$X_{max} = L_2 \ (\cos \theta_{2min} + \ell),$$
$$X_{min} = L_2 \ (\cos \theta_{2max} + \cos \bar{\gamma}_{max}),$$

The unsolved problems remaining in this investigation are :

- Development a synthesis procedure to get the suitable kinematic dimensions and location of the robot arm which will enclose within its accessible region a set of specified working positions.
- Presentation of a kinematic performance criterion which can be used in practic for evaluation of robot based on accessible region.

These problems will be discussed in Part 2, a continuation of this paper, which is in preparation.

CONCLUSION

The present investigation deals with the study of determining the accessible region for two-link robotic arm in planar case. Based on the analytical and geometrical reprasentations of the loci-curves tranced by a two-link robotic arm, the effect of the kinematic dimensions on the accessible region are discussed and design charts are developed. Following the analysis of the accessible region, the paper presents quntitavely evaluation of the boundary contour of the accessible region and its area.

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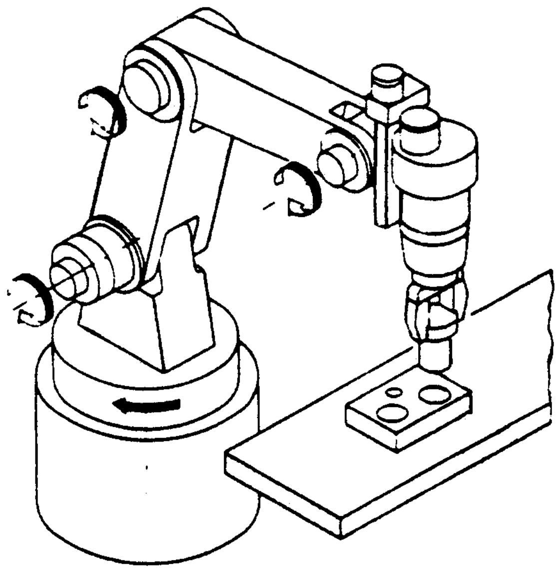


ig. 1. Industrial Robot

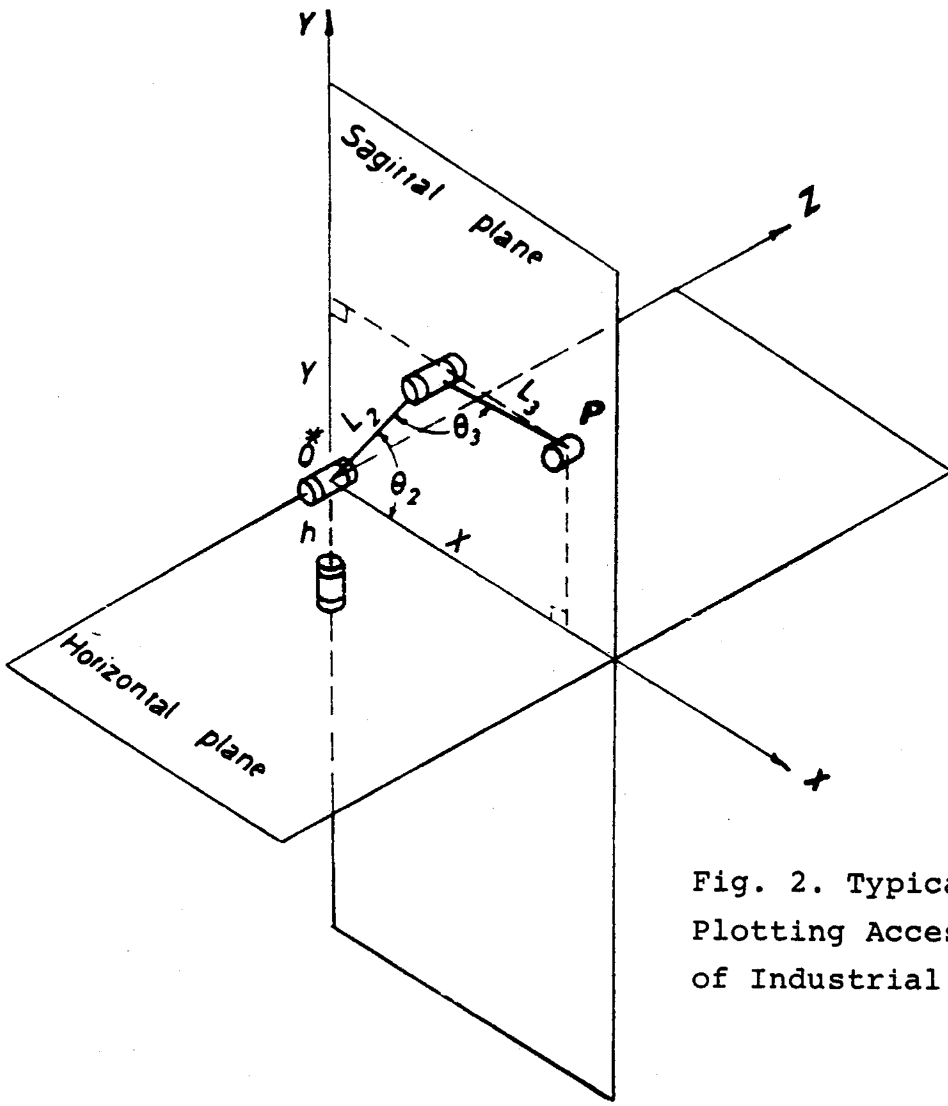


Fig. 2. Typical Plane for
Plotting Accessible Region
of Industrial Robot.

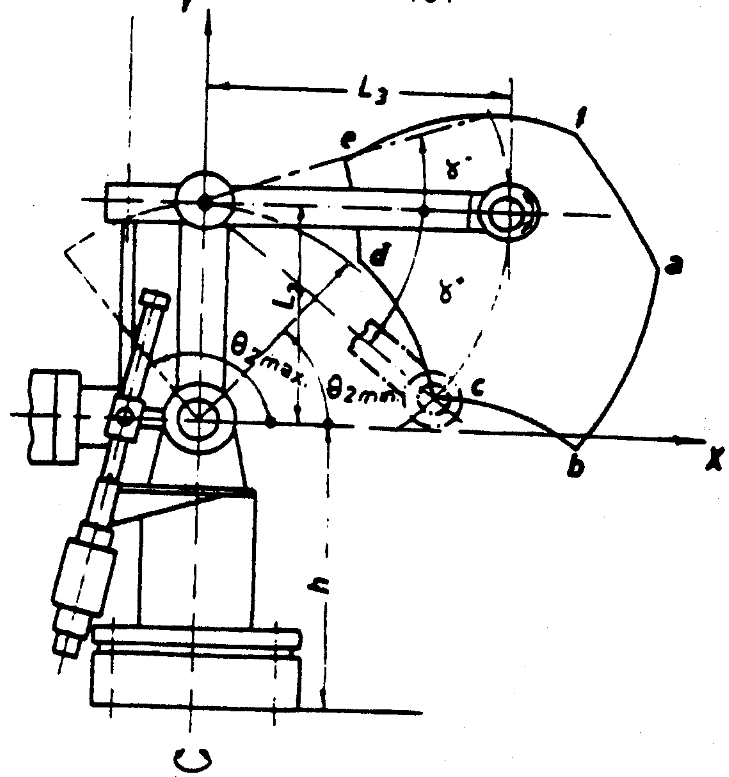
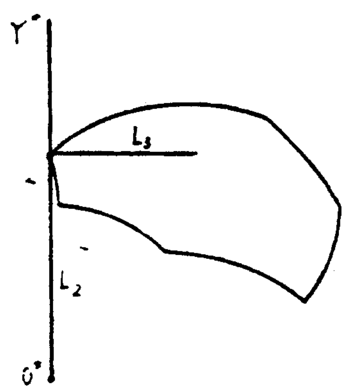
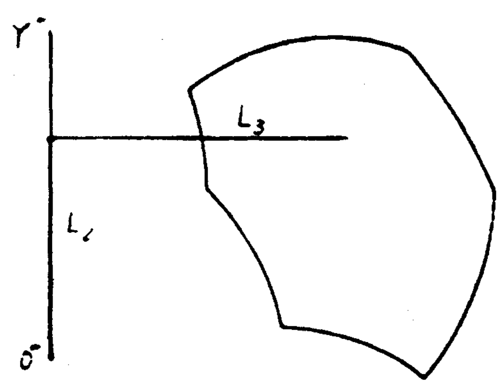


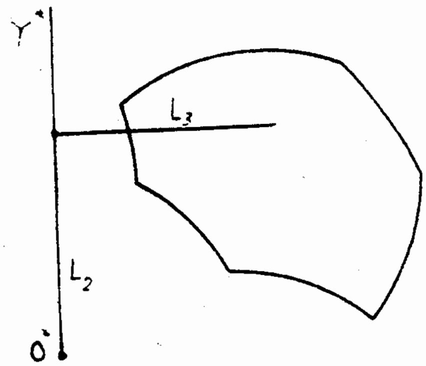
Fig. 3. Accessible Region of Industrial Robot.



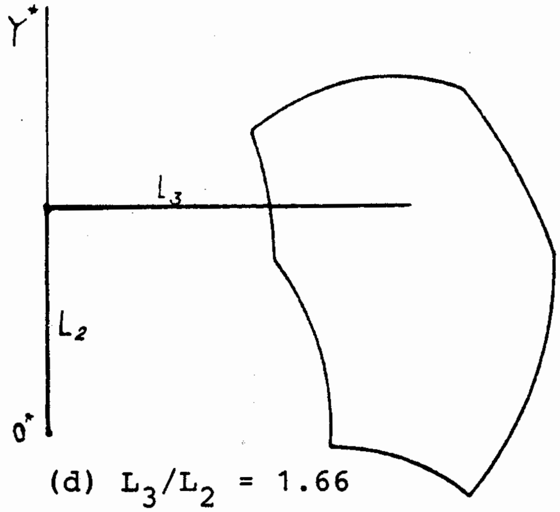
(a) $L_3/L_2 = 0.66$



(b) $L_3/L_2 = 1$



(c) L_3/L_2 = 1.33



(d) L_3/L_2 = 1.66

Fig. 4. Area Covered by the Accessible Region

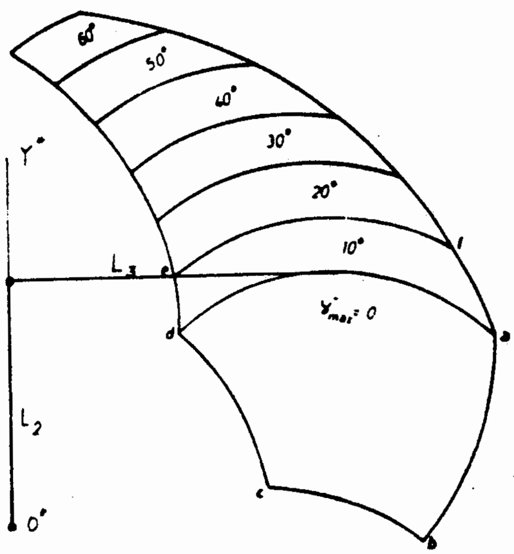


Fig.5. Accessible Region with γ^-_{max} as a variable.

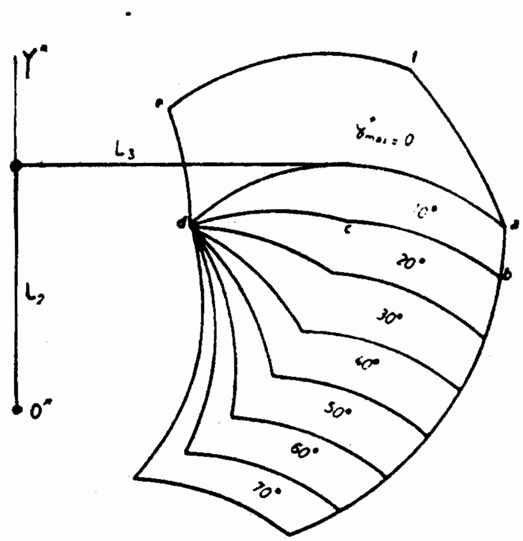


Fig.6. Accessible Region with γ^+_{max} as a variable.

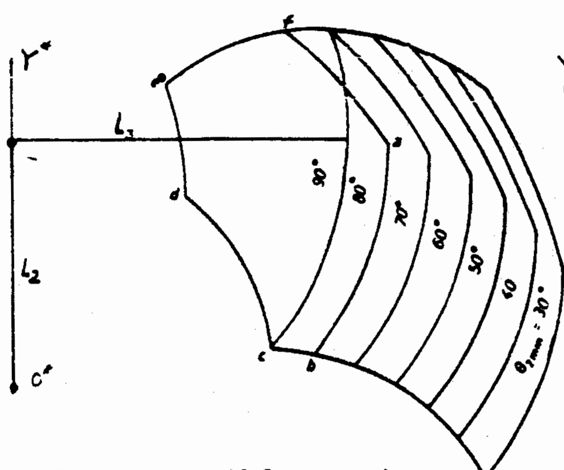


Fig.7. Accessible Region with θ_{2min} as a variable.

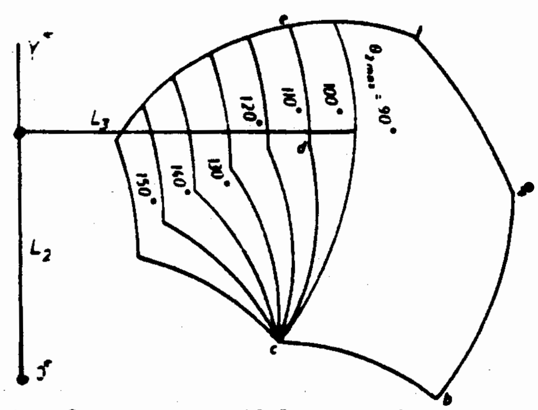


Fig.8. Accessible Region with θ_{2max} as a variable.

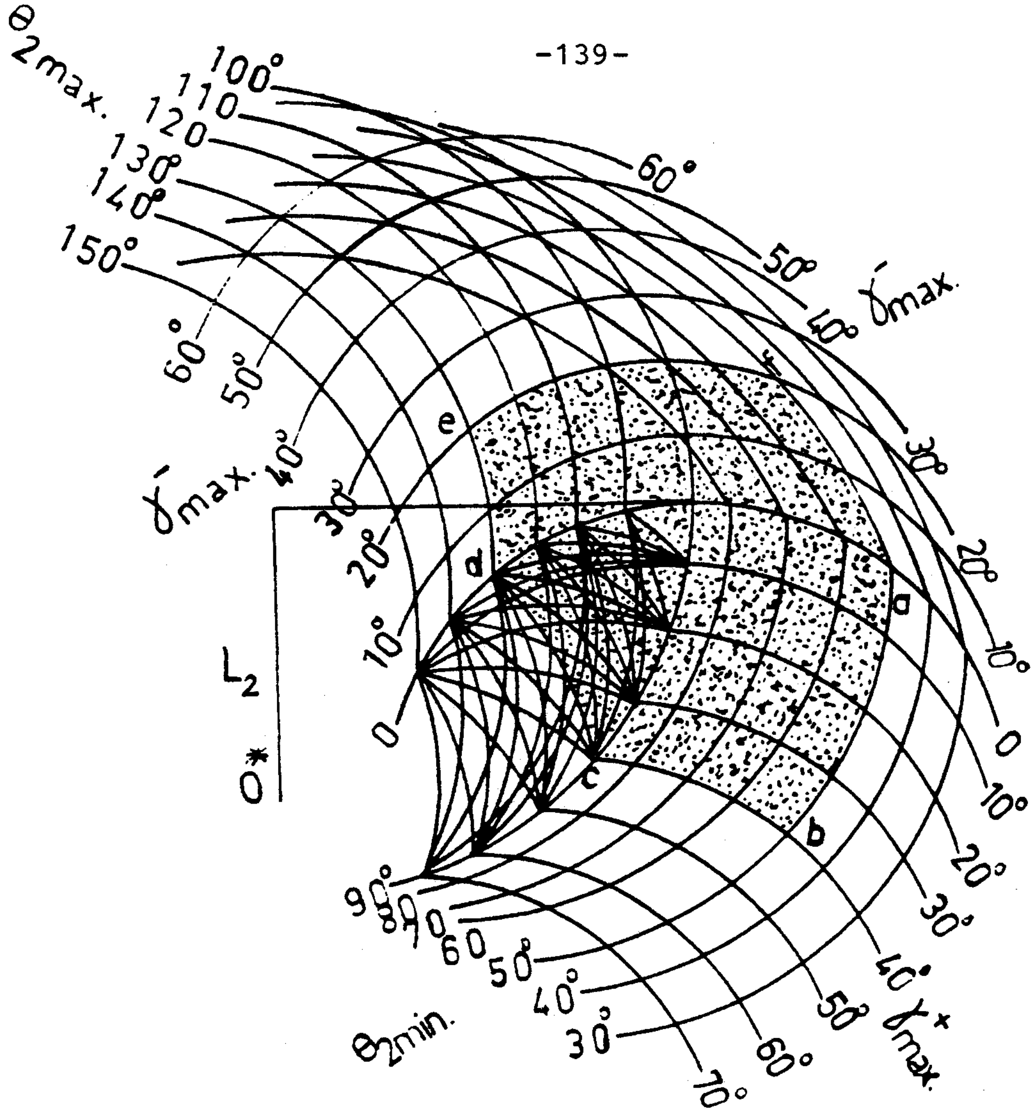


Fig. 9. General Design Chart for $L_2/L_3 = 1.33$

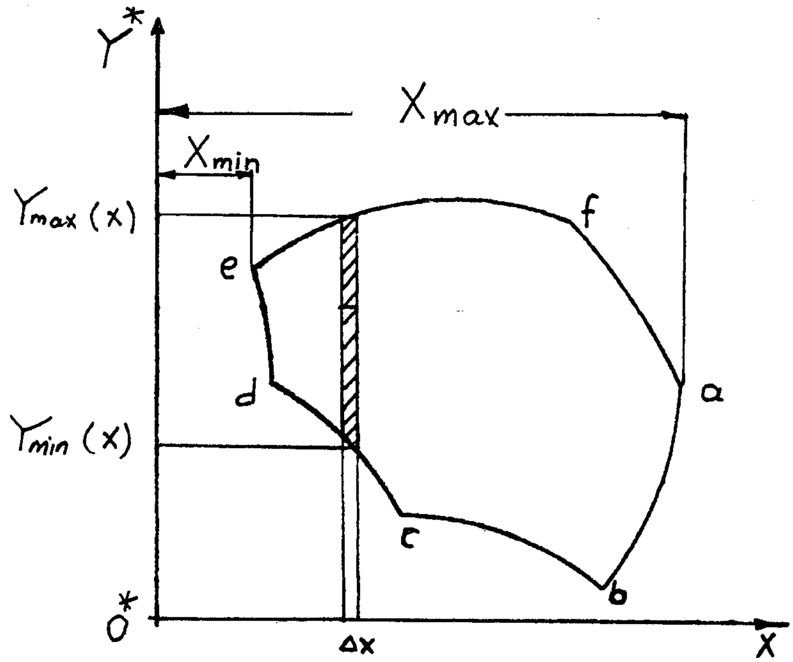


Fig.10. An Illustration of the integration of Accessible Region Area.