عامر

Mansoura University
Faculty of Engineering
Math. & Eng. Physics. Dept.

Mathematics (3)

1st year,
Final Exam, Dec. , 28, 2013
Time Allowed: 3 Hours.

يتألف الإختبار من 4 أسئلة في ورقتين. برجاء بدء إجابة كل فرع من إحدى نهايتي ورقة الإجابة.

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[1]-(a) [**20 pts**] Solve by any method

1. $(\sin x \cosh y)\,dx - (\cos x \sinh y)\,dy = 0, \quad y(0) = 0,$

2. $\dfrac{dy}{dx} = \dfrac{y}{x + xy + x^2 y},$

3. $x^2\,y'' - 2x\,y' + 2y = x^3 \sin x.$

4. $y'' + y = \tan x.$

(b) [**5 pts**] Find the orthogonal trajectories of

$$y^2\,(2C - x) = x^3.$$

(c) [**5 pts**] Determine the proper form of $y_p(x)$ for

$$\left(D^2 - 3\,D\right) y = 6\,e^{3x} - 5 \sin x$$

but do not solve for the undetermined coefficients.

(d) [**5 pts**] Find the domain of the function

$$f(x, y) = \sqrt{y - x}\,\ln(y + x)$$

[2]-(a) [**5 pts**] Find a general solution for the homogeneous differential equation with constant coefficients whose auxiliary equation is

$$(r-1)^3\,(r-2)\,\left(r^2 + r + 1\right)\,\left(r^2 + 6\,r + 10\right)^3 = 0.$$

(b) [**5 pts**] If

$$z = y^2 \cos\left(\frac{y - x}{y + x}\right) + 2\,x\,y \tan^{-1}\frac{y}{x}$$

Prove that

$$x^2\,z_{xx} + 2\,x\,y\,z_{xy} + y^2\,z_{yy} = 2\,z$$

(c) [**5 pts**] Find a quadratic

$$f(x, y) = \sin x \sin y \qquad \text{near the origin.}$$

(d) [**5 pts**] Find the local extreme values of the function

$$f(x, y) = x\,y - x^2 - y^2 - 2\,x - 2\,y + 4$$

Assoc. Prof. Dr. El-Gamel

[3] (a) **[20 pts]** Aya is waiting to receive mail telling her whether she has been accepted to a certain college. She estimates that her probability of being accepted 0.6. She estimates that the conditional probabilities of receiving notification on each day of next week (working days only), given that she is accepted and that she is rejected are as follows:

Day	P(mail/accepted)	P(mail/rejected)
Saturday	0.15	0.05
Sunday	0.2	0.1
Monday	0.25	0.1
Tuesday	0.15	0.15
Wednesday	0.1	0.2

i) What is the probability that she receives mail on Saturday?

ii) What is the conditional probability that she received mail on Sunday given that she does not receive mail on Saturday?

iii) If there is no mail until Monday, what is the conditional probability that she will be accepted?

iv) What is the conditional probability that she will be accepted if mail comes on Tuesday?

v) What is the conditional probability that she will be accepted if no mail arrives?

(b) **[3 pts]** A person has 8 friends, of whom 5 will be invited to a party. How many choices if 2 of the friends will only attend together?

(c) **[7 pts]** For two symmetric dice, If two of their sides painted red, two painted black, one painted yellow, and the other painted white. When this pair of dice is rolled, the random variable X represents the color appears on two dice. Find E[X] and Var[X].

[4] (a)**[8 pts]** Using Newton Raphson Method find the second positive root of equation

$\tan(x) = x + 2$ Note: $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

(b) **[8 pts]** Given the function f at the following values,

x	0	0.25	0.5	0.75	1
$f(x)$	1	1.118034	1.224745	1.322876	1.414214

approximate $\int_0^1 f(x)dx$ using Simpson Rule. $\int_a^b f(x)dx = \frac{h}{3}\sum_{\substack{i=0 \\ i=i+2}}^{n-2}(f(x_i) + 4f(x_{i+1}) + f(x_{i+2}))$

(c) **[9 pts]** Solve the following system of equation using Gauss-Seidel Method [start by $(x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0)$]

$-x_1 + 11x_2 - x_3 + 3x_4 = 25$ $\quad 2x_1 - x_2 + 10x_3 - x_4 = -11$

$10x_1 - x_2 + 2x_3 = 6$ $\quad 3x_2 - x_3 + 8x_4 = 15$