Mansoura University
Faculty of engineering
Department: Engineering Math. & Phys
Year: preparatory

Course: Mathematics (1)
Code: BAS1011
Date: 16-1-2014 Time: 3 hrs
Full Mark: 130

يتألف الامتحان من ٤ أسئلة. برجاء بدء إجابة كل فرع من إحدى نهايتي ورقة الإجابة.

Question1: [34 pints]

a) [6 pints] Determine the value of k so that that the function

$$f(x)=\begin{cases}\dfrac{\ln(1+x\sin x)}{1-\cos x} & x\neq 0\\ k & x=0\end{cases}$$

is continuous at $x=0$.

b) [6 pints] Find the equation of the curve that has vertical asymptotes $x=2$ and $x=3$ and horizontal asymptote at $y=1$ and pass through the points (0,0) and $(1,-2)$.

c) [6 pints] Find the equation of the tangent line to the curve $x=e^{-t}$, $y=te^{-t}$ at $t=0$.

d) [6 pints] Prove that $\tanh^{-1}(x)=\ln\left(\sqrt{\frac{1+x}{1-x}}\right)$ and hence deduce that

$$\frac{d}{dx}\tanh^{-1}x=\frac{1}{1-x^2}.$$

e) [10 pints] Sketch the functions $y=\tanh x$ and $y=\tanh^{-1}x$. Find the domain, range and equations of asymptotes for each function.

Question2: [31 pints]

a) [6 pints] Find $y^{(100)}$ of the function $y=x^2\ln(x)$.

b) [8 pints] Find $\frac{dy}{dx}$ if:

(i) $y=\dfrac{e^{\sin x}\ln(\cos x)}{\tanh^4(x^2)\,\sin(e^x)}$.

(ii) $e^y x^{\sec x}=\tanh^{-1}(3^{\sin x})+(\ln x)^{\tanh^{-1}x}$.

c) [9 pints] Evaluate the following limits

(i) $\lim_{x\to 0}\left(\frac{1}{x}-\cot x\right)$ (ii) $\lim_{x\to 0}(x\cot 2x)$ (iii) $\lim_{x\to 0}(e^x+x)^{\frac{1}{x}}$

d) [8 pints] Prove that The Maclaurin seris expansion of the function $f(x)=\cosh x$ is given by

$$\cosh x=1+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots.$$

Hence, deduce The Maclaurin seris expansion of the function

$$f(x)=\sinh\left(\frac{3x}{2}\right)\sinh\left(\frac{x}{2}\right).$$

Useful relation: $\cosh(x\pm y)=\cosh x\cosh y\pm\sinh x\sinh y$.

With my best wishes

Dr. Mohamed Soror

Linear Algebra Jan 2014 This part counts for 70 marks

answer as many as you can

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3. [40 marks] Answer the following:

   (a) [8 marks] Use partial fractions to decompose $\dfrac{2}{x^3 - 3x^2 + 2x}$

   (b) [4 marks] **True or False:** The curve $f(x) = x^3 + x^2 + x + 1$ intersects the x-axis at *exactly two points*(Justify your answer)

   (c) [4 marks] When the polynomial $f(x)$ is divided by $x^2 - 3x + 2$, the remainder is $6x - 2$. What is the **remainder** when $f(x)$ is divided by $(x - 2)$

   (d) [4 marks] Given the quintic polynomial equation (polynomial of degree 5)

   $$x^5 - ax^4 + bx^3 - cx^2 + dx = 0,$$

   where a , b, c, and d are real constants. Find these constants if the equation has *a purely imaginary* root $x = i$ of *multiplicity* 2.

   (e) [4 marks] Given the two polynomial functions

   $$f(x) = 2x^4 + 13x^3 + 18x^2 + x - 4 \text{ and } g(x) = x^2 + 5x + 2$$

   i. [2 marks] $f(x)$ is divided by $g(x)$ to write $f(x) = g(x)Q(x) + R(x)$. Find the quotient $Q(x)$ and the remainder $R(x)$

   ii. [2 marks] Show that $f(x)$ and $g(x)$ have no common roots

   (f) [16 marks] Given the following matrices

   $$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} -1 & 0.5 \\ 0.2 & -1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 0.2 \end{bmatrix}, \text{ and } \mathbf{E} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

   i. [4 marks] For the matrix $\mathbf{C}$, find the eigenvalues λ_1 and λ_2 and the corresponding eigenvectors $V_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ and $V_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$.

   ii. [4 marks] Let $\mathbf{F} = \mathbf{E}^T\mathbf{A} + \mathbf{B}$ and $\mathbf{G} = 2\mathbf{C}^{-1}\mathbf{B} - \mathbf{A}$. Only one of those two matrices is possible to find. which one? then find it?

   iii. [4 marks] **Complete:** $\mathbf{E}^T\mathbf{C}^m\mathbf{E}$ equals .... when $m \to \infty$.

   iv. [4 marks] Find a 2×2 matrix $\mathbf{Y}$ such that $\mathbf{C}(A - \mathbf{Y})\mathbf{B}^{-1} = 2\mathbf{I}_2$

Please Question (4) in the back side

4. [30 marks] Answer the following

(a) [10 marks] Let λ and the vector $\mathbf{V}$ be the eigenvalue and the corresponding eigenvector of an $n \times n$ nonsingular matrix $\mathbf{A}$ (simply $\mathbf{AV} = \lambda\mathbf{V}$). If we define the matrices $\mathbf{B}$ and $\mathbf{C}$ as

$$\mathbf{B} = \mathbf{A}^3 - 2\mathbf{A}^{-1} \qquad \textit{and} \qquad \mathbf{C} = \mathbf{A}^T + 3\mathbf{I}_n$$

i. [5 marks] Show that the vector $\mathbf{V}$ is an eigenvector of the matrix $\mathbf{B}$

ii. [2 marks] If $\lambda = 2$ what is the corresponding eigenvalue of $\mathbf{V}$ as an eigenvector of the matrix $\mathbf{B}$

iii. [3 marks] **True or False:** $\lambda - 3$ is an eigenvalue of the matrix $\mathbf{C}$

(b) [10 marks] For the homogeneous system $\mathbf{A}\,\mathbf{X} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}; \qquad \mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}.$$

i. [5 marks] Find the $rref(\mathbf{A})$, then determine $rank(\mathbf{A})$

ii. [3 marks] Specify the *free* variable(s) and the *pivot* variable(s)

iii. [2 marks] Write the solution $\mathbf{X}$ in the parametric form

(c) [10 marks] Consider the following linear system

$$\begin{array}{rrrcl} x_1 & & +x_3 & = & 1 \\ & -x_2 & +2x_3 & = & \omega \\ x_1 & -x_2 & +3x_3 & = & 2 \\ x_1 & +x_2 & -x_3 & = & 1-\omega \end{array}$$

where ω is an arbitrary real constants. Answer the following

i. [2 marks] Rewrite the system in the matrix form then express it in the augmented matrix form

ii. [5 marks] Determine the value(s) of ω that results in a consistent system

iii. [3 marks] **True or False:**

1. The rank of coefficient matrix does not depend on the parameter ω and equals three
2. When the system is consistent, the variable x_2 is the free variable.
3. The system never has a unique solution

With best wishes, *Examiner: Dr. Samir Shams Eldeen*