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On hkg-Closed Sets and Weakly Hausdor® Spaces

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ABSTRACT

The ¯rst aim of this paper is to introduce four classes of generalized closed sets called ®±g-closed sets, s±gclosed sets, ®µg-closed sets and sµg-closed sets. We discuss their properties and several examples are provided to illustrate the behavior of these new types of generalized closed sets.

The second aim is to obtain new characterizations of weakly T2-spaces by using the same types of these sets.
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INTRODUCTION

In 1970, Levine [19] introduced the notion of generalized closed sets in topological spaces whose closure belongs to every open superset, and de¯ned the notion of T1=2-space which is between T0-space and T1-space. Since then, many concepts related to generalized closed sets were de¯ned and investigated. Recently, there are several researches are puplished in that ¯eld [1-4,6,7,12,25,27,30]. By using the semi-regularization of a given topology and the associated ±-closure operator, Dontchev and Ganster [10] introduced and studied the concept of ±g-closed sets which is a slightly stronger form of g-closedness properly placed between ±-closedness and g-closedness and introduced the notion of T3=4-spaces as the spaces where every ±g-closed set is ±-closed, i.e. closed in the semi-regularization topology. The example of such space is the digital line or the so called the Khalimsky line [14,16] which is widely used in the applications of point-set topology in computer graphics. Dontchev et al. [11] introduced and studied the notions of g±-closed and ±g¤-closed sets and used these sets to give characterizations of almost weakly T2-spaces [11]. Park et al. [25] in 2007 introduced and studied the concept of g±s-closed sets, which is slightly weaker form of ±g-closedness and ±-semiclosedness [24]. They use g±sclosed sets and ±gs-closed sets to obtain new characterizations of almost weakly T2-spaces. In this paper we introduce and study the concepts of ®±g-closed sets, s±g-closed sets, ®µg-closed sets and sµg-closed sets which are slightly weaker form of ±g-closedness and ®-closedness [23]. We use these types of generalized closed sets to de¯ne new separation axioms namely, T®±g; Ts±g; T®µg and Tsµg respectively, where we will discover that both T®±g and Ts±g are identical, also both T®µg and Tsµg are identical. Moreover as applications, using their axioms, we obtain many characterizations of weakly Hausdor® spaces (weakly T2-spaces, for short) [10].

In computational topology for geometric design and molecular design [20], digital topology information system, particle physics [17], one can observe the innocence made in these realms of applied research by general topological spaces, properties and structure. Thus, we may stress once more the importance of the four types of generalized closed sets and the possible application in digital topology, computer graphics [14,15].

Through this paper, hkg-closed set denotes to any one of the four types of generalized closed sets, i.e. h 2 H = f®; sg and k 2 K = f±;µg, where we will denote to the word of semi by the symbols brie°y. The spaces (X; ¿) and (Y; ¾) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space (X; ¿): The closure of A and the interior of A are denoted by cl(A) and int(A); respectively. A subset A is said to be regular open (resp. regular closed) if A = int(cl(A)) (resp. A = cl (int(A))): Since the intersection of two regular open sets is regular open, the collection of all regular open sets forms a base for a coarser topology ¿s than the original one ¿: The family ¿s is called the semi-regularization [21] of ¿: A space (X; ¿) is called semiregular

if ¿ = ¿s: The ±-interior [29] of a subset A of X is the union of all regular open sets of X contained in A and is denoted by ±-int(A): The subset A is called ±-open [29] if A = ±-int(A); i.e. a set A is ±-open if it is the union of regular open sets. The complement of a ±-open set is called ±-closed. Alternatively, a set A µ X is called ±-closed [29] if A = ±-cl(A), where ±- cl(A) = fx 2 X : int(cl(U)) \ A 6= Á;U 2 ¿ and x 2 Ug: The family of all ±-open sets forms a topology on X and is denoted by ¿±: It is well known that

¿s = ¿±:

The µ-interior [29] of a subset A of X is the union of all open sets of X whose closures are contained in A; and is denoted by µ-int(A): The subset A is called µ-open [29] if A = µ-int(A): The complement of a µ-open set is called µ-closed. Alternatively, a set A µ X is called µ-closed [29] if A = µ-cl(A); where µ-cl(A) = fx 2 X : cl(U) \ A 6= Á; U 2 ¿ and x 2 Ug: The family of all µ-open sets forms a topology on X and is denoted by ¿µ.

A subset A of X is called semi-open [18] (resp. ®-open [23], ±-semiopen [24], semi-preopen [5]) if A µ cl(int(A)) (resp. A µ int(cl(int(A)));A µ cl(±-int(A)), A µ cl(int(cl(A)))) and the complement of a semi-open (resp. ®-open, ±-semiopen, semi-preopen) set is called semi-closed (resp. ®-closed, ±-semiclosed, semi-preclosed). The intersection of all semi-closed (resp. ®- closed) sets containing A is called the semi-closure [8] (resp. ®-closure) of A and is denoted by s-cl(A) (resp. ®-cl(A)): Dually, the semi-interior (resp. ®-interior) of A is de¯ned to be the union of all semi-open (resp. ®-open) sets contained in A and is denoted by s-int(A) (resp. ®-int(A)): A subset A of a topological space (X; ¿) is said to be ±-generalized closed [10] (brie°y, ±g-closed) if ±-cl(A) µ G whenever A µ G and G is open set in X:

We will using the symbol h-cl(A); Where h 2 H = f®; sg; to denote to any of the ®-closure or s-closure of A in the most part of this paper when we need to this.

2. Basic Properties of hkg-closed Sets. Let (X; ¿) be a topological space and h 2 H; k 2 K: A subset A µ X is called hkg-closed if h-cl(A) µ G where A µ G and G is k-open set in X: Note that each type of generalized closed sets is de¯ned to be hkg-closed for some h 2 H and k 2 K: Namely; the hkg-closed set A is:

(1) ®±g-closed if h = ® and k = ±:
(2) ®µg-closed if h = ® and k = µ:
(3) s±g-closed if h = s and k = ±:
(4) sµg-closed if h = s and k = µ:

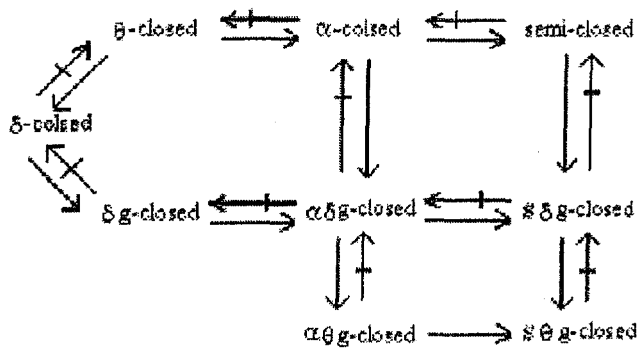

Figure 1.

Fig 1 shows the relationships between the four types of generalized closed sets, where the remark (6!) indicates that the reversible relation is not possible as shown by examples of [9, 10] and the following examples.

Example 2.1: Let X = fa; b; c; dg and ¿ = fX; Á; fag; fbg; fa; bg; fa; b; cgg:

Then:

(1) A = fcg is ®±g-closed but not ±g-closed.

(2) B = fbg is s±g-closed but not ®±g-closed.

(3) C = fa; bg is sµg-closed but not s±g-closed.

(4) D = fa; b; cg is ®±g-closed but not ®-closed.

(5) E = fa; b; dg is s±g-closed but not semi-closed.

Example 2.2: Let X = fa; b; c; dg and ¿ = fX; Á; fag; fb; cg; fa; b; cgg: Then A = fa; cg is ®µg-closed but not ®±g-closed.

A partition space [22] is a space where every open set is closed.

Theorem 2.3: For a subset A of a partition space (X; ¿) the following are equivalent:

(1) A is ±g-closed.

(2) A is hkg-closed for each h 2 H and k 2 K:

Proof: (1))(2) are clear.

(2))(1) Let A µ U; where U is open in X: Since X is a partition space, then U is k-open for each k 2 K: Since in partition space ±-cl(A) = ®-cl(A) = scl(A) for any set A; by (2), ±-cl(A) µ U: Hence A is ±g-closed. 2

Theorem 2.4: For a topological space (X; ¿); h 2 H and k 2 K; the following are equivalent:

(1) Every k-open set of X is h-closed.

(2) Every subset of X is hkg-closed.

Proof: (1))(2) Let A µ U, where U is k-open and A is an arbitrary subset of X: By (1), then U is h-closed and thus h-cl(A) µ h-cl(U) = U: Hence A is hkg-closed.

(2))(1) If U µ X is k-open, then by (2) h-cl(U) µ U or equivalently U is h-closed. 2

The intersection of two ®±g-closed (resp. s±g-closed) sets need not be ®±gclosed (resp. s±g-closed) and the union of two s±g-closed sets need not be s±g-closed as shown by the following examples.

Example 2.5: Let (X; ¿) be the space given in Example 2.1. Consider A = fa; b; dg and B = fa; b; cg; then A and B are ®±g-closed and then they are s±g-closed but A \ B = fa; bg is neither ®±g-closed nor s±g-closed.

Example 2.6. Let X = fa; b; cg with ¿ = fX; Á; fag; fbg; fa; bgg: If A = fag and B = fbg; then A and B are s±g-closed but A[B = fa; bg is not s±g-closed in X:

Proposition 2.7: The intersection of a hkg-closed set and a k-closed set is hkg-closed, for each h 2 H and k 2 K:

Proof: Let A be a hkg-closed set and F be a k-closed set of (X; ¿): If U is k-open set with A\F µ U; then A µ U[(XnF) and thus h-cl(A) µ U[(XnF): Then we have h-cl(A \ F) µ h-cl(A) \ F µ U and hence A \ F is hkg-closed. 2

Proposition 2.8: Let A be a subset of a topological space (X; ¿), h 2 H and k 2 K: Then the following are ful¯led:

(1) If A is hkg-closed, then h-cl(A) n A does not contain any non-empty k-closed set.

(2) If A is a subset of X such that h-cl(A)nA does not contain any non-empty h-closed set, then A is hkg-closed.

(3) If A is hkg-closed in X and A µ B µ h-cl(A); then B is hkg-closed in X:

Proof:

(1) Let F be a k-closed subset of h-cl(A) n A: Then A µ (X n F):
Since A is hkg-closed and X n F is k-open, we have h-cl(A) µ (X n F); i.e. F µ (X n h-cl(A)): Thus F µ h-cl(A) \ (X n h-cl(A)) = Á: This shows that F = Á:

(2) Suppose that A µ U and that U is k-open. If h-cl(A) 6µ U; then h-cl(A) \ (X n U) is a non empty h-closed subset of h-cl(A) n A:

(3) Clear. 2

Corollary 2.9. A hkg-closed subset A of X is h-closed if and only if h-cl(A)nA is k-closed.

Proof: Let A be a hkg-closed subset of X: Since h-cl(A) n A is k-closed, by Proposition 2.8(1), h-cl(A) n A = Á and hence A is h-closed.
Conversely, if hkg-closed set A is h-closed, then h-cl(A) nA = Á and hence h-cl(A) n A is k-closed. 2

Proposition 2.10: If A is k-open and hkg-closed in X; then A is h-closed and hence regular open, for each h 2 H and k 2 K:

Proof: If A is k-open and hkg-closed, then h-cl(A) µ A and so A is hclosed. Thus A is regular open, since every k-open and h-closed set is regular open. 2

3. Some Basic Properties On hkg-open Sets. Let (X; ¿) be a topological space and h 2 H; k 2 K: A subst A µ X is called hkg-open if its complement X n A is hkg-closed.

Proposition 3.1: A subset A of a topological space (X; ¿) is hkg-open if and only if F µ h-int(A) whenever F is k-closed and F µ A; for each h 2 H and k 2 K:

Proof: Obvious. 2

Proposition 3.2: If a subset A of a topological space (X; ¿) is hkg-open, then U = X whenever U is k-open and h-int(A)[(X nA) µ U; for each h 2 H and k 2 K:

Proof: Let U be k-open subset of X and h-int(A) [(X n A) µ U: Then (X n U) µ (X n h-int(A)) \ A; i.e. (X n U) µ h-cl((X n A) n (X n A): Since X n A is hkg-closed, by Proposition 2.8(1), X n U = Á and hence U = X: 2

Proposition 3.3.: If A is a hkg-open subset of a topological space (X; ¿) and h-int(A) µ B µ A; then B is hkg-open, for each h 2 H and k 2 K:

Proof: Let F µ B and F be a k-closed subset of X: Since A is hkg-open and F µ A; then F µ h-int(A) and then F µ h-int(B): Hence B is hkg-open. 2

Proposition 3.4: If a subset A of a topological space (X; ¿) is hkg-closed, then h-cl(A) n A is hkg-open, for each h 2 H and k 2 K:

Proof: Let F µ h-cl(A)nA; where F be k-closed in X: Then by Proposition 2.8(1), F = Á and so F µ h-int((h-cl(A) n A): This shows that h-cl(A) n A is hkg-open. 2

Now we are going to introduce and study the notion of k-separated sets related to this types.

Let (X; ¿) be a topological space, k 2 H [K and A;B µ X: We call A and B are k-separated if k-cl(A) \ B = A \ k-cl(B) = Á:

Proposition 3.5: If A and B are k-separated hkg-open sets, then A [B is hkg-open, for each h 2 H and k 2 K:

Proof: Let F be a k-closed subset of A [B: Then F \ k-cl(A) is k-closed and F \ k-cl(A) µ A and hence by Proposition 3.1, F \ k-cl(A) µ h-int(A): Similarly, F \ k-cl(B) µ h-int(B): Now we have
F = F \ (A [B) µ (F \ k-cl(A)) [(F \ k-cl(B)) µ h-int(A) [h-int(B) µ hint(A[B): Hence F µ h-int(A[B) and by Proposition 3.1. A[B is hkg-closed. 2

The union of two ®±g-open (resp. s±g-open) sets is generally not ®±gopen(resp. s±g-open). For, consider the following example.

Example 3.6: Let (X; ¿) be a space given in Example 2.1. Then fcg and fdg are ®±g-open and then s±g-open but their union fc; dg is neither ®±g-open nor s±g-open.

Corollary 3.7.: If A and B are hkg-closed sets such that X n A and X n B are k-separated, then A \ B is hkg-closed, for each h 2 H and k 2 K:

4. OnThkg-spaces. A topological space (X; ¿) is called Thkg if every hkgclosed set is h-closed, where h 2 H and k 2 K: Note that each type of Thkgaxioms is de¯ned to a Thkg-space for some h 2 H and k 2 H: Specially, a Thkg-space (X; ¿) is said to be:

(1) T®±g-space if h = ® and k = ±:
(2) T®µg-space if h = ® and k = µ:
(3) Ts±g-space if h = s and k = ±:
(4) Tsµg-space if h = s and k = µ:

Obviously, every T®µg-space is T®±g; but the converse is not true by the following example.

Example 4.1. Let X = fa; b; cg and ¿ = fX; Á; fag; fbg; fa; bgg: Then (X; ¿) is T®±g-space but not T®µg: Indeed the subset fa; bg is ®µg-closed but not ®-closed.

To discover the identical of T®±g and Ts±g spaces and the identical of T®µg and Tsµg spaces we introduce the following theorems.

Theorem 4.2: For a topological space (X; ¿); the following conditions are equivalent:

(1) (X; ¿) is T®±g-space.

(2) Every singleton of X is either ®-open or ±-closed.

(3) Every singleton of X is either open or ±-closed.

(4) Every singleton of X is either semi-open or ±-closed.

(5) (X; ¿) is Ts±g-space.

Proof: (1))(2) Let x 2 X and assume that fxg is not ±-closed. Then clearly X n fxg is not ±-open and X n fxg is trivially ®±g-closed. By (1), it is ®-closed and thus fxg is ®-open.

(2))(1) Let A be ®±g-closed and let x 2 ®-cl(A):We consider the following two cases:

Case 1: Let fxg be ±-closed. By Proposition 2.8(1), ®-cl(A) n A does not contain fxg: Since x 2 ®-cl(A); then x 2 A:

Case II: Let fxg be ®-open. Since x 2 ®-cl(A), fxg \ A 6= Á: Thus x 2 A:

So, in both case, x 2 A and hence A is ®-closed.

(2),(3),(4) Note that every singleton is ®-open if and only if it is open if and only if it is s-open. 2

(4),(5) Similar to the Proof of the parts (1))(2),(2))(1).

Theorem 4.3: For a topological space (X; ¿); the following conditions are equivalent:

(1) (X; ¿) is T®µg-space.

(2) Every singleton of X is either ®-open or µ-closed.

(3) Every singleton of X is either open or µ-closed.

(4) Every singleton of X is either semi-open or µ-closed.

(5) (X; ¿) is Tsµg-space.

(6) (X; ¿) is Ts±g-space and every ±-closed singleton of X is µ-closed.

Proof: (1),(2),(3),(4),(5) The Proof is similar to that of Theorem 4.2.

(5))(6) Let A µ X be s±g-closed. Then A is sµg-closed. Since X is Tsµg; then A is semi-closed and hence X is Ts±g: Assume that x 2 X and fxg is ±-closed in X: Since X is Ts±g; then by Theorem 4.2 , fxg is not semi-open.

Hence by (4), fxg is µ-closed.
(6))(5) Obvious. 2

A subset A of a topological space (X; ¿) is called h-preopen (resp. hnowhere dense) if A µ h-int(cl(A)) (resp. h-int(cl(A)) Á); where h 2 H:
Lemma 4.4. Let (X; ¿) be a topological space and h 2 H: Then the following are ful¯lled:
(1) Every singleton is h-preclosed or h-open in X:
(2) Every singleton is h-nowhere dense or h-preopen.

Proof: (1) Let x 2 X and suppose that fxg is not h-open. Thus fxg is not open and hence int(fxg) = Á: This implies that h-cl(int(fxg)) = Á µ fxg and hence fxg is h-preclosed.
(2) Let x 2 X and suppose that fxg is not h-preopen. Then x =2 h-int(cl(fxg)) and hence h-int(cl(fxg)) = Á: Thus fxg is h-nowhere dense.
In fact, h-int(cl(fxg)) 6= Á is impossible, since a h-nowhere dense set is not h-preopen. 2
Theorem 4.5: Let (X; ¿) be a topological space and let h 2 H, k 2 K: Then the following are equivalent:
(1) (X; ¿) is Thkg-space.
(2) Every h-preclosed singleton of X is k-closed.
(3) Every non-h-open singleton of X is k-closed.

Proof: (1))(2) Let x 2 X and fxg be h-preclosed in X: By Lemma 4.4, we have fxg is not h-open and hence by Theorem 4.2 and Theorem 4.3, fxg is k-closed.
(2))(1) If fxg is not h-open for some x 2 X; then by Lemma 4.4, fxg is h-preclosed and by (2), it is k-closed. Hence X is Thkg:
(2),(3) Obvious. 2

From Theorem 4.2, Theorem 4.3, Theorem 4.5 and the fact that, every singleton is ±-closed if and only if it is regular closed if and only if it is ±-semiclosed, we have the following characterizations of the T®±g-spaces and the T®µg-spaces.
Theorem 4.6: For a topological space (X; ¿); the following are equivalent:
(1) (X; ¿) is T®±g-space.
(2) Every singleton of X is either ®-open or ±-closed.
(3) Every singleton of X is either ®-open or regular closed.
(4) Every singleton of X is either ®-open or ±-semiclosed.
(5) Every singleton of X is either open or ±-closed.
(6) Every singleton of X is either open or regular closed.
(7) Every singleton of X is either open or ±-semiclosed.
(8) Every singleton of X is either semi-open or ±-closed.

(9) Every singleton of X is either semi-open or regular closed.
(10) Every singleton of X is either semi-open or regular closed.
(11) Every ®-preclosed singleton of X is ±-closed.
(12) Every ®-preclosed singleton of X is regular closed.
(13) Every ®-preclosed singleton of X is ±-semiclosed.
(14) Every semi-preclosed singleton of X is ±-closed.
(15) Every semi-preclosed singleton of X is regular closed.
(16) Every semi-preclosed singleton of X is ±-semiclosed.

Theorem 4.7: For a topological space (X; ¿); the following are equivalent:
(1) (X; ¿) is T®µg-space.
(2) Every singleton of X is either ®-open or µ-closed.
(3) Every singleton of X is either open or µ-closed.
(4) Every singleton of X is either semi-open or µ-closed.
(5) Every ®-preclosed singleton of X is µ-closed.
(6) Every semi-preclosed singleton of X is µ-closed.
(7) (X; ¿) is Ts±g-space and every ±-closed singleton of X is µ-closed.
(8) (X; ¿) is Ts±g-space and every regular closed singleton of X is µ-closed.
(9) (X; ¿) is Ts±g-space and every ±-semiclosed singleton of X is µ-closed.

5. Characterizations of Weakly T2-spaces. Recall that a topological space (X; ¿) is callad weakly T2 if its semi-regularization is T1. Another alternative de¯nition of a weakly T2 is if for each two di®erent point x and y we have x 2 U and y =2 U for some regular open set U: Interesting generalizations of weakly T2-spaces were studied by Fukutake [13], Umehara and Maki [28]. Dontchev and Ganster [10] obtained characterizations of weakly T2-space. To ¯nd other Characterizations, we start with the following lemma.

Lemma 5.1. For a topological space (X; ¿) and for each h 2 H and k 2 K; the following are equivalent:
(1) Every h-preopen singleton is k-closed
(2) Every singleton is h-nowhere dense or k-closed.

Proof: (1))(2) By Lemma 4.4, every singleton is either h-nowhere dense or h-preopen. In the ¯rst case we are done, in the second case k-closedness follows from assumption.

(2))(1) Let fxg be h-preopen. Assume that fxg is not k-closed. Then by (2) it is h-nowhere dense. Thus fxg µ h-int(cl(fxg)) = Á; which is impossible. 2

Theorem 5.2. For a topological space (X; ¿), the following are equivalent:
(1) (X; ¿) is weakly T2-space.
(2) Every singleton is ±-closed.
(3) (X; ¿) is T®±g-space and every singleton is either ®-nowhere dense or ±-closed.

(4) (X; ¿) is T®±g-space and every ®-preopen singleton is ±-closed.

Proof. (1),(2) is proved in [4].

(2))(3) Obvious.

(3))(4) If a singleton is not ±-closed, then by (3) it must be ®-nowhere dense. Since a non-empty ®-nowhere dense set cannot be ®-preopen at the same time, then (4) is true.

(4))(2) Obvious. 2

Theorem 5.3. For a topological space (X; ¿), the following are equivalent:

(1) (X; ¿) is weakly T2-space.

(2) Every singleton is ±-closed.

(3) (X; ¿) is Ts±g-space and every singleton is either semi-nowhere dense or ±-closed.

(4) X is Ts±g-space and every semi-preopen singleton is ±-closed.

Proof: Similar to the proof of Theorem 5.2. 2

A topological space (X; ¿) is called T3=4 [10] if every ±g-closed subset of X is ±-closed. Dontchev and Ganster [10] showed the following implication: X is weakly T2-space , X is T3=4 and each singleton is ±g-closed. From this implication, Theorem 4.2, Theorem 5.2 and Theorem 5.3, we obtain the following characterizations of the weakly T2-space.

Theorem 5.4. For a topological space (X; ¿); the following are equivalent:

(1) (X; ¿) is weakly T2-space.

(2) Every singleton is ±-closed.

(3) (X; ¿) is T3=4-space and every singleton is ±g-closed.

(4) (X; ¿) is T®±g-space and every singleton is either ®-nowhere dense or ±-closed.

(5) (X; ¿) is Ts±g-space and every singleton is either semi-nowhere dense or ±-closed.

(6) (X; ¿) is T®±g-space and every ®-preapen singleton is ±-closed.

(7) (X; ¿) is Ts±g-space and every semi-preopen singleton is ±-closed.

From the property that, every singleton is ±-closed if and only if it is regular closed if and only if it is ±-semiclosed, we can obtain another characterizations of the weakly T2-space.

Theorem 5.5. For a topological space (X; ¿); the following are equivalent:

(1) (X; ¿) is a weakly T2-space.

(2) Every singleton is ±-semiclosed.

(3) (X; ¿) is T3=4-space and every singleton is ±g-closed.

(4) (X; ¿) is T®±g-space and every singleton is either ®-nowhere dense or ±-semiclosed.

(5) (X; ¿) is Ts±g-space and every singleton is either semi-nowhere dense or

±-semiclosed.
(6) (X; ¿) is T®±g-space and every ®-preopen singleton is ±-semiclosed.
(7) (X; ¿) is Ts±g-space and every semi-preopen singleton is ±-semiclosed.
Theorem 5.6: For a topological space (X; ¿); the following are equivalent:
(1) (X; ¿) is weakly T2-space.
(2) Every singleton is regular closed.
(3) (X; ¿) is T3=4-space and every singleton is ±g-closed.
(4) (X; ¿) is T®±g-space and every singleton is either ®-nowhere dense or regular closed.
(5) (X; ¿) is Ts±g-space and every singleton is either semi-nowhere dense or regular closed.
(6) (X; ¿) is T®±g-space and every ®-preopen singleton is regular closed.
(7) (X; ¿) is Ts±g-space and every semi-preopen singleton is regular closed.

DISCUSSION

We could in this research to study new types of the generalized closed sets and we studied the di®erent relations between them.

According to these types we could de¯ne new separation axioms called T®±g,Ts±g,T®µg and Tsµg. Also we made many characterizations for these axioms. By using these separation axioms we made new characterizations to weakly T2- spaces.

In this word we have a problem, we suggest to be a point of new research in the future, where we could prove that each ®µg-closed set is sµg-closed, while we failed to ¯nd an example to prove that the opposite is not correct, at the same time we could not prove the identity of the two concepts.

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