Natural Frequencies and Modes of Vibrations for Cable Roofs

By

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"الترددات الطبيعية وأشكال الإهتزازات للأسقف ذات الكابلات "

الخلاصة : يتناول هذا البحث فى البداية عرض مبسط لطرق حساب الترددات الطبيعية وأشكال الإهتزازات وأهمية إستخدامها فى التحليل الديناميكى للمنشآت وتركز هذا العرض حول ثلاث طرق شائعة لتكثيف المصفوفات وذلك فى المنشآت ذات الوصلات الكثيرة لإمكانية حلها على الحاسب الآلى وتم تطبيق هذه الطرق الشائعة على إطار ذو ستة طوابق وعمل مقارنات بين هذه الطرق . وقد تم حساب الترددات الطبيعية وأشكال الإهتزازات لثلاثة أنواع من الأسقف وهى الأسقف المحدبة والأسقف المقعرة والأسقف المقعرة المحدبة ذات بحور مختلفة أخذا فى الإعتبار تأثير الشد الإبتدائى فى الكابلات ونسب إرتفاع وترخيم الكابلات إلى بحور هذه الكمرات وكذلك المسافة بين الكابلات الرأسية وحالات الأحمال المتماثلة والغير متماثله مع إستعراض الأمثلة التوضيحية لذلك مع ذكر أهم النتائج التى تم إستنتاجها .

Abstract.

The free vibration mode shapes have certain speeial properties whieh are very useful in structural dynamic analysis . For structures with a large number of joints , it is often necessary to reducc the number of degrees of freedom hy simplifying the strueture or by condensing the stiffness and mass matrices . Three popular methods of condensation applied in dynamie structures are presented . The natural frequencies and corresponding modal shapes for convex , eoneave and concave-convex cable roofs are studied . Also, effect of initial tensions in cables , both sag and rise to span ratios , distance between vertical ties , and symmetric and asymmetric of loads on the frequeneies for all three types of roofs are outlined . All eomputer programs for construction of overall stiffness and mass matrices , solution of free vibration equation using exact tcchnique and eondensation are constructed by the author .

1- Introduction

Pretensioned cable roof structures are in general lighter and their roof more flexible than other forms of constructions . Also , their height-to-span ratios are usually relatively smaller . The cable roofs will be quite stiff if tensioned to a level which ensures that both sagging and rising cables remain in tension under any combination of loading . The author [1,2,3] studied the optimum shape and dimensions for convex , concave and concave-convex cable roofs and recommended to complete the study by studying the eigenproblem analysis for these types of cable roofs .

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A complete dynamic analysis of a structure must involve the frequency analysis .

Also , the applieation of the response spectrum for earthquake design of structures requires the determination of the natural frequencies and modal shapes . The dynamic response of any linear structure can readily be obtained after its vibration mode shapes and frequencies have been determined . The natural frequencies of a structure will , if the frequency spectrum of the dynamic loading is known , indicate whether or not the structure is likely to rcspond dynamically. The shapes of the modes will indicate in which way the structure is likely to respond and the best position for placing artificial dampers if required .

For large structures , the solution of the corresponding eigenproblem to determine natural frequencres and modal shapes will be difficult and expensive . Also , in most practical cases , only a relatively small number of modes need be considered in the analysis to obtain adequate accuracy . So that , the reduction of the number of degrees of freedom before determination of natural frequencies and modal shapes is needed . A brief review of reduction techniques have been mentioned in the following item . A numerical example for multi-story shear building to compare between the common condensation methods used in this paper is proposed . Finally , the natural frequencies and modal shapes for three types of cable roofs shown in Figs. [1-a], [1-b] and [1-c] are proposed . The influence of lumped and consistent mass matrices on vibration characteristics of three dimensional cable roofs have been examined . Also , the influence of design parameters on the natural frequencies of cable roofs is taken into considerations [4] .

2- A brief Review of Eigenproblem Analysis

Numerous techniques for the reduction of the number of coordinates used in modeling structural dynamic systems have been proposed . The practical application of these techniques , however requires judicious selection of both the particular coordinates to be reduced and the specific reduction process . A survey of the published literature shows that the first major step towards a method of reducing or condensing the dimension of the eigenproblem of a structural dynamic system , appeared in the paper published by [5] . The relation between the primary and unwanted (secondary) degrees of freedom in the static condensation proposed by [5] is found by establishing the static relation between them . Also , in order to reduce the mass and stiffness matrices , it was considered that the same static relation remain valid for the dynamic problem and the reduced matriees have the property of considering both the potential and kinetic energies of the system . An equivalent algorithm to reduce the stiffness and mass matrices is presented by[6] . Also , a technique which reduces the dynamic problem using the flexibility method is presented by [7] .

The algorithm proposed by [8] describes the determination of the number of frequencies lower than any chosen frequencies . Also , the paper published by

[9] describes a bound algorithms to measure the accuraey of eigenvalue problem solutions obtained after system reductions . By expanding in series the expression containing the eigenvalue , and by neglecting the higher-order terms , Geradin [9] derived an expression for the transformation of coordinates as well as for the reduced stiffness and mass matrices . To obtain the modal shape in terms of all coordinates in the original modeling of the structure using the back-transformation , the exact relation between primary and secondary coordinates is recommended by [10] . This transformation is affected by the errors introducecl in the calculation of the eigenvalue of the reduced system . The methods proposed by [11,12] give more accurate results after reducing coordinates . To minimize the error introduced in the reduction process , a number of authors [10,13,14,15,16,17] have developed algorithms for that purpose . Various techniques for coordinate reduction of structural dynamic problem , are presented in a number of books as [18,19,20,21,22] .

3- Methods of Computation Eigenproblem

If it is assumed that the natural frequencies and mode shapes are not significantly affected by the amplitude of vibration , then both eigenvalues and eigenvectors may be found by the eigenproblem equation :

$$[[k]-\omega^2[M]]\{Y\} = \{0\} \qquad (1)$$

where

[K] = the tangent stiffness matrix at the static equilibrium position ;

[M] = the lumped or consistent mass matrix ;

{Y} = the mode shape vector ; and

ω^2 = an N*N diagonal matrix of the square of the natural angular frequencies corresponding to the mode shape .

The formulation of eq. (1) is an important mathematical problem for eignproblem and its nontrivial solution requires that the determinant of the matrix factor of {Y} be equal to zero as :

$$\left| [k]-\omega^2[M] \right| = 0 \qquad (2)$$

In large cable structures , it is sometimes necessary to divide a structure into a large number of elements because of change in geometry , loading , and or material properties . In this case , the number of degrees of freedom , may be quite large . As a consequence , the stiffness and mass matrices will be of large dimensions . The solution of the corresponding eigenproblem will be difficult and expensive . In such eases , it is desirable to reduce the size of these matrices in order to make the solution manageable and economical . Such reduction is referred to as condensation . Three popular methods of condensation are mentioned below .

3-1 Static Condensation Applied to Dynamic Problems [5,20,22,23]

The static condensation method is proposed by [5] . In order to describe this method , consider s are the secondary eoordinates to be condensed and p are the

primary coordinates (remaining coordinates) . With this arrangement , the stiffness equation for the structure may be written using partition of matrices as :

$$\left|\begin{array}{c|c}[K_{ss}] & [K_{sp}] \\ \hline [K_{ps}] & [K_{pp}]\end{array}\right| \left\|\begin{array}{c}\{Y_s\} \\ \hline \{Y_p\}\end{array}\right\| = \left\|\begin{array}{c}\{0\} \\ \hline \{F_p\}\end{array}\right| \qquad (3)$$

where $\{Y_s\}$ and $\{Y_p\}$ are the displacement vectors corresponding to s and p degrees of freedom , respectively . Expanding eq.(3) into two equations , it can be written as

$$[K_{ss}]\{Y_s\} + [K_{sp}]\{Y_p\} = \{0\} \qquad (4)$$

and

$$[K_{ps}]\{Y_s\} + [K_{pp}]\{Y_p\} = \{F_p\} \qquad (5)$$

Equation (4) is equivalent to

$$\{Y_s\} = [\bar{T}]\{Y_p\} \qquad (6)$$

where $[\bar{T}]$ is the transformation matrix given by

$$[\bar{T}] = -[K_{ss}]^{-1}[K_{sp}] \qquad (7)$$

Substitution eq.(6) and using eq.(7) in eq.(5) results in the reduced stiffness equation as :

$$[\bar{K}]\{Y_p\} = \{F_p\} \qquad (8)$$

where $[\bar{K}]$ is the reduced stiffness matrix given by

$$[\bar{K}] = [K_{pp}]-[K_{ps}][K_{ss}]^{-1}[K_{sp}] \qquad (9)$$

Equation (6) can be rewritten in the form

$$\{Y\} = [T]\{Y_p\} \qquad (10)$$

where

$$\{Y\} = \left|\begin{array}{c}\{Y_s\} \\ \hline \{Y_p\}\end{array}\right| \text{ and } [T] = \left|\begin{array}{c}[\bar{T}] \\ \hline [I]\end{array}\right| \qquad (11)$$

Substituting eq. (10) and (11) into eq. (3) and pre multiplying by the transpose of [T] results in :

$$[T]^T[K][T]\{Y_p\} = [[\bar{T}]^T \; [I]] \left|\begin{array}{c}\{0\} \\ \hline \{F_p\}\end{array}\right| \qquad (12)$$

and using eq. (8)

$$[\bar{K}] = [T]^T[K][T] \qquad (13)$$

At this stage of elimination process the stiffness equation (3) has been reduced to

$$\left|\begin{array}{c|c}[I] & -[\bar{T}] \\ \hline [0] & [\bar{K}]\end{array}\right| \left\|\begin{array}{c}\{Y_s\} \\ \hline \{Y_p\}\end{array}\right\| = \left|\begin{array}{c}\{0\} \\ \hline \{F_p\}\end{array}\right| \qquad (14)$$

In this way , the Gauss-Jordan elimination process yields both transformation matrix $[\bar{T}]$ and reduced stiffness matrix $[\bar{K}]$. There is , thus no need to calculate $[K_{ss}]^{-1}$ in order to reduce the secondary coordinates of the system . Also , the static condensation method is valid to use in dynamic problem . Hence , the same transformation based on static condensation for the reduction of the stiffness matrix is also used in reducing the mass matrices . Specifically , if [M] is the mass matrix of the system , then the reduced mass matrix is given by

$$[\bar{M}] = [T]^T[M][T] \tag{15}$$

3-2 Dynamic Condensation Method

This method has been recently proposed by [24,25,26] as an extension of the static condensation method . The eigenproblem eq.(1) can be rewritten in the form :

$$\left|\begin{array}{c|c} [K_{ss}] - \omega_i^2[M_{ss}] & [K_{sp}] - \omega_i^2[M_{sp}] \\ \hline [K_{sp}] - \omega_i^2[M_{pp}] & [K_{pp}] - \omega_i^2[M_{pp}] \end{array}\right| \left|\begin{array}{c} \{Y_s\} \\ \hline \{Y_p\} \end{array}\right| = \left|\begin{array}{c} \{0\} \\ \hline \{0\} \end{array}\right| \tag{16}$$

Where ω_i^2 is the approximation of the ith eigenvalue which was calculated in the preceding step of the process . To start the process one takes an approximate or zero value for the first eigenvalue ω_1^2

The following three steps are executed to calculate the ith eigenvalue ω_i^2 and the corresponding eigenvector $\{Y\}_i$ as well as an approximation of the eigenvalue of the next order ω_{i+1}^2 .

Step 1. The approximation of ω_i^2 is introduced in eq. (16) ; Gauss- Jordan elimination of coordinates {Ys} is then used to reduce eq. (16) to

$$\left|\begin{array}{c|c} [I] & -[\bar{T}] \\ \hline [0] & [\bar{D}_i] \end{array}\right| \left|\begin{array}{c} \{Y_s\} \\ \hline \{Y_p\} \end{array}\right| = \left|\begin{array}{c} \{0\} \\ \hline \{0\} \end{array}\right| \tag{17}$$

Step 2. The reduced mass and stiffness matrices are calculated respectively as

$$[\bar{M}_i] = [T_i]^T[M][T_i] \tag{18}$$

and

$$[\bar{K}_i] = [\bar{D}_i] + \omega_i^2[\bar{M}_i] \tag{19}$$

where the transformation matrix [Ti] is given by eq. (11) and the reduced dynamic matrix $[\bar{D}_i]$ is defined in eq.(16) .

Step 3. The reduced eigenproblem

$$[\bar{K}_i] - \omega^2[\bar{M}_i]\{Y_p\} = \{0\} \quad (20)$$

is solved to obtain an improved eigenvalue ω_i^2 , and also an approximation for the next order eigenvalue ω_{i+1}^2 .

This three-step process may be applied iteratively . Experience has shown that one or two sueh iterations will produce virtually exact eigensolutions . Once an eigenvector {Yp} for the reduced system is found , the ith modal shape is given by eq. (10) .

3-3 Modified Dynamic Condensation Method [27]

In this modification , the reduced stiffness matrix $[\bar{K}]$ is calculated only one by simple elimination of s displacements in eq. (16) after setting $\omega^2 = 0$, Also , the reduced mass matrix for any mode i is calculated from

$$[\bar{M}_i] = \frac{1}{\omega_i^2}[[\bar{K}]-[\bar{D}_i]] \quad (21)$$

As can be seen , the modified method requires , for each eigenvalue calculated , only the application of the Gauss-Jordan process to eliminate s unknowns in a linear system of equations such as the system in eq. (16) .

3-4 Mass Properties [19,20] .

3-4-1 Lumped-Mass Matrix .

The simplest procedure for defining the mass properties of any structure is to assume that the entire mass is concentrated at the points at which the translational displacements are defined . The lumped-mass matrix is diagonal .

In which m' is the unit mass per unit length of the element , the lumped-mass matrix for cable element is given by

$$[M_l] = \frac{\bar{m}\,L}{2}\begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix} \quad (22)$$

where ($\lceil\rfloor$) is a special symbol used for diagonal matrices .

3-4-2 Consistent - Mass Matrix .

The dynamic analysis of a consistent-mass system generally requires considerably more computational effort than a lumped-mass system does , for two reasons :

(1) The lumped-mass matrix is diagonal , while the consistent-mass matrix has many off-diagonal terms .

(2) The rotational degrees of freedom can be eliminated from a lumped-mass

analysis , whereas all rotational and translational degrees of freedom must be included in a consistent-mass analysis .

The consistent-mass matrix for cable element is given by :

$$[M_c]=\frac{m'L}{6}\begin{vmatrix} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 \end{vmatrix} \qquad (23)$$

3-5 System Stiffness Matrix [19]

The stiffness matrix for pin-jointed pretensioned link is given by :

$$[K]=\frac{EA-T_0}{L_0}\begin{vmatrix} GG^T & -GG^T \\ -GG^T & GG^T \end{vmatrix}+\frac{T_0}{L_0}\begin{vmatrix} I & -I \\ -I & I \end{vmatrix} \qquad (24)$$

where E is the modules of elasticity , A is an area , T0 is the initial tension , L_0 is the initial length , I is a unit matrix of dimension (3*3), and $G^T = \{l \quad m \quad n\}^T$ and l , m , n are the direction cosines of the member .

4- Numerical Example .

A Simple type of structure , known as a shear building is selected to demonstrate the effectiveness of popular methods of condensations . The number of degrees of freedom is equal to the number of stories in the building . The six-stories analyzed frame with the stiffness and mass properties are shown in Fig. [2-a] . The multimass spring modeled and the free body diagram for the frame are shown in Figs. [2-b] and [2-c] , respectively .

The stiffness [K] and mass [M] matrices for the frame are given as :

$$[K] = \begin{vmatrix} 22000 & -10000 & 0 & 0 & 0 & 0 \\ -10000 & 18000 & -8000 & 0 & 0 & 0 \\ 0 & -8000 & 14000 & -6000 & 0 & 0 \\ 0 & 0 & -6000 & 10000 & -4000 & 0 \\ 0 & 0 & 0 & -4000 & 6000 & -2000 \\ 0 & 0 & 0 & 0 & -2000 & 2000 \end{vmatrix}$$

and $[M] = \lceil 21 \quad 18 \quad 15 \quad 12 \quad 9 \quad 6 \rfloor$

The number of degrees of freedom is condensed from six to three only . The reduced coordinates were numbers 1,3 and 5 . The results shown in Table (1) indieates the following (1) Statie condensation inethod produces only the fundamental frequency with an acceptable value ; hugher frequencies have errors

of about 7% ; and (2) Dynamic and modified dynamic condensation methods give virtually the same values for the natural frequencies as those obtained using the complete model without condensation .

5- Eiegenvaiues Eigenvectors for Cable Roofs .

5- Analysis Considerations .

The eigenproblem analysis for convex , concave , and concave-convex cable roofs shown in Figs. [1-a] , [1-b] and [1-c] , respectively for both types are taken into considerations [1,2,3]. Lumped or consistent mass was considered . Three cases of loads are considered as static loads as :

Case A : uniformly distributed dead load of 0.3 KN/m' .

Case B : A combination of case A with uniformly live load on full span with intensity of 1.1 KN/m' .

Case C : A combination of case A with uniformly live load on half span with intensity of 1.1 KN/m' .

5-2 Natural Frequencies and Model Shapes .

First the influences of many factors on eigenvalues for all three mentioned types of cable roofs are observed . These factors were summarized as , the initial tension in cables , spacing between vertical ties , both sag and rise to span ratios , span and loads . To carry out the influence of any factor on the eigenvalues , it is considered that the other factors are kept constant as the initial tensions of 10% of breaking loads , spacing between vertical ties of 3m , and both sag and rise to span ratios of 5% . The results given in Figs. [3 to 11 lcan be summarized as :

1) An inerease of initial tensions in cables produces a slight increase for low frequencies and observable increase for high frequencies, Figs. [3,4,5] .
2) In case of increasing the spacing between vertical ties , Fig. [6] had remote ehanges in the frequeneies up to 6 m , and then the frequencies decreases slightly .
3) In view of Figs. [8 to 10] , an increase of span produces a decrease of natural frequencies .
4) With rises of loads , the natural frequencies decrease Fig. [11] .

Also , with reference to the results given in Table [3] , it noticed that the effect of using consistent mass matrix instead lumped mass matrix gave a remote changes in low frequencies and these changes increase in the higher frequencies . Also, using diagonal elements (type B) causes an increase of natural frequencies .

Finally , the natural frequencies for all three mentioned types of roofs having a spans of 30m , 60m , and 90m (both eases A and B) with all cases of

gravitational loading are given in Figs. [12 to 20]. Also, the first few modal shapes corresponding to case of gravitational loading A (type A) for all types of cable roofs are shown in Figs. [21 to 23].

6- Conclusions :

The general conclusions can be summarized as :

1- The natural frequencies of convex, concave, and concave-convex cable roofs vary with span, gravitational loading and initial tensile forces in cables, as well as with the spacing between vertical ties and both sag and rise to span ratios.
These variations are :

a) The natural frequencies of cable roofs increase with increasing cable pretensions and decrease with increasing spans, gravitational loading, and both sag and rise to span ratios.
b) An increasing of spacing between vertical ties produces a remote decrease of natural frequencies up to spacing of 6m and a slight decrease of spacing greater than 6m.
c) Using diagonal elements (roofs type B) produces an increase of natural frequencies.
d) Case of dead load only (case of loading A) gave the higher values of natural frequencies in comparison with other cases.
e) The comparison of the natural frequencies in complete similar cases for all types of cable roofs shows that the smallest in convex roof but they are the biggest in concave-convex roofs.
f) Using a lumped mass matrix instead the consistent mass matrix, remote changes in low frequencies is observed, whereas with high frequencies these changes are considerably noticeable.

2- In application of structural dynamics as only the first few lower frequencies are of interest, the lumped mass matrices could be significantly used with advantage in vibration problem. These lumped matrices give computational advantages because of they are diagonal

3) The reduction of unwanted or secondary degrees of freedom is usually accomplished in practice by the static condensation method. This method introduces errors when applied to the solution. These errors are very small in low frequencies. So that, this method is applicable for use in cable roofs.

4) The dynamic and modified dynamic condensations methods are valid to use in structures having a small numbers of degrees of freedoms and need a more carefulness for choosing the unwanted degrees of freedom to be condensed in large structures, especially in cable roofs.

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Table 1. Natural frequencies (cps) for the frame shown in Fig. [2-a] .

Mode numbers	No Condensation	Methods of Condensation					
		Static		Dynamic		Modified dynamic	
	Frequency	Freq	%error	Freq	%error	Freq.	%error
1	1.11	1.125	1.35%	1.11	0%	1.11	0%
2	2.409	2.499	3.73%	2.409	0%	2.41	0.04%
3	3.649	3.899	6.85%	3.649	0%	3.653	0.1%

Table (2) : Cables Properties for Cable Roofs .

span m	Type	Diameter, cm	Area , cm2	Breaking load ,(KN)	Weight N/m'	Modules,E KN/mm
30m	BS5896	1.8	2.23	380	17.17	150
60m	spiral	3.31	6.78	910	54.8	169.7
90m	locked	4.8	15.3	2001.3	125.7	158.4

Table [3] : Some Results of Natural Frequencies for Cable Roofs .

Frequency number (cps)	Type of Mass	Convex Roof				Concave Roof				Concave-Convex Roof			
		Type A		Type B		Type A		Type B		Type A		Type B	
	Case ⇨	(A)	(B)	(A)	(B)	(A)	(B)	(A)	(B)	(A)	(B)	(A)	(B)
1	Lumped	0.45	0.55	0.52	0.63	0.46	0.56	0.62	0.74	0.46	0.56	0.52	0.64
	Consistent	0.45	0.55	0.53	0.64	0.46	0.56	0.63	0.75	0.46	0.67	0.52	0.64
2	L	0.86	1.14	1.02	1.35	0.89	1.21	1.06	1.54	0.90	1.26	1.17	1.55
	C	0.89	1.17	1.05	1.44	0.92	1.28	1.09	1.63	0.93	1.32	1.21	1.59
3	L	1.0	1.16	1.4	1.72	1.19	1.4	1.52	1.86	1.23	1.37	1.27	1.56
	C	1.03	1.23	1.42	1.74	1.22	1.44	1.57	1.91	1.25	1.41	1.29	1.65
4	L	1.18	1.45	1.48	1.78	1.27	1.61	1.66	2.04	1.29	1.67	1.62	2.07
	C	1.22	1.54	1.59	1.97	1.37	1.71	1.8	2.25	1.39	1.83	1.76	2.32
5	L	1.21	1.53	1.86	2.22	1.57	1.69	2.01	2.55	1.46	2.08	1.92	2.48
	C	1.3	1.69	2.12	2.67	1.63	1.86	2.29	3.05	1.57	2.39	2.2	3.04
10	L	1.98	2.33	2.69	4.86	2.12	2.73	3.0	5.29	2.34	2.91	2.88	5.15
	C	2.59	3.26	4.22	5.93	2.60	3.63	2.98	6.07	2.89	3.68	4.48	6.62
15	L	2.40	3.78	11.62	11.76	2.65	4.49	13.18	13.18	2.59	4.39	12.13	14.91
	C	3.82	5.0	12.24	13.89	3.93	6.25	13.72	15.52	4.34	6.17	12.24	15.93
20	L	8.82	9.81	14.45	14.62	14.45	14.45	16.62	16.8	16.41	16.52	16.77	18.28
	C	9.43	9.93	18.4	18.42	16.23	16.13	20.64	20.95	17.3	17.45	21.73	23.21
25	L	15.7	16.95	21.45	27.6	24.05	26.77	24.55	33.87	21.72	24.86	21.6	30.99
	C	19.46	22.39	22.97	30.22	25.91	34.49	34.25	36.46	27.07	28.7	27.76	36.91

Fig. (1-a) Convex Roof - Type A

Fig. (1-a) Convex Roof - Type B

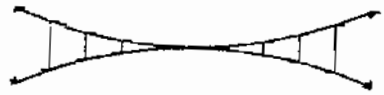

Fig. (1-b) Concave Roof - Type A

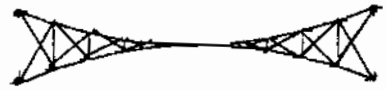

Fig (1-b) Concave Roof - Type B

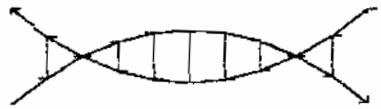

Fig. (1-c) Concave-Convex Roof - Type A

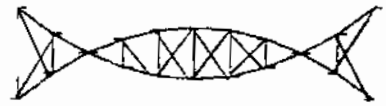

Fig. (1-c) Concave -Convex Roof - Type B

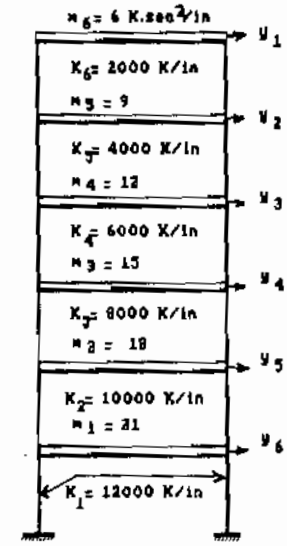

Fig. (2-a) Analysed Frame

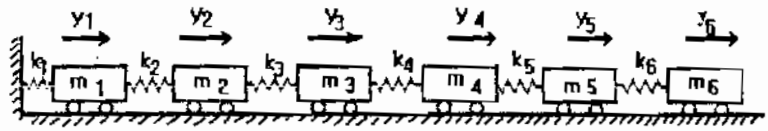

Fig. (2 b The multimass spring modeled for the frame .

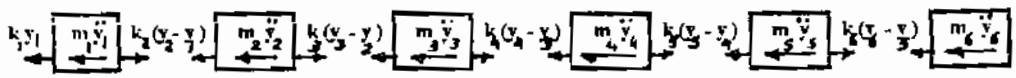

Fig. (2-c) The free body diagram for the analysed frame.

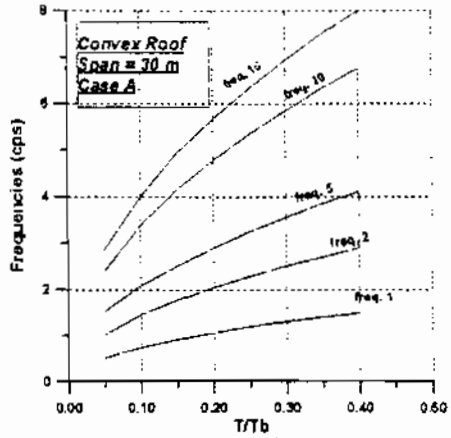

Fig. (3) Frequencies with initial tension.

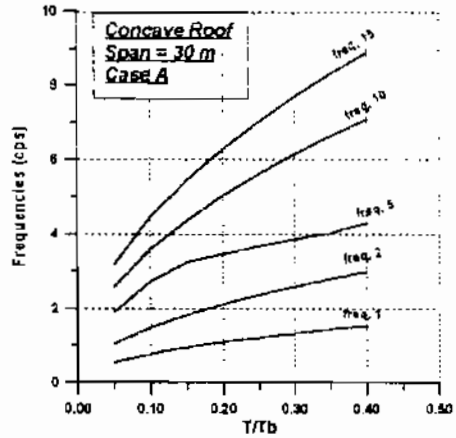

Fig.(4) Frequencies with inetial tension

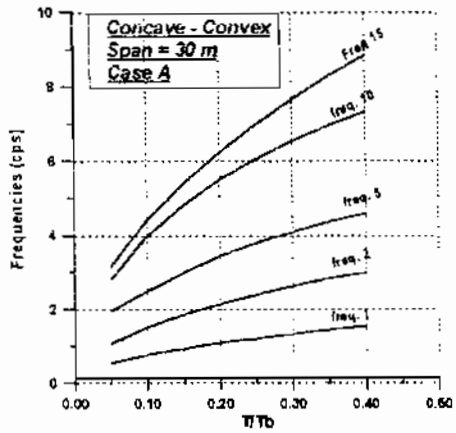

Fig. (5) Frequencies with initial tension

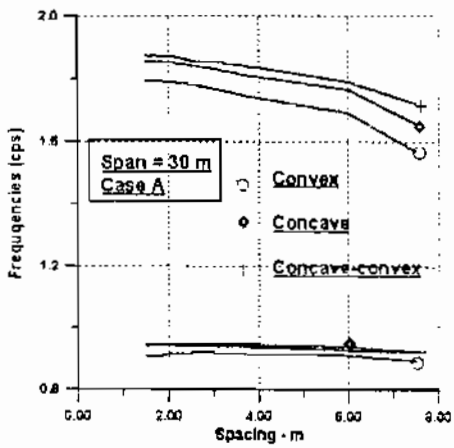

Fig. (6) Variation of freq. with spacing.

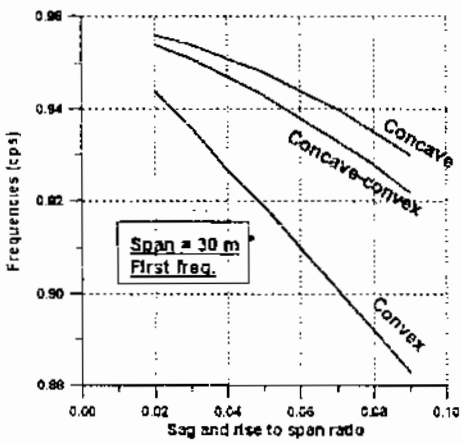

Fig. (7) Variation of freq. with sag

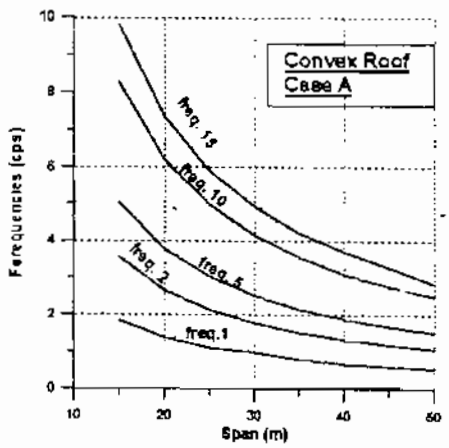

Fig. (8) Variation of span with frequencies

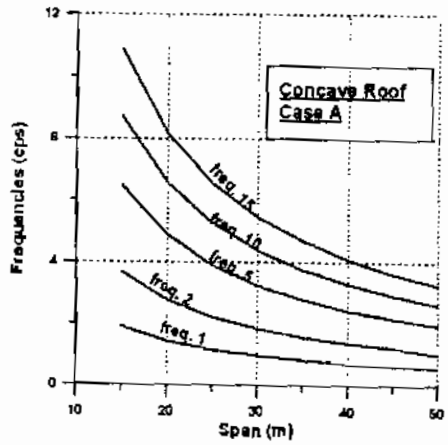

Fig. (9) Variation of span with frequencies.

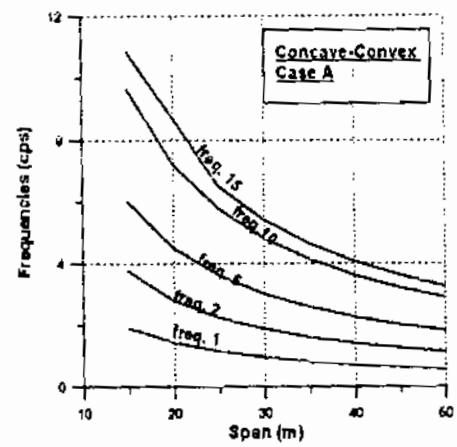

Fig. (10) Variation of span with frequencies

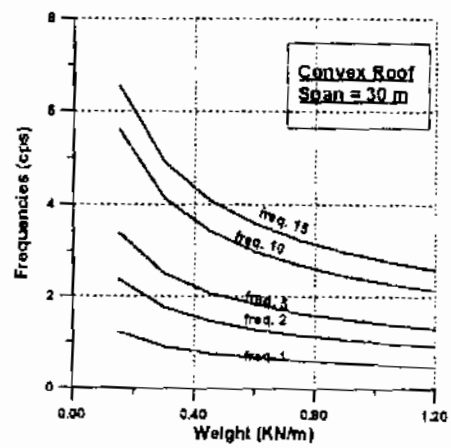

Fig. (11) Variation of weights with frequencies

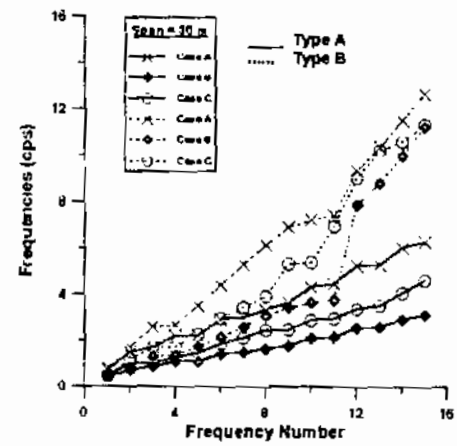

Fig. (12) Convex Roof

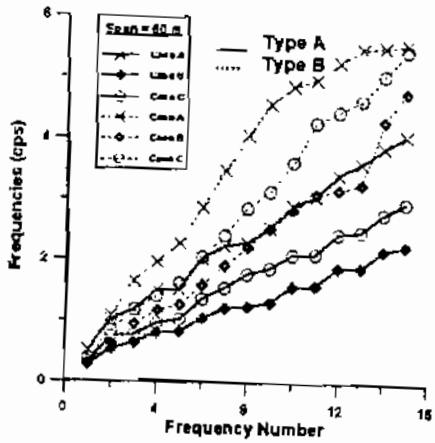

Fig. (13) Convex Roof

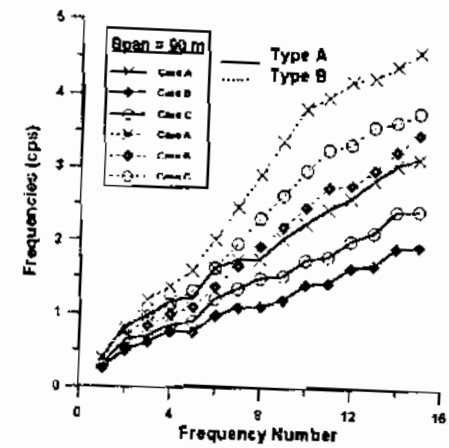

Fig. (14) Convex Roof

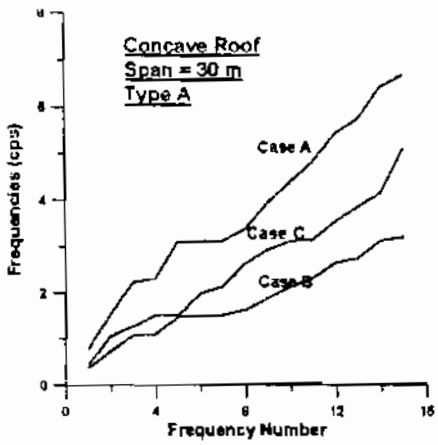

Fig. (15) First 15th frequencies.

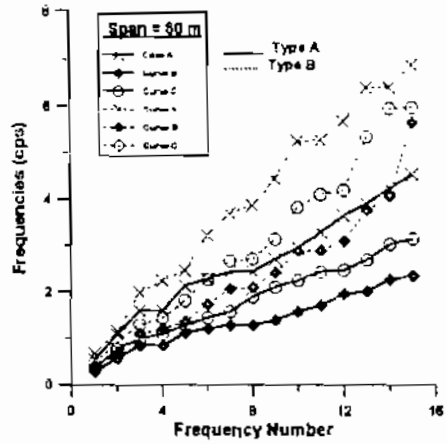

Fig. (16) First 15th freq. for concave.

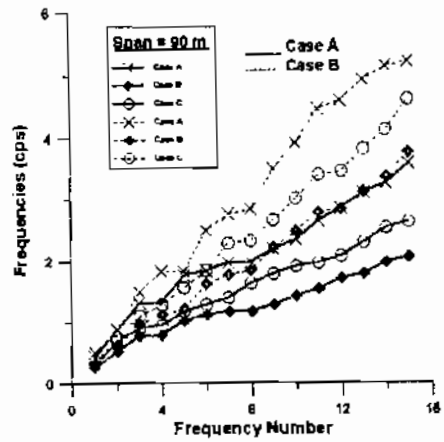

Fig. (17) Concave Roof.

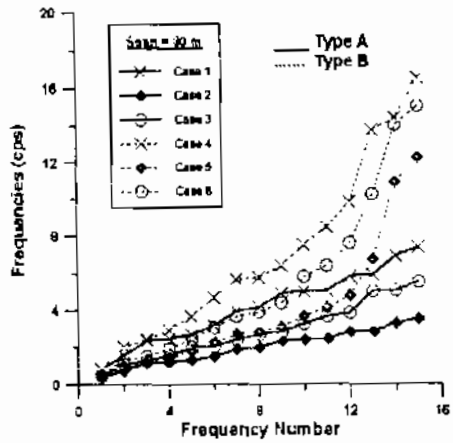

Fig. (18) Concave - Convex Roof

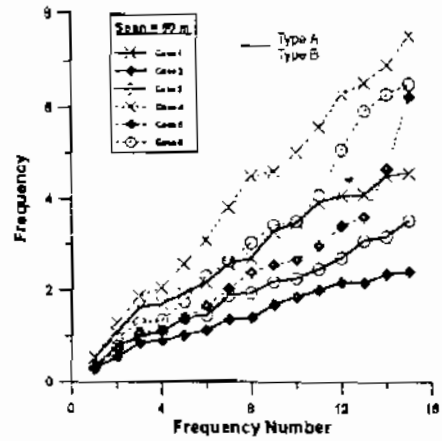

Fig. (19) Concave-Convex Roof

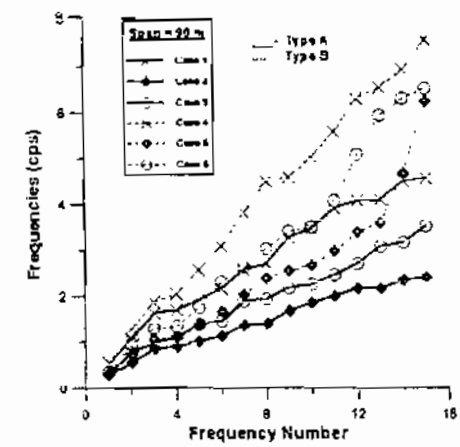

Fig. (20) Concave - Convex Roof

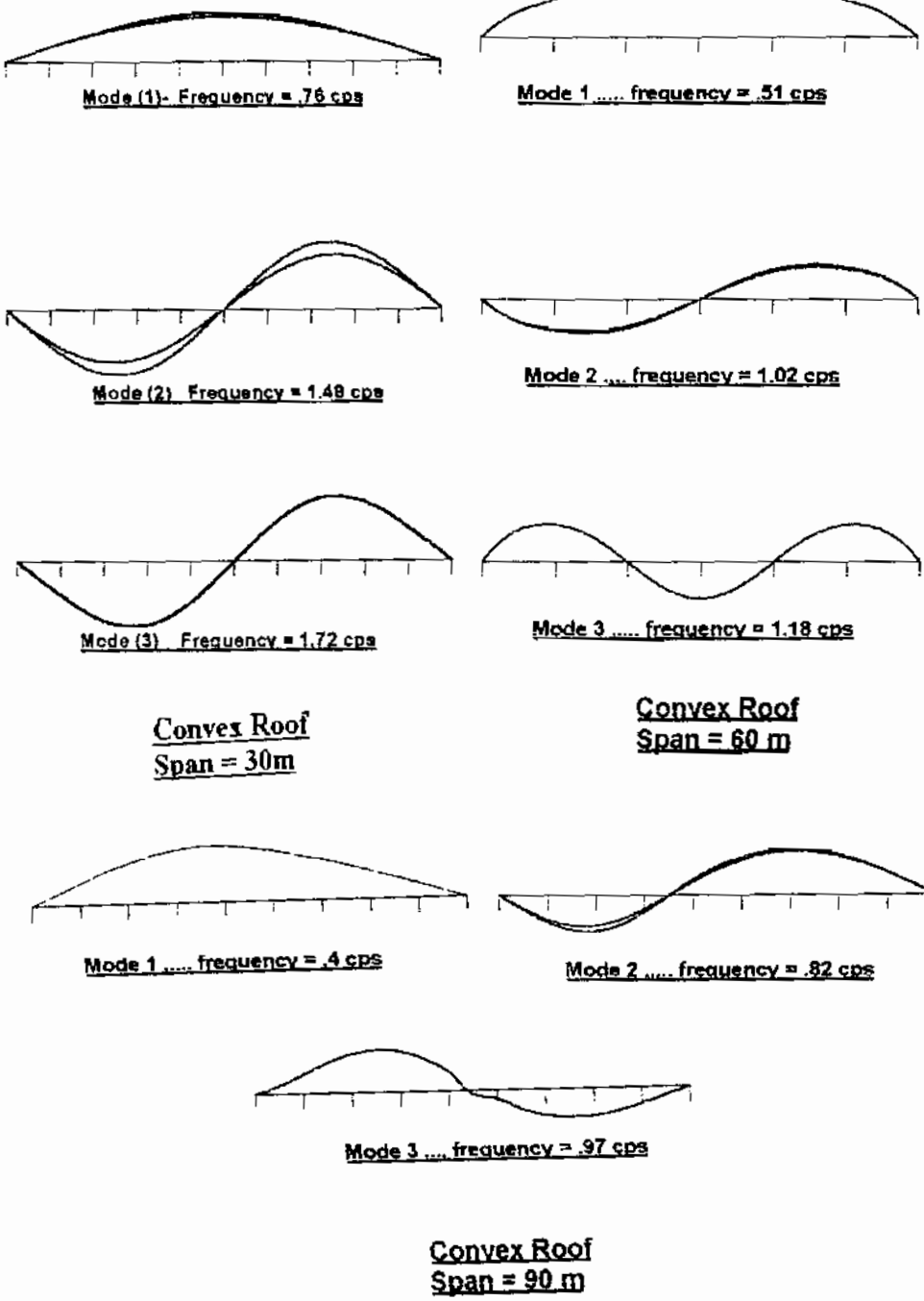

Fig. (20) First three mode shapes for convex roofs .

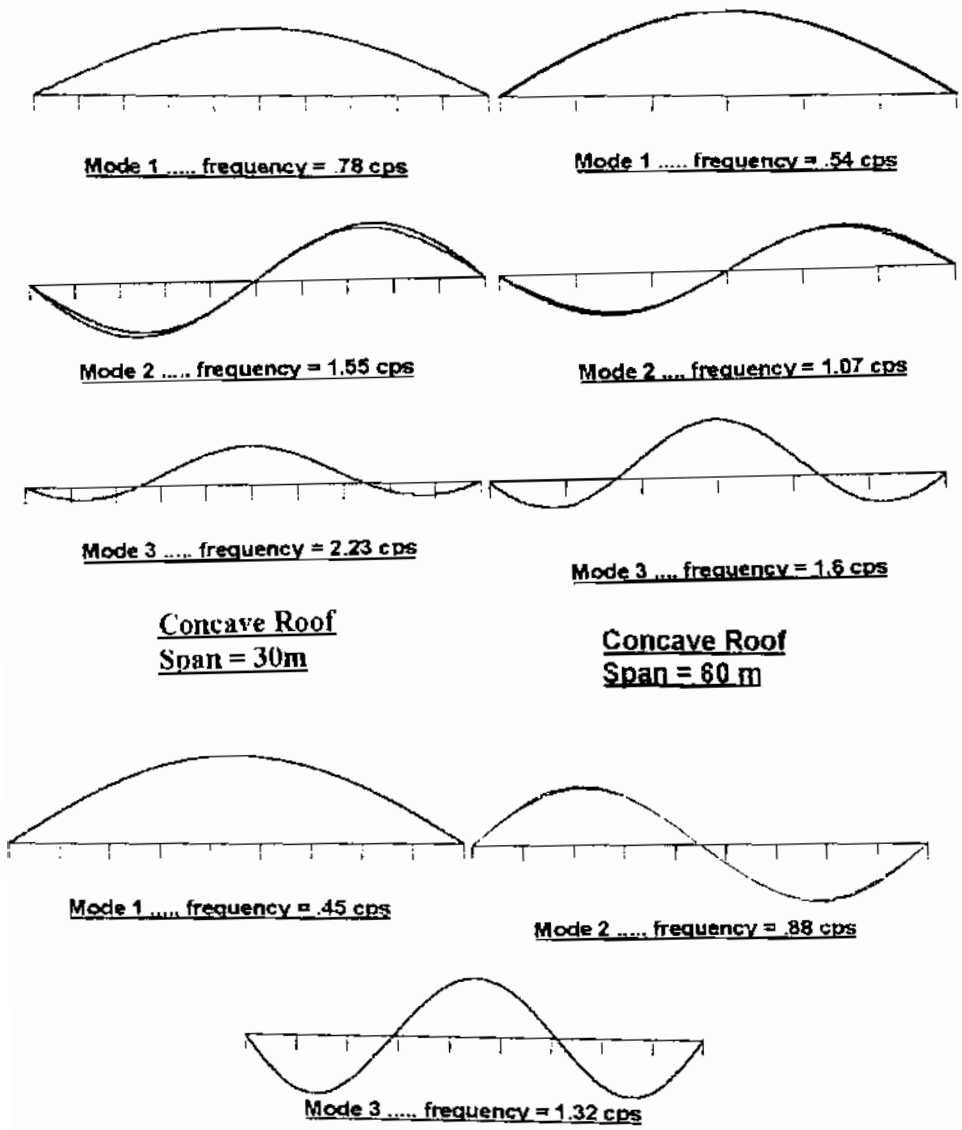

Fig. (21) First three mode shapes for concave roofs .

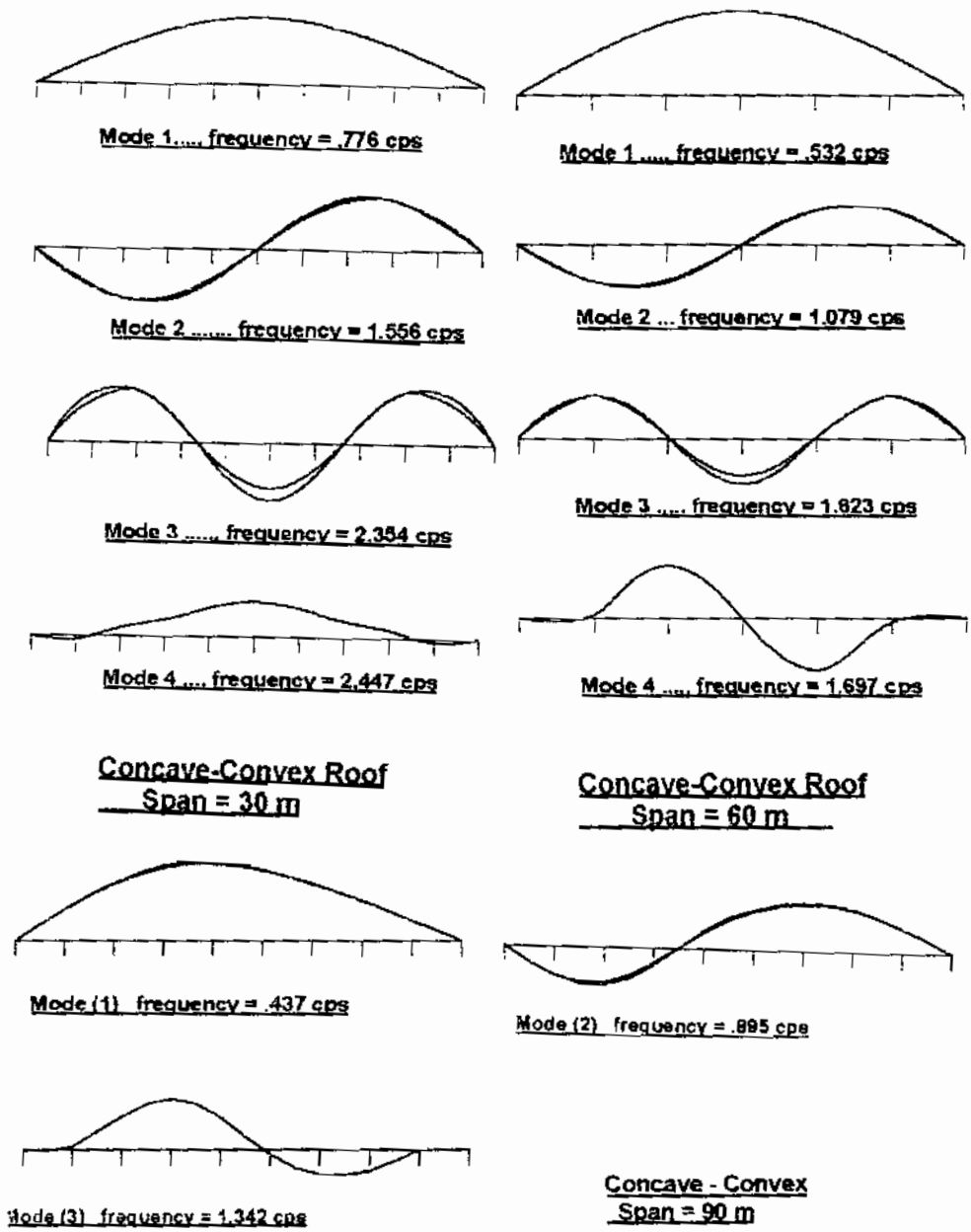

Fig. (22) First four mode shapes for concave-convex roofs .