
Solve the following questions

Question 1 (27 marks)

(a) Solve the initial value problem

$$\left(y\,e^{-y}\cos x\right)dx+\left(1+e^{-y}\sin x\right)dy=0,\qquad y(0)=1.$$

(b) Solve the differential equation $\cos x\ y'''+\cos x\ y'=\sec x$.

(c) A body of mass *2 kg* is thrown vertically upward with initial velocity of $v_0 = 100\ m/sec$. Assume that the air resistance is twice the velocity of the body.
Find **(i)** the equation of motion,
(ii) the velocity of the body at any time t,
(iii) the time at which the body reach its maximum height,
(iv) the maximum height.

Question 2 (28 marks)

(a) Solve the integro-differential equations

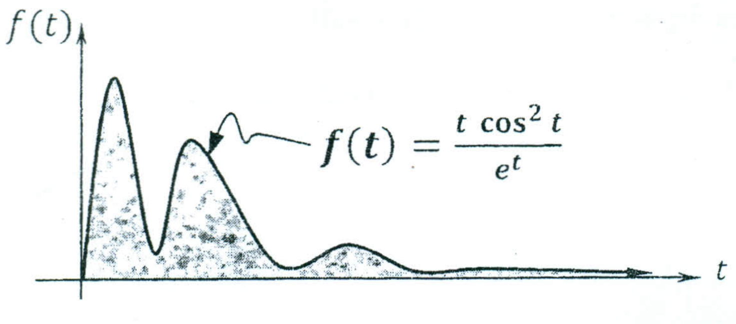
$$y'(t)=e^{t}-\int_0^t y(x)\cosh(t-x)\,dx\,,\quad y(0)=1.$$

(b) Find $L^{-1}\left\{e^{-\pi s}\times\ln\left(\frac{e^{\tan^{-1}s}}{\sqrt{s^2+1}}\right)\right\}$

(c) Find Laplace transform of $f(t)$, where

$$f(t)=\begin{cases} e^{t}, & 0\le t\le\pi\\ \sin t, & t>\pi\end{cases}$$

(d) Evaluate the following shaded area



3. (a) [5 pts] Suppose that the temperature T at a point (x, y) is $T = \frac{x^2}{2} + \frac{y^2}{2}$.

   i. Find the rate of change of T at $p(1,1)$ in the direction of $v = 3i + 4j$.

   ii. In what direction from p does T increase most rapidly?

   iii. What is the maximum rate of change of T at p?

   (b) [5pts] Find the maximum and minimum values of $f(x, y) = x^2 + 2y^2$ on the disk $x^2 + y^2 \leq 1$.

   (c) [5pts] Find the value of $\int_0^\infty x^2 e^{-ax^2} dx$ if $\int_0^\infty e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$.

   (d) Consider the function $f(x, y) = \sin xy$.

   i. [4 pts] Expand $f(x, y)$ in a Taylor series about $\left(\frac{\pi}{2}, 1\right)$.

   ii. [2 pts] Expand $f(x, y)$ in a Taylor series about $(0,0)$.

   iii. [2 pts] Apply the second derivative test to $f(x, y)$ at the points $\left(\frac{\pi}{2}, 1\right)$ and $(0,0)$.

   iv. [2 pts] Explain how one can use Taylor series obtained in i. and ii. to indicate the type of the extreme values of $f(x, y)$ at the given points.

   (e) [5pts] Use the chain rule to find u_{tt} at the point $x = 1, y = 2, t = 0$ if $u = r^2 s^2$, $r = y + x\cos t$, $s = x + y\sin t$.

4. (a) Consider $F = xj + z^2 k$, and the solid R bounded by $z = 3 - \sqrt{x^2 + y^2}$, $z = 0$.

   i. [4 pts] Verify Green's theorem for the vector field F and the lower surface of R.

   ii. [2 pts] Verify Stokes' theorem for the vector field F and the upper surface of R.

   iii. [9 pts] Verify divergence theorem for the vector field F and the solid R.

   (b) i. [5 pts] Evaluate $\int_0^{\pi^2} \int_{\sqrt{x}}^{\pi} \sin(y^3)\, dy dx$.

   ii. [5 pts] Find $\iint_S \left(\frac{\frac{3}{4}y^2}{\sqrt{4x^2 + \frac{y^2}{4} + 1}} + \sqrt{4z - \frac{3}{4}y^2 + 1} \right) dS$, $S: z = x^2 + \frac{y^2}{4}$, $z \leq 1$.

   iii. [5 pts] A solid E lies below $x^2 + y^2 + z^2 = z$ and above $z = \sqrt{x^2 + y^2}$. By using the spherical coordinates, find the volume of E.

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