



Answer all the following questions:

- Q.1** (A) Define the following terms and mention some examples for each one: Mathematical modeling, physical modeling, computational modeling, dynamic System, probabilistic model and dimensional analysis.
- (B) Show the steps of constructing a mathematical model then, draw the flow chart that describes the relation between physical and mathematical model.
- (C) What is the difference between the deterministic dynamic system and probabilistic dynamic system?
- (D) State the evolution rule of dynamic system.
- (E) Write a mathematical model to describe one of the following:
- The motion of a projectile of mass (m) and diameter (d) with launching speed (Vp) with launching angle (α).
 - Three phase induction motor.
- (F) What are the aims of physical modeling and What are its classes.

[Q.1 (50 mark)]

- Q.2** (A) Use Euler's method with step size $h = 0.1$ to Solve the following system:

$$\frac{dx}{dt} = -2tx + 3y^2, \quad \frac{dy}{dt} = -3x^2(1-y)$$

from $t = 0.1$ to $t = 0.3$ with the initial conditions are $x(0) = -1$ and $y(0) = 2$.

- (B) Find the solution of the following heat equation:

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2};$$

where $0 < x < h, t > 0, u(0,t) = u(h,0) = 0$ and $u(x,0) = f(x)$.

Q.2 (C) Write what you know about Riccati equation.

(D) When monitoring the variations of the price of a product, it is observed that a high price for the product in the market attracts more suppliers thus increasing the quantity of items sold. In particular, the price of above 50 dollars attracts more suppliers increasing the quantity sold for 0.5 items per dollar. However, increasing the quantity of the product tends to drive the price down. In particular, producing more than 200 items decreases the price by 0.1 dollars per item. If the current price is 30 dollars and 100 items are produced, write down a model that can be used to predict the price and quantity in subsequent years. Then find the steady states of the system and explain its behavior.

(E) Explain chaos theory, then what are the properties that a dynamical system must have to be classified as chaotic dynamic system?

(F) If $u_n = ku_{n-1}$; where k is constant and $n \geq 2$, prove that $u_n = k^{n-1}u_1$ then solve $u_n = 5u_{n-1}$; where $u_1 = 2$, and find u_9 .

Q.2 (50 mark)]

With my best wishes

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