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## CALCULATION OF ANOMALOUS MAGNETO- RESISTANCE AS A FUNCTION OF EFFECTIVE MASSES OF CHARGE CARRIERS

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It is well Known that in the crystalline semiconductors, which have cubic structures such as m 3 m groupe ( $\mathrm{Pbs}, \mathrm{Ge}$, and Si ), the measurements of magnetoresistivity $\Delta \delta / \delta_{\mathrm{o}}$ through any axis of symmetry are isotropic when the direction of $B$ was reversed 1-3). In this case the $\Delta \delta / \delta_{0}$ has a quadratic dependent on magnetic field strength. Also they found that at low electric and magnetic field, the $\Delta \delta / \delta_{0}$ at $\pm \mathrm{B}$ are Symmetric, when the weak current folws along the axis of symmetry 4), according to Onsager relation's ${ }^{5}$ ) ;

$$
B_{i j k}=-B_{i j k}, \delta_{i j k}=-\delta_{i j k}
$$

Generally Johnson ${ }^{6}$ ) found that the theoreticl calculations predicts a much samller magnetoresistive effect than, it actually observed in cubic semiconductors. This variation was attributed to (I) scattering of conduction electron by impurity ions as well as by lattice, and (II) conduction by both holes and electrons in high temperature semiconductors 7). This considerations proved that, the presence of impurity scattering decrease the magnitude of effects produced by B and thus increases the gap between the theoritical and experimental values.

In this work, the relation between the efective masses of charge carriers ( $\mathrm{m}^{*}$ ) and anomalous $\Delta \delta$ / $\delta$ o was studied, in the case of deviating the current vector from any cubic axes of crystal.

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The effect of weak $B$ on the $m^{*}$ of charge carriers during the intravalley scattering in one ellipsoid related to the set of ellipsoirs along any axis of cubic crystal was studied;

$$
\begin{equation*}
m^{*}=m^{*}(B) \tag{1}
\end{equation*}
$$

The population of current density ( j ) is the same in each ellipsoid having different $\mathrm{m}^{*}$. That is the charge carriers during the scattering remain on the same kind of energy ellipsoids, in K-space, this is the case with intravalley scattering;

$$
\begin{equation*}
j=\Sigma_{i} j^{(i)}=\left(\Sigma_{i} \delta^{(i)}\right) E \sim\left(\Sigma_{i} m_{i}^{*-1}\right) E \tag{2}
\end{equation*}
$$

Where $j(i)$ is the current contribution of the $i$-th energy ellipsoid, $\delta(i)$ is the contribution of the i -th ellipsoid to the conductivity tensor.

The resistivity tensor $\delta$ is proportional to the $\mathrm{m}^{*}$, hence the effective masses variation are strongly dependent on the actual direction of the motion of charge carriers in k-space;

$$
\begin{gather*}
\delta \frac{j m^{*} j}{j}  \tag{3}\\
\delta-m^{*} L^{-1}
\end{gather*}
$$

where $\mathrm{mJ}^{*}$ is the projection of the $\mathrm{m}^{*}$ in j direction, and L is mean free path. This means that the square of the radii of ellipsoids is related to the $\mathrm{m}^{*}$ as shown in Fig. 1. So the value of $m^{*}$ is changing due to the variation of the $m^{*}$ direction. When the direction of $\mathrm{m}^{*}$ makes an angle $\theta$ with the direction of quadratic rdius (r) of ellipsoid (Fig.2), $m_{L}{ }^{*}$ and $m_{t}{ }^{*}$ are the effective masses components along the major and minor

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axes of the ellipsoid,

$$
\begin{align*}
& \mathrm{m}^{*}(\theta) \sim \mathrm{r}^{2}(\theta) \\
& \mathrm{K}_{\mathrm{m}}=\mathrm{m}_{\mathrm{L}}^{*} / \mathrm{m}_{\mathrm{t}}^{*} \tag{4}
\end{align*}
$$

The ratio $K_{m}$ is related to the square radius ( $\left(r^{2}\right)$ of the ellipsoid as the following relation.

$$
\begin{align*}
\mathrm{r}^{2} & =\frac{\mathrm{K}_{\mathrm{m}}}{\cos ^{2} \theta+\mathrm{K}_{\mathrm{m}} \sin ^{2} \theta}  \tag{5}\\
& =\frac{1}{\mathrm{~K}_{\mathrm{m}}^{-1} \cos ^{2} \theta+\sin ^{2} \theta}
\end{align*}
$$

The effect of the magnetic field perpendicular to the current $j$ on the charge carriers can be written by the Lorentz force as;

$$
\begin{equation*}
F_{L}=e \mu / B \times E /=e \mu E \tag{6}
\end{equation*}
$$

where $\mu$ is the mobility. So the magnetic field modifies the direction of the charge carriers by an angle $\delta \theta$ (Fig.2) :

$$
\Delta \theta=\arctan \mu \mathrm{B}=\theta-\theta_{\mathrm{O}}
$$

Where, $\tan \theta=F_{L} / F_{E}+\frac{e \mu B E}{e E}=\mu B$ and $\theta \circ$ is the deviation angle of current from the axis of symmetry of ellipsoid, so from expressions 1 and 2 it can be seen that,
$\mathrm{m}^{*}(\mathrm{~B})=1 / \mathrm{K}_{\mathrm{m}}{ }^{-1} \cos ^{2}\left(\theta_{\mathrm{o}}+\arctan (\mu \mathrm{B})+\sin ^{2}\left[\theta_{\mathrm{o}}+\arctan (\mu \mathrm{B})\right]\right)$
The variation in the anomalous $\Delta \delta / \delta_{0}$ means that the change in the fraction

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function of mean free path $L(B)$ or the anomalous function through $m^{*}$, i.e,

$$
\begin{equation*}
\delta \sim L^{-1} \mu_{j} m^{*} \underline{\mu}_{j} \tag{8}
\end{equation*}
$$

where $\mu_{\mathrm{j}}$ is the unit vector as a function of vector direction and L is given in terms of $\mathrm{L}_{\mathrm{O}}$ as the following;

$$
\begin{equation*}
\mathrm{L}=\mathrm{L}_{\mathrm{o}}\left(1+\mu^{2} \mathrm{~B}^{2}\right)^{-1} \tag{9}
\end{equation*}
$$

$\mu \mathrm{B}$ is called intravalley scattering of charg carriers. This means that the conductivity or specific resistivty according to equations 3 and 8 . have a dependence on the direction, and the relative change of anomalous $\Delta \delta$ / $\delta_{0}$ can be determined from the following equation;

$$
\begin{align*}
& \frac{\Delta \delta}{\delta^{\circ}}=\frac{\delta(\mathrm{B})-\delta(\mathrm{o})}{\delta(\mathrm{O})}=\frac{\delta(\mathrm{B})}{\delta o}-1 \sim \mathrm{mj}(\mathrm{~B})-\mathrm{mj}^{*}(\mathrm{o}) \\
& \frac{\Delta \delta}{\delta^{\circ}}=\frac{(1+\mu 2 \mathrm{~B} 2)\left(\mathrm{K}_{\mathrm{m}}^{-1} \cos 2 \theta+\sin 2 \theta\right.}{\left(\mathrm{k}_{\mathrm{m}}^{-1} \cos ^{2} \theta \mathrm{o}+\sin ^{2} \theta \mathrm{o}\right.}-1 \tag{10}
\end{align*}
$$

From the abve equation, it can be seen that the $\Delta \delta / \delta_{0}$ depends on the direction of both magnetic field and square radii of ellipsoids, i.e. equation (10) give us the reasonble of appearance of anomalous $\Delta \delta / \delta_{0}$, in spite of, still the symmetry relations of the cubic crystal. If the current flows along the symmetry axis, the anomaly vanishes. The fundamental calculation of anomalous $\Delta \delta / \delta_{0}$ it is appeared when examin the resultant directions of $\pm F_{L}$ at $\pm B$ in the case of deviating the electric field (eE) vector from the axis of symmetry as shown in'Fig.3. On the other hand if the direction of $e E$ is coinside on the axis of symmetry, the induced $F_{L}$ at $+B$ are
is coinside on the axis of symmetry, the induced $\mathrm{F}_{\mathrm{L}}$ at +B are symmetrical as shown in Fig. 4.

The theretical calcuations of anomalous $\Delta \delta / \delta_{0}$ as afunction of both $\theta$ at different ratios of $\mathrm{k}_{\mathrm{m}}$ and $\mu \mathrm{B}$ at different $\theta$ are shown in Figs. 5 and 6. These calculations were done by using equations 7 and 10 with the aid of TI - 59 program. In Figs. 5 and 6 many types of anomalous $\Delta \delta / \delta_{0}$ were observed. These figures show that the transport ellipsoidal energy surfaces are proportional with the squar radii of ellipsoids. where $\mathrm{m}^{*} \mathrm{~L} / \mathrm{m}_{\mathrm{t}}$ reflects the anisotriopy of constant - energy surfaces $\mathrm{k}_{\mathrm{m}}=$ ( $m_{\mathrm{L}}{ }^{*} / \mathrm{m}_{\mathrm{t}}$ ). As is well known $\mathrm{m}^{*} \alpha \mathrm{r}^{2} \theta$, so the average ratio of ellipsoids radii lengths is given by

$$
\mathrm{r}_{\theta}=\sqrt{\mathrm{m}_{\mathrm{l}}^{*} / \mathrm{m}_{\mathrm{t}}^{*}}
$$

As shown in Fig. 5. the calculated $\Delta \delta / \delta_{0}$ at fidderent ratios of $\mathrm{k}_{\mathrm{m}}$ are positive at $\theta=0^{\circ}$ and then as $\theta$ increses $\Delta \delta / \delta_{0}$ decreases unitl reached to zero at different values of $\theta$. This means that the shape of surfaces energy is spherical found, i.e there are no anomalous $\Delta \delta / \delta_{0}$ due to the appearance of symmetry relations in the Brillouin Zone boundary ;
$\left(\Delta \delta / \delta_{0}\left(\theta_{\mathrm{n}}\right) \mathrm{k}_{\mathrm{m}} \cong 0=\right.$ spherical surfacs energy.

From Fig. 5 it can be seen that the effective periodical angle on the anomalous behaviours of calculated $\Delta \delta / \delta_{\mathrm{O}}$ is $\theta=180^{\circ}$, where $\Delta \delta / \delta_{\mathrm{O}}$ for Km are positive at $\theta=0^{\circ}$ and $360^{\circ}$, but at $\theta=180^{\circ}$ are completely negative;

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$$
\left.\left.\Delta \delta / \delta_{0} \quad\left(\theta_{\mathrm{n}}\right)\right) \mathrm{K}_{\mathrm{m}} \quad \# \quad \Delta \delta / \delta_{0}\left(\theta=180^{\circ}\right)\right) \mathrm{Km}
$$

It is well known that as thr temperature increases, the effective mass ratio decrses. So in fig. 5 at low ratios the anomalous $\Delta \delta / \delta_{0}$ is clear, but as $\mathrm{k}_{\mathrm{m}}$ increases, the anmalous $\Delta \delta / \delta_{0}$ tend to decrease as result of decrasing the temperature. This shows that the anomalous $\Delta \delta / \delta_{\mathrm{O}}$ is strongly dependent on the $\mathrm{m}^{*}$ variation.

The calculated $\Delta \delta / \delta_{\mathrm{O}}$ as a function of $\pm \mu \mathrm{B}$ at different $\theta$ (Fig. 6) shows that the anomalous behaviours of $\Delta \delta / \delta_{\mathrm{O}}$ is controlled y the value of $\theta$. wher the $(\mu \mathrm{B}-\Delta \delta /$ $\left.\delta_{0}\right) \theta_{\mathrm{n}}$ is not obeying onsager relation 's5) at $\pm \mu \mathrm{B}$;

$$
\Delta \delta / \delta_{\mathrm{O}} \mu \mathrm{~B}(\theta) \# \Delta \delta / \delta_{\mathrm{o}} \mu \mathrm{~B}\left(\theta+80^{\circ}\right) .
$$

Also the negative areas of $\Delta \delta / \delta_{0}$ for all $\theta$ at $\pm \mu \mathrm{B}$ are not symmetrical in spite of the parabolic characters of $\Delta \delta / \delta_{0}$ curves are still present. Therfrom it can be found that $50-50 \%$ considering anomalous and notmal components participation of the relative change of resistivity.

From the above mentioned it was found that these expressions taking into account of calculations of $\Delta \delta / \delta_{0}$ the influnce of the magnetic field on the effective mass, which isnot symmetry when the magnetic field was reversed $8-21$ )

$$
\mathrm{m}_{\mathrm{i}}^{*}(\mathrm{~B}) \# \mathrm{~m}_{\mathrm{j}}^{*}(-\mathrm{B})
$$

In the case of a current density vector oriented along a certain crystallographic directions, the effects mentioned abve are much weaker or they vanish.

fig. 1 : The relation between the radius of ellipsoid and the effective mass of charge carriers in cubic crystal.


Fig. 2 : Asymmetry directions among the Lorentz force ( $\mathrm{F}_{\mathrm{L}}$ ), deviated electric field (E) from the axis of symmetry, and the weak magnetic field ( $\mathrm{F}_{\mathrm{B}}$ ).






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fig. 5 : The calculated magnetoresistivity $\left(\Delta \delta / \delta_{0}\right)$ as a function of rotational angle ( $\theta$ ) at different values of effective mass ratio ( $\mathrm{K}_{\mathrm{m}}=\mathrm{m}_{1}^{*} / \mathrm{m}_{\mathrm{l}}{ }^{*}$ ).


Fig. 6: The calculated $\Delta \delta / \delta_{0}$ as a function of intravalley scattering of carriers ( $\mu \mathrm{B}$ ) at different $\theta$.

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## حساب المحاومه المغناطيسيه الفير متماثله كدالة فى الكت الهعاله لحواهل الشيهنات

كلية العلوم (بنين) - قسمى الطبيعه - فكيد إبراميم حجازیى الازهر

من المعلدم لدينا أنه توجد علاقات رياضيه بين معامل التوصيل والمقاومه لوحدة







 المقاومة المفناطيسية .

تم إختيار هذه الصيغه حيث رسمت العلاقة بين المقاومة المغناطيسية وزاوية

 كداله فى معامل التشتت الداخلى للشتحنات زوايا دوران مختلفه .

من هذه الحسابات ثبت وجود عدم تماثل مختلف الشده للمقاومة المغناطيسية يعتمد على الكتل الفعاله للشحنات وكذلك على قيمة زاوية اللوران .

