



Answer the following questions

(50 marks)

Question 1:

a) Solve the following differential equations:

i) $\frac{dy}{dx} = \frac{x(2\ln x + 1)}{\sin y + y \cos y}$

ii) $\frac{dy}{dx} = \frac{3x + 2y}{3x + 2y + 2}$

iii) $\cos^2 x \sin x \frac{dy}{dx} + (\cos^3 x)y = 1$

iv) $y - 2x \left(\frac{dy}{dx} \right) - \left(\frac{dy}{dx} \right)^2 = 0$

v) $(D^3 - 5D^2 + 7D - 3)y = e^{2x} \cosh x$

vi) $x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 20y = (x+1)^2$

b) Solve the initial value problem by using Laplace transform:

$y'' - 2y' - y = e^{2t} - e^t$ $y(0) = 1, y'(0) = 3$

c) Find the center of the mass of a thin plate bounded by the parabola $y = 6x - x^2$ and the straight line $y = x$, given that the density as $\rho(x,y) = 1$

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d) Evaluate the following integrals:

i) $\iiint_D 2z e^{-x^2} dy dx dz$ where D is given by $D: 0 \leq y \leq x, 0 \leq x \leq 1, 1 \leq z \leq 4$

ii) $\iint_D \frac{2}{(x+1)(y+1)} dx dy$ where D is given by $D: 0 \leq x \leq y, 0 \leq y \leq 4$

Question 2

(50 marks)

a) Find the Laplace transform of the functions:

i) $f(t) = (1 + e^{2t})^2 \sin 2t \cos 2t$

ii) $f(t) = e^{-t} t \sin 2t + \cos^2 t$

b) Find the inverse Laplace transform of the functions:

$$i) F(s) = \ln \frac{s^2 + 9}{s^2 + 1}$$

$$ii) F(s) = \frac{-2s^2 - 3s - 2}{s(s+1)^2}$$

$$iii) F(s) = \frac{s-3}{(s-\sqrt{3})(s+\sqrt{3})}$$

c) Find $L^{-1}\{F(s)\}$ by using the Convolution theorem:

$$F(s) = \frac{1}{(s+1)(s+2)}$$

d) Test the convergence of the following series:

$$i) \sum \frac{4n^2 - n - 3}{n^3 + 2n}$$

$$ii) \sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^3 + 1}$$

e) Find the interval of convergence : $\sum \frac{n(x-1)^n}{2^n(3n-1)}$

f) Draw the periodic function and then find its Laplace transform

$$f(t) = e^t \quad 0 < t < 2\pi$$

With my best wishes

Dr. Eng. Rizk Masoud

This exam measures the following ILOs											
Question Number	Q 1-a	Q 1-a	Q 1-b	Q 5-c	Q4-c	Q5-b			Q3-b	Q 5-a	
	Q 2-a	Q 2-b	Q 2-c	Q3-c	Q3-a	Q4-b			Q4-a		
Skills	Knowledge & understanding Skills			Intellectual Skills			Professional Skills				