Engineering Math. & Phys. Dept.
Faculty of Engineering
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Numerical Analysis
Math. PGS –Year 1
11-9-2013

Instructions:

- Books or any notes are **NOT** allowed
- You must justify your answers for full credit
- Exam contain FOUR questions , and do **ALL** problems
- Time limit THREE hours

Question 1

1) The cubic equation $x^3 - 1.70x^2 - 11.44x + 23.66 = 0$ has a double root. Find this root correct to five decimal places by using the Newton-Raphson method.

2) Suppose the simple iteration method is used to find the point of intersection of the two curves $y_1 = x^3$ and $y_2 = e^x$. Suggest an iteration form $x = \varphi(x)$ that will converge to the rquired point . (Don't iterate to find approximations of the point)

3) Assuming that the equation $f(x) = 0$ has a root in the interval $(-3,4)$ and it is required to find this root within an absolute error of 10^{-8} using the bisection method. Determine the minimum number of iterations required to guarantee that accuracy.

4) Consider the root-finding problem $g(x) = x^2 - 5 = 0$. Let $x_0 = 2.00$, $x_1 = 2.10$, use the Secant method to find x_2.

Question 2

1) Determine the constants $a, b,$ and c that make the function

$$S(x) = \begin{cases} S_0(x) = a + b\,x + 1.5\,x^2 - 0.5x^3, & 1 \le x \le 2 \\ S_1(x) = -2.5 + 11.5x - 0.5x^2 + c\,x^3, & 2 \le x \le 3 \end{cases}$$

a cubic spline. Is it a natural cubic spline?Why or why not?

2) Prove that if $g(x)$ interpolates the function $f(x)$ at $x_0, x_1, \ldots, x_{n-1}$ and if $h(x)$ interpolates $f(x)$ at $x_1, x_2, \ldots, x_n$, then the function

$$q(x) = g(x) + \frac{x_0 - x}{x_n - x_0}[g(x) - h(x)]$$

interpolates $f(x)$ at $x_0, x_1, \ldots, x_n$.

Question 3

1) Given the linear agebraic system $\boldsymbol{A}_{2\times 2}\, \boldsymbol{X}_{2\times 1} = \boldsymbol{B}_{2\times 1}$, where the coefficient matrix $\boldsymbol{A} = \begin{bmatrix} 4 & 1 \\ 2 & 5 \end{bmatrix}$, and the two column vectors $\boldsymbol{X} = \begin{bmatrix} x \\ y \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$. This system can be solved iteratively using the iteration process

$$\boldsymbol{X}^{(k+1)} = \boldsymbol{T}\, \boldsymbol{X}^{(k)} + \boldsymbol{C}, \quad k = 0,1,2, \ldots,$$

where $\boldsymbol{T}$ is the iteration matrix . For this problem, use the marix norm subordinate to the infinity norm whenever you need to compute a norm. Then answer " **True** " **or** " **False** " , with explanation , to the following statements:

a) $\boldsymbol{A}$ is not a strictly diagonally dominant matrix .

b) $\boldsymbol{AX} = \boldsymbol{B}$ is an *ill-conditioned* system.

c) The Jacobi and Gauss-Seidel iteration matrices , $\boldsymbol{T_J}$ and $\boldsymbol{T_{GS}}$ respectively, have equal norms.

d) The Gauss-Seidel method converges twice as fast as Jacobi method.

2) Use the Cholesky factorization method to determine the lower triangular matrix $\boldsymbol{L}$ such that $\boldsymbol{A} = \boldsymbol{L}\,\boldsymbol{L}^T$, where

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 2 & 1 \\ 1 & 2 & 1 & 6 \end{bmatrix}$$

3) Consider the following linear system

$$\begin{bmatrix} -1 & 6 & 2 \\ 4 & 2 & 0 \\ 2 & 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 16 \\ 20 \end{bmatrix}$$

Determine the first two itrates using Jacobi method, and only the first iterate for Gauss-Seidel and the SOR (with a relaxation parameter $\omega = 1.25$) methods . Use the zero vector as an intial approximation for the solution.

Question 4

1) Use Taylor's method of order 2 to the initial IVP

$$y' - y = 1 - x^2, \qquad y(0) = 0.5,$$

to find y at $x = 0.2$ and 0.4. Then use Adams-Bashforth three-step method to find $y(0.6)$, use a step size $h = 0.2$.

Note: Adams-Bashforth three-step explicit formula is

$$y_{n+1} = y_n + \frac{h}{12}[23\, y_n' - 16\, y_{n-1}' + 5\, y_{n-2}'], \qquad n = 2,3, \ldots$$

2) Consider the second order IVP

$$\frac{d^2y}{dx^2} + y = 0, \qquad y(0) = 1, \qquad y'(0) = 0$$

a) Rewrite the problem as a system of first order equations, with initial conditions

b) Then use RK4 method with step size $h = 1$, to approximate the the solution $y(1)$. Compute the absolute relative error at $x = 1$.

End of Exam, Good luck