ERJ
Engineering Research Journal
Faculty of Engineering
Minoufiya University

STRESS ANALYSIS OF FGM FOR I-SECTION BEAMS

A. El Megharbel and H. Abd El-Hafez

Suez Canal University, Faculty of Engineering, Production Engineering and Mechanical Design Department, Port Said 42523, Egypt

ABSTRACT

A theoretical analysis for functionally gradient materials (FGMs) of I-section beams is introduced in the present study. Analytical methods are set in the form of equations, in order to provide a method for predicting the normal stress distribution of the FGMs beam under axial load and bending moment; using the effective principal axes. Considering the elastic modulus to be a power function, the effect of the non-homogeneity parameter on the distribution of the normal stress, as well as on the position of the neutral axis along the beam height, is discussed. The results obtained show that the non-homogeneity parameter has a great effect on the normal stress distribution and on the position of the neutral axis.

يهتم هذا البحث بدراسة الإجهادات الواقعة على عارضة لها مقطع على شكل (I) و من مواد غير متجانسة الخواص الميكانيكية (FGMs) وذلك خلال دراسة تحليلية في صورة معادلات رياضية. لذلك أهتم البحث بدراسة الإجهاد العمودي على العارضة و ذلك تحت تأثير حمل محوري و عزم انحناء مع استخدام المحور الأساسي الفعال خلال البحث. و تم اعتبار معامل المرونة (E) في هذه الدراسة متغير من خلال دالة أسية. كما أهتم البحث بدراسة معامل عدم التجانس على توزيع الإجهاد العمودي و كذلك موضع محور التعادل بالنسبة لإرتفاع العارضة. وأظهرت نتائج البحث التأثير الواضح لمعامل عدم التجانس على الإجهاد العمودي و كذلك على موضع محور التعادل لعارضة من مواد غير متجانسة الخواص الميكانيكية (FGMs).

***Keywords*:** Bending moment, Functionally gradient materials, I-Section beam, Neutral axis, Normal stress

1. INTRODUCTION

Functionally gradient materials (FGMs) have been widely used in modern industries including aviation and aerospace, mechanical, transportation, energy, electronic, chemical, biomedical and civil engineering. The functionally gradient materials (FGMs) are a class of advanced composites characterized by the gradual variation in composition, microstructure and material properties.

The study of the functionally gradient material (FGM) problems has already been tackled by many investigators [1-3].

In thermal problems, Dai et al. [4] have introduced a method of lines to solve the temperature field of FGM while Hosseini [5] has studied the coupled thermoelasticity behaviour of functionally graded thick hollow cylinders. An elastic–plastic stress analysis of FGM plates under a transient thermal loading cycle that consists of heating followed by cooling is carried out by Nemat-Alla et al. [6]. Whereas, the mechanical and thermal stresses in a functionally graded rotating disk with variable thickness due to radially symmetry loads are discussed in [7].

The static and kinematic shakedown of a plate made of functionally graded materials (FGMs) is analyzed by Peng et al. [8]. Theotokoglou and Stampouloglou [9] have discussed the elastostatic problem of a hollow nonhomogeneous cylindrical tube under internal loading, analytically.

The effect of the weak-discontinuity and micro-discontinuity on the dynamic fracture behaviour of the coating–substrate interface are discussed in [10]. However, Li et al. [11] have studied the transient response of FGMs with a finite crack under anti-plane shear impact. Moreover, the anti-plane impact fracture analysis was performed for a weak-discontinuous interface in a symmetrical functionally gradient composite strip by Li et al. [12].

Ozturk and Erdogan [13] are calculated the stress intensity factors as functions of the nonhomogeneity parameter for various loading conditions. The anti-plane fracture analysis for a functionally graded coating–substrate system with a crack inclined to the weak/micro-discontinuous interface was performed by Li and Lee [14].

The stress distribution in rotating two composite structures of functionally graded solid disks is discussed in [15], while the characterization of laser powder deposited Ti–TiC have studied by Zhang et al. [16].

The mechanical property gradient of the examined material, such as the elastic modulus or the shear modulus in the pervious studies was assumed to be some certain function such as a linear function, a power function, an exponential function or even a hyperbolic function.

The most wide-used in engineering components is the beam, which mechanical behaviour has an important value for studying. Thus, the aim of this paper is to present a theoretical analysis for the FGMs of I-section beams under axial load and bending moment. In addition, studying the effect of the non-homogeneity parameter on the normal stress distributions is also one of the targets of this research.

2. THEORETICAL ANALYSIS

The mechanical model of the FGMs beam, of I-section considered in this study is shown in Fig. 1. In which H, B, h and b are the height, width of the flange, thickness of the flange and width of the web of the beam, respectively. The beam is subjected to an axial load N_x and a bending moment M. The elastic modulus is assumed to vary continuously in the direction of the height only. Thus the elastic non-homogeneity is assumed to be in the following exponential form [3]

$$E(Y) = E_o e^{mY} \tag{1}$$

where E_o is the elastic modulus at the origin point O; (m) is the non-homogeneity parameters.

For the above-mentioned FGMs beam, the traditional centroidal principal axes are no longer suitable for the analysis of the normal stress. The following analysis is based on the effective principal axes through a new coordinate system *oxy* with the point o(0, aH) in OXY as a new origin where

$$y = Y - aH \tag{2}$$

is the effective vertical principal axis, whereas (a) is the position parameter of the effective principal axis. The value of the parameter (a) is related to the non-homogenous parameter (m) which will be determined later on. In the new coordinate *oxy,* the elastic modulus of the beam may be expressed as follows:

$$E(y) = E_1 e^{my} \tag{3}$$

where E_1 is the elastic modulus at the point o(0,aH) and may be expressed as

$$E_1 = E_o e^{amH} \tag{4}$$

The normal stress subjected to the FGMs beam may be expressed as:

$$\sigma_x = E_1 e^{my} \varepsilon_x \tag{5}$$

The beam is assumed to be stressed by an axial load (N_x) and a bending moment (M). Assuming the linear strain produced by the axial force are the same for the different points in the cross section. Moreover, the curvature (ρ) produced by the bending moment is the same too. Therefore, the equation of compatibility for the beam is expressed as follows:

$$\varepsilon_x = \varepsilon_o + \frac{y}{\rho} \tag{6}$$

The previous equation assumed that the beam is loaded by the axial force and the bending at the same time. ε_o is the homogenous axial linear strain in the cross section, ρ is the radius of curvature of the beam.

Thus the equilibrium equations of the above-mentioned beam, under axial load and bending moment, may be expressed as [17]:

$$N_x = \int_A \sigma_x dA \qquad M = \int_A \sigma_x y dA \tag{7}$$

Substituting in the previous equation with Eqs.(5-6), yields to:

$$\begin{aligned} N_x = & \int_{-B/2}^{B/2} \int_{(-aH)}^{(-aH+h)} E_1 \left(\varepsilon_o + \frac{y}{\rho} \right) e^{my} dy dz \\ & + \int_{-B/2}^{B/2} \int_{(-aH+h)}^{(H-aH-h)} E_1 \left(\varepsilon_o + \frac{y}{\rho} \right) e^{my} dy dz \\ & + \int_{-B/2}^{B/2} \int_{(H-aH-h)}^{(H-aH)} E_1 \left(\varepsilon_o + \frac{y}{\rho} \right) e^{my} dy dz \end{aligned} \tag{8}$$

$$\begin{aligned} M = & \int_{-B/2}^{B/2} \int_{(-aH)}^{(-aH+h)} E_1 \left(\varepsilon_o y + \frac{y^2}{\rho} \right) e^{my} dy dz \\ & + \int_{-B/2}^{B/2} \int_{(-aH+h)}^{(H-aH-h)} E_1 \left(\varepsilon_o y + \frac{y^2}{\rho} \right) e^{my} dy dz \\ & + \int_{-B/2}^{B/2} \int_{(H-aH-h)}^{(H-aH)} E_1 \left(\varepsilon_o y + \frac{y^2}{\rho} \right) e^{my} dy dz \end{aligned} \tag{9}$$

As in [1], assuming that:

$$\int_{(-aH)}^{(-aH+h)} y e^{my} dy + \int_{(-aH+h)}^{(H-aH-h)} y e^{my} dy + \int_{(H-aH-h)}^{(H-aH)} y e^{my} dy = 0 \tag{10}$$

Using the previous assumption into Eqs (8-9) the following is obtained:

$$N_x = \varepsilon_o E_1 A' \tag{11}$$

$$M = \frac{E_1 I'}{\rho} \tag{12}$$

Where

$$A' = \frac{B\left[\left(e^{m\gamma H} - 1\right)\left(e^{mH} + e^{m\gamma H}\right) + \delta e^{m\gamma H}\left(e^{m(1-\gamma)H} - e^{m\gamma H}\right)\right]}{m e^{m(a+\gamma)H}} \tag{13}$$

$$I' = \frac{BH^2}{m e^{amH}} \cdot C_1 \tag{14}$$

$$\begin{aligned} C_1 = & \left(1-a\right)^2 e^{mH} + \left(1-\delta\right)\left(a-\gamma\right)^2 e^{m\gamma H} \\ & - \left(1-\delta\right)\left(1-a-\gamma\right)^2 e^{m(1-\gamma)H} - a^2 \end{aligned} \tag{15}$$

$$\delta = \frac{b}{B}, \qquad \gamma = \frac{h}{H} \tag{16}$$

In which δ and γ are non-dimensional parameters, moreover; the position parameters (a) of principal axes y may be determined from Eq. (10) as

$$a = \frac{\left[1 + e^{mH}(mH - 1)\right]}{mH\left(e^{mH} - 1\right)} \tag{17}$$

The normal stress in the beam may be obtained from Eqs. (5, 6, 11, 12) as

$$\sigma_x = \sigma_x^N + \sigma_x^M \tag{18}$$

where σ_x^N is the normal stress due to the axial force N_x (Eq. 4, 11) and σ_x^M is the normal stress due to the bending moment M (Eq. 4, 12), therefore

$$\sigma_x^N(y) = \frac{N_x}{A'} e^{my} \tag{19}$$

$$\sigma_x^M(y) = \frac{M}{I'}(Y - aH)e^{my} \tag{20}$$

It could be assumed that, the normal stress σ_x (Eq. 18) has a zero value at the point of neutral axis, therefore, the neutral axis equation may be determined as

$$Y = aH - \frac{I'}{A'}\frac{N_x}{M} \tag{21}$$

3. RESULTS AND DISCUSSION

The mathematical models for the normal stress of FGMs for I-sections are introduced in the previous section. The effect of the non-homogeneity parameter on the distribution of the normal stress and on the position of the neutral axis is substantial. The distribution of the normal stress along the height of the FGMs beam is displayed in Figs. 2-7. The effect of the absolute value of the non-homogeneity parameter (m) on the normal stress σ_x^N is illustrated in Figs. 2-3. When the beam is loaded by the axial force N_x only, it was noticed that the normal stress σ_x^N in the less hard region of the cross section is much lower than that in the harder region. Whereas, with the increase of the absolute value of (m), the σ_x^N in the less hard region decreases, however, that in the harder region increases. In Figs. 4-5, the effect of the non-homogeneity parameter (m) on the normal stress σ_x^M distribution along the beam height is shown when the FGMs beam is stressed by bending moment (M) only. As the absolute value of the non-homogeneity parameter (m) increases, the σ_x^M in the less hard region decreases, whereas, that in the harder region increases. The effect of the non-homogeneity parameter (m) on the normal stress σ_x distribution along the beam height is shown in Figs. 6-7. These results correspond to the FGMs I-beam stressed by axial load (N) and bending moment (M) at the same time. The results demonstrate similar general trends between Figs 4-6 for m<0 and Figs. 5-7 for m>0; except that the values are different due to the effect of the normal force (N). It can be indicated from Figs. 6-7 that, the value of (m) has almost no effect on the normal stress along the beam height at two points (0.08, 0.4) for m<0 and (0.14, 0.51) for m>0. It can be concluded from Figs. 2-7 that if the absolute value of the non-homogeneity parameter (m) is 0.01, which is almost a zero value, the normal stress in the FGMs beam is varying straightly with the beam height (H), which means that the beam is related to a homogenous material. As the non-homogeneity parameter (m) increases, the normal stress varies non-linearly along the beam height.

In Figs 8-9, the effect of non-homogeneity parameter (m) on the position of the neutral axis along the beam height is displayed for FGMs beam under axial load and bending. As the absolute value of the non-homogeneity parameter (m) increases, the position of the neutral axis transfers towards the hard region. Moreover, it can be found that, the position of the neutral axis within the beam height (0.6) is about 0.217 for m=|0.01| which relate to m≈0 (i.e. the homogenous beam). This demonstrates the method of the effective principal axes for the FGMs beam.

4. CONCLUSION

The stress analysis of the functionally gradient material (FGM) for I-section beam has been carried out to investigate the effect of the non-homogeneity parameter (m) on the normal stress distribution along the beam height with different loading cases. The beam was subjected to an axial load with a bending moment and it was assumed that the elastic modulus to be a power functions. The effect of the non-homogeneity parameter (m) on the position of the neutral axis has also been examined. The present work could be applied to any FGMs of I-section, which can provide reference for the design of the functionally gradient materials beams in engineering.

5. REFERENCES

[1] Y.D. Li, H. Zhang, N. Zhang, Y. Dai, "Stress analysis of functionally gradient beam using effective principal axes", Int. J. Mech. Mater. Des., pp. 157-164, (2005).

[2] J. Zhen-yi, "The general solution for axial symmetrical bending of nonhomogenous circular plates resting on an elastic foundation", Applied Mathematics and Mechanics Vol. 12, No. 9, pp. 871-879 (1991).

[3] N. Konda, F. Erdogan, "The mixed-mode crack problem in a nonhomogeneous elastic medium", Engng. Fract. Mech. 47(3), 533–545 (1994).

[4] Y. Dai, W .Tan, Q .Sun, Y.D. Li, "Effect of different thermal conductivity functions on temperature felids in FGM", Journal of Materials Processing Technology 187~188, pp. 212–214 (2007).

[5] S. M. Hosseini, "Coupled thermoelasticity and second sound infinite length functionally graded thick hollow cylinders (without energy dissipation)", Materials and Design 30, pp. 2011–2023(2009).

[6] M. Nemat-Alla , K. Ahmed, I. Hassab-Allah, "Elastic–plastic analysis of two-dimensional functionally graded materials under thermal loading", International Journal of Solids and Structures 46 , pp. 2774–2786 (2009).

[7] M. Bayat, M. Saleem, B.B. Sahari, A.M.S. Hamouda, E. Mahdi, "Mechanical and thermal stresses in a functionally graded rotating disk with variable thickness due to radially symmetry loads", International Journal of Pressure Vessels and Piping 86, pp. 357–372 (2009).

[8] X. Peng, H. Zheng, N. H u, C. Fang, "Static and kinematic shakedown analysis of FG plate subjected to constant mechanical load and cyclically varying temperature change", Composite Structures 91, pp. 212–221(2009)

[9] E.E. Theotokoglou, I.H. Stampouloglou, "The radially nonhomogeneous elastic axisymmentric problem", International Journal of Solids and Structures 45, pp. 6535–6552 (2008).

[10] Y. D. Li, B. Jia, N. Zhang, L. Q. Tang, Y. Dai, "Dynamic stress intensity factor of the weak/micro-discontinuous interface crack of a FGM coating", International Journal of Solids and Structures 43, pp. 4795–4809 (2006)

[11] C. Li, G. J. Weng, Z. Duan, Z. Zou, "Dynamic stress intensity factor of a functionally graded material under antiplane shear loading", Acta Mechanica 149, pp.1-10 (2001).

[12] Y. D. Li, K. Y. Lee, "Fracture analysis of a weak-discontinuous interface in a symmetrical functionally gradient composite strip loaded by anti-plane impact", Arch Appl Mech 78, pp. 855–866 (2008).

[13] M. Ozturk, F. Erdogan, "The axisymmetric crack problem in a nonhomogeneous medium", Lehigh University, final project report (1992).

[14] Y. D. Li, K. Y. Lee, "Effects of the weak/micro-discontinuity of interface on the fracture behavior of a functionally graded coating with an inclined crack", Arch Appl. Mech. (2008).

[15] A.M. Zenkour , "Stress distribution in rotating composite structures of functionally graded solid disks", journal of materials processing technology 209, pp. 3511–3517 (2009).

[16] Y. Zhang, Z. Wei, L. Shi, M. Xi, "Characterization of laser powder deposited Ti–TiC composites and functional gradient materials", journal of materials processing technology 206, pp. 438–444 (2008).

[17] A. El Megharbel, G.A. El Nasser, Aly El Domiaty, "Bending of tube and section made of strain-hardening materials", journal of materials processing technology 203, pp. 372–380(2008).

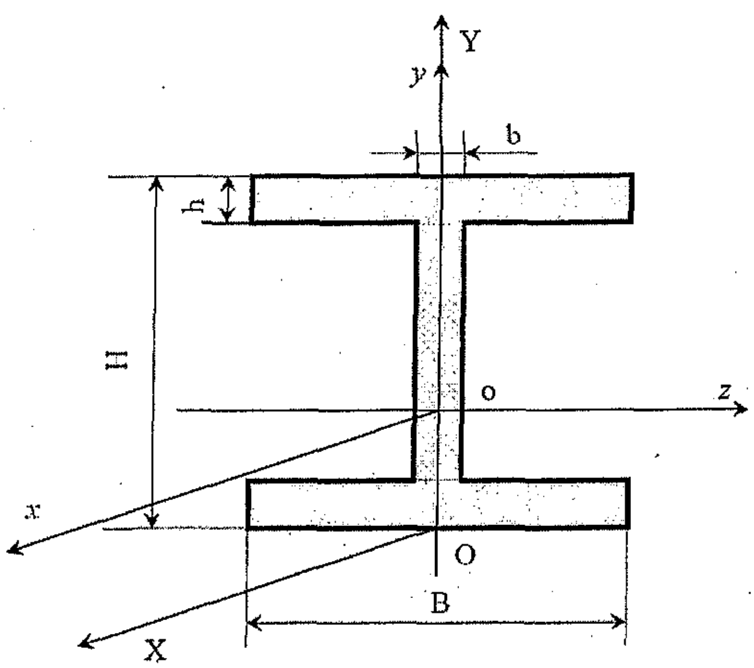

Fig. 1 The mechanical model of FGMs beam of I-section

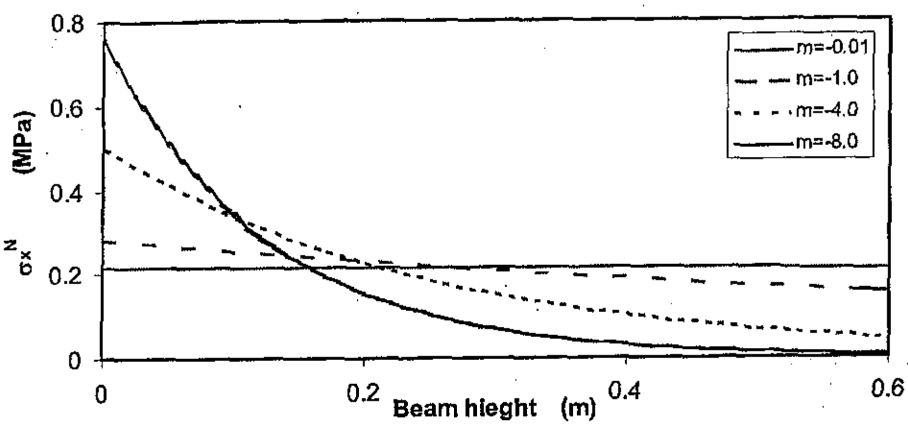

Fig. 2 The σ_x^N distributions along the FGMs beam height with different (m) values (H=0.6m, B=0.3m, δ=γ=0.2, N_x=20kN, M=0)

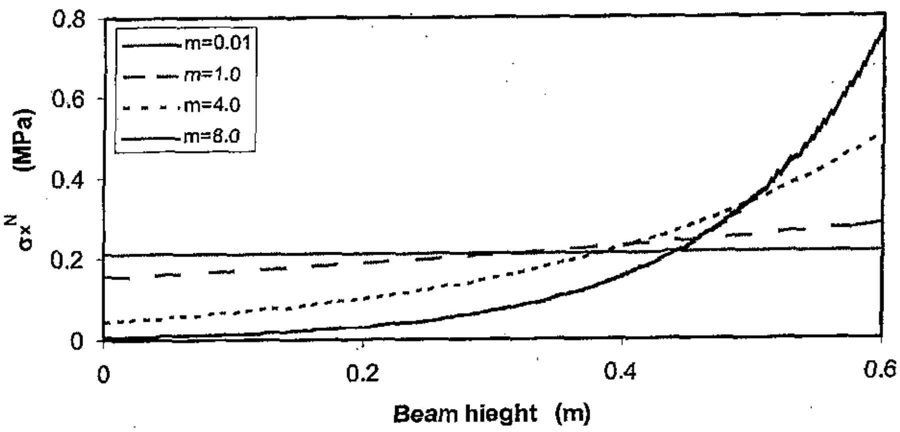

Fig. 3 The σ_x^N distributions along the FGMs beam height with different (m) values (H=0.6m, B=0.3m, δ=γ=0.2, N_x=20kN, M=0)

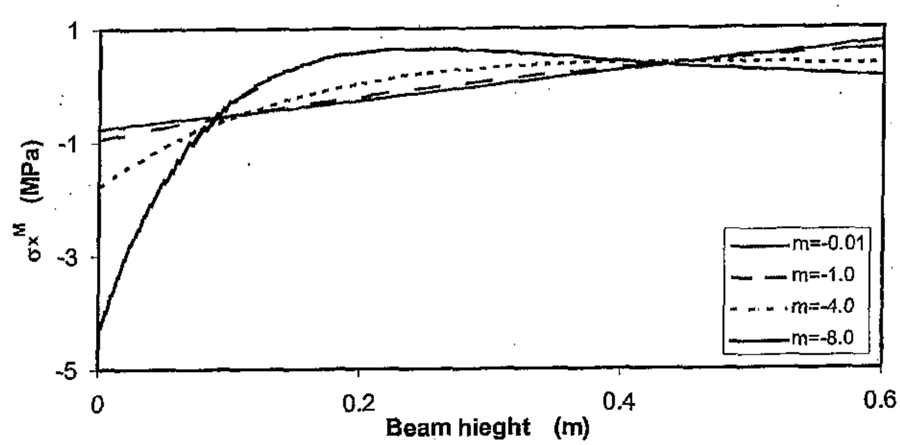

Fig. 4 The σ_x^M distributions along the FGMs beam height with different (m) values (H=0.6m, B=0.3m, δ=γ=0.2, N_x=0, M=20kN.m)

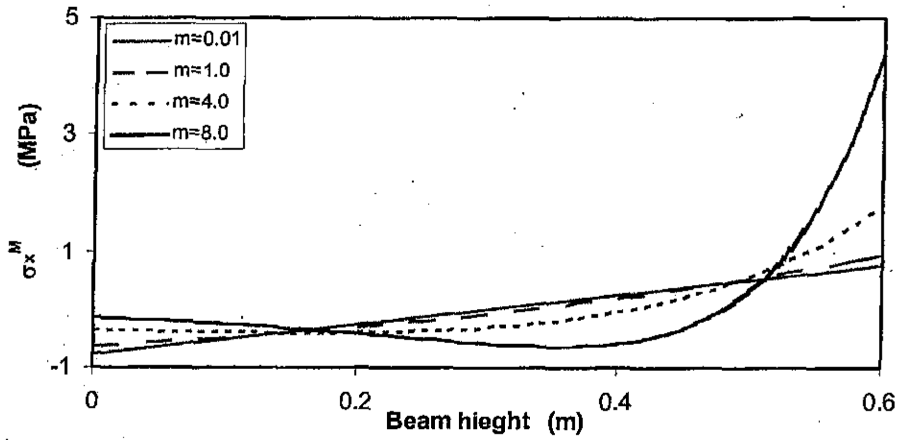

Fig. 5 The σ_x^M distributions along the FGMs beam height with different (m) values (H=0.6m, B=0.3m, δ=γ=0.2, N_x=0, M=20kN.m)

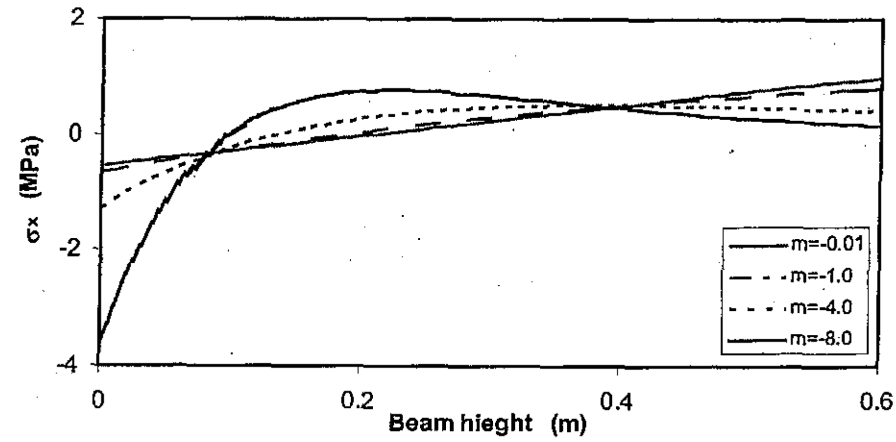

Fig. 6 The σ_x distributions along the FGMs beam height with different (m) values (H=0.6m, B=0.3m, δ=γ=0.2, N_x=20kN, M=20kN.m)

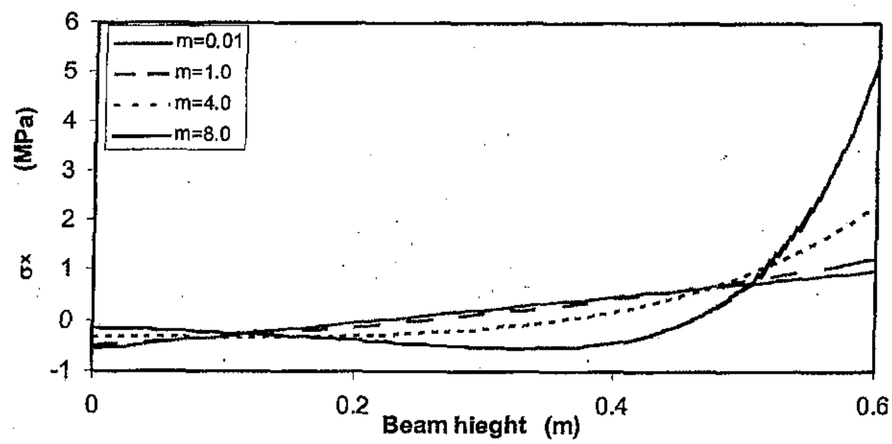

Fig. 7 The σ_x distributions along the FGMs beam height with different (m) values (H=0.6m, B=0.3m, δ=γ=0.2, N_x=20kN, M=20kN.m)

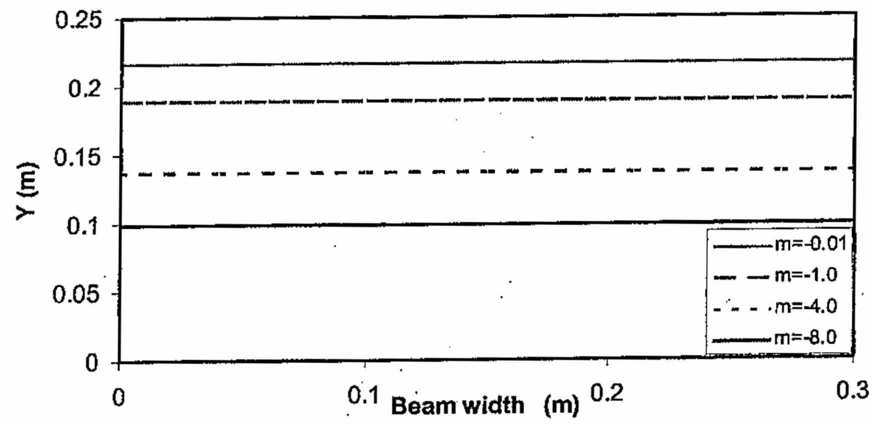

Fig. 8 The relationship between the position of neutral axis in the cross section of the FGMs beam and the beam width (B) with different (m) values (H=0.6m, B=0.3m, δ=γ=0.2, N_x=20kN, M=20kN.m)

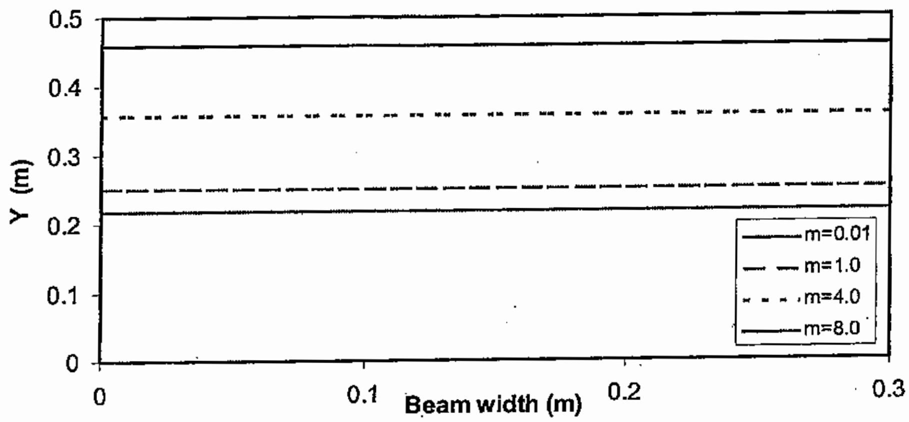

Fig. 9 The relationship between the position of neutral axis in the cross section of the FGMs beam and the beam width (B) with different (m) values (H=0.6m, B=0.3m, δ=γ=0.2, N_x=20kN, M=20kN.m)