

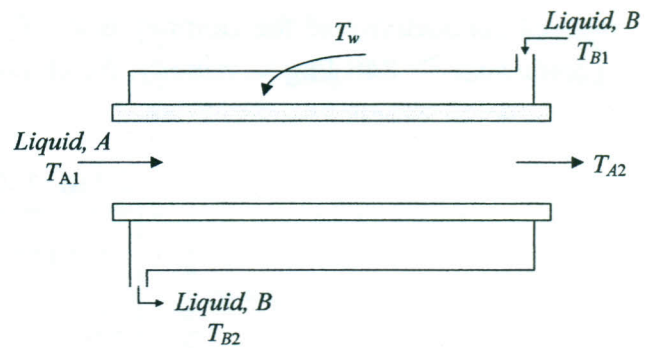


**Answer the following questions, and assume any missing data:**

**Q1. Fundamentals [15 mark]**

a) State the steps of the general procedure for building up a mathematical model. [5 mark]

b) Develop the lumped model (state the assumptions clearly) and make the degree of freedom analysis for a system of a tube heat exchanger shown in the figure. Liquid A of density  $\rho_A$  and specific heat  $c_{pA}$  is flowing through the inner tube and is being heated from temperature  $T_{A1}$  to  $T_{A2}$  by liquid B of density  $\rho_B$  and specific heat  $c_{pB}$  flowing counter-currently around the tube. The temperature of liquid B decreases from  $T_{B1}$  to  $T_{B2}$ .



[10 mark]

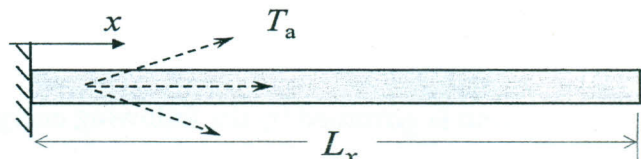
**Q2 ODE [20 mark]**

The conservation of heat can be used to develop a heat balance for a long, thin rod. If the rod has a uniform heat generation and is not insulated along its length and the system is at steady state. The equation that results is:

$$\frac{d^2T}{dx^2} = h(T - T_a) + S$$

with boundary conditions

$$T(0) = T_0, \quad \text{and} \quad \left. \frac{dT}{dx} \right|_{x=0} = q$$



where  $h$  is a heat transfer coefficient ( $\text{m}^{-2}$ ) that parameterizes the rate of heat dissipation to the surrounding air,  $T_a$  is the temperature of the surrounding air ( $^{\circ}\text{C}$ ), and  $S$  is the heat source ( $^{\circ}\text{C}/\text{m}^2$ ).

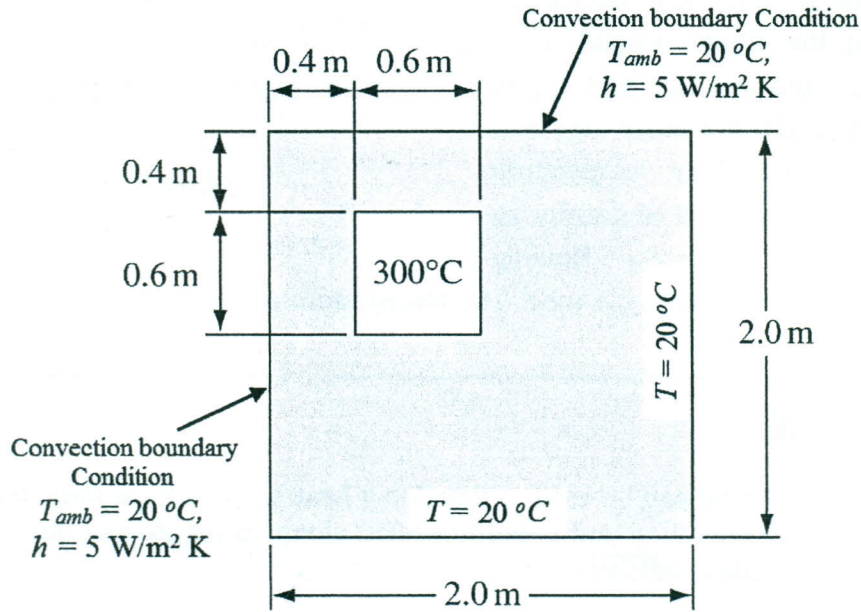
Take:  $L_x = 1.0 \text{ m}$ ,  $T_0 = 20 \text{ C}$ ,  $q = 20 \text{ C/m}$ ,  $h = 0.025 \text{ m}^{-2}$ ,  $T_a = 20\text{C}$ , and  $S = 20 \text{ C/m}^2$

- By hand calculation, using Euler method calculate the temperature distribution along the fin, take step size  $\Delta x = 1/3 \text{ m}$ . [5 mark]
- Write a MATLAB function to solve a system of ordinary differential equations by Euler method. [5 mark]
- Write a MATLAB function to define the model and its parameters and constants. [5 mark]
- Write a MATLAB script to solve for the temperature distribution in the fin described above. [5 mark]



**Q3 PDE [25 mark]**

The horizontal cross section of an industrial chimney is shown in the accompanying sketch. Flue gases maintain the interior surface of the chimney at 300 °C, and the outside northern and western surfaces are exposed to an ambient temperature of 20 °C through a heat transfer coefficient of 5 W/m<sup>2</sup> K, while the southern and eastern surfaces are maintained at 20 °C. The thermal conductivity of the chimney is  $k = 0.5$  W/m K, the density  $\rho = 1800$  kg/m<sup>3</sup>, and the specific heat  $c = 840$  J/kg K. initially, the chimney is at temperature of 20 °C.



If the system is governed by the following energy (unsteady conduction) equation

$$\frac{\partial T}{\partial t} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

- a) Derive the finite difference equations (FDE) and the stability conditions for:
  - i. The interior (non-boundary) nodes [3 mark]
  - ii. The boundary nodes [7 mark]
- b) Develop MATLAB code to determine:
  - i. How much time is required to reach the steady state, [5 mark]
  - ii. The temperature distribution in the chimney at any time, and [5 mark]
  - iii. The rate of heat loss from the flue gases per unit length of the chimney at steady state. [5 mark]

Good luck,  
 Dr. Hossam S.S. AbdelMeguid