

# CIVIL ENGINEERING

**SOLAR RADIATION AND A SOLUTION  
FOR  
TRIGONOMETRIC LEVELLING ATMOSPHERIC  
REFRACTION**

"الاشعاع الشمسي وحل الانكسار الجوي في الميزانيات المثلثية"

BY

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الخلاصة:

الميزانية المثلثية واحدة من أهم الطرق في الأعمال المساحية ولكن نتائجها تعتمد أساسا على الدقة في تعيين خطأ الانكسار الجوي. ولحذف أو تقليل تأثير خطأ الانكسار الجوي المنتظم على الأرصاد يجب الأخذ في الاعتبار مصادر هذا الخطأ. يعتبر الإشعاع الشمسي مصدرا رئيسيا لخطأ الانكسار الجوي. ويهدف هذا البحث إلى شرح كيفية تقليل تأثير خطأ الانكسار الجوي المنتظم على أرصاد الميزانيات المثلثية وذلك بتطبيق انساب معادلة رياضية تعتمد أساسا على أرصاد الطاقة الشمسية المقيسه. وقد تناول البحث بالتحليل نتائج المعادلات الرياضية المستخدمة وانسبها بالاضافة إلى التوصيات اللازمة.

**ABSTRACT**

Trigonometric levelling is one of the most important techniques in surveying works , however its results depends mainly on the systematic refraction error determination . In order to totally or partially eliminate the influence of systematic refraction error on observations , more attention should be paid to the source of such error. Daily solar radiation is considered the main source of the refraction error .

The main objective of this paper is to demonstrate the elimination of the systematic refraction effect on trigonometric levelling by applying an appropriate mathematical model based on the available data of solar radiation. The results predicted by the new solution are presented and analyzed .

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attempted to estimate the solar radiation theoretically using different models. Sometimes, this theoretical estimation of hourly values of direct, diffuse and total solar radiation data can be used when the measuring equipment's are not available .

In the last few years , Modern Technology in electronics has been used for the accurate measuring of the net and diffused solar radiation . the present paper deals only the measuring data of solar radiation .

### 3. Relation between atmospheric refraction effect on trigonometric levelling and solar radiation .

To obtain the best possible relation between Refraction and solar radiation several observations were carried out from sunrise to sunset during 1993 and 1994 . The observations illustrate the refraction effect for different atmospheric conditions , Also the solar radiation was recorded parallel to the trigonometric levelling observations.

All observations were measured using EDM instrument DI 20 built up one second wild theodolite T2 . The height differences between the instrument point and receiver points are obtained precisely by precise levelling . The measuring refraction error presents the difference between trigonometric height and height difference given by precise levelling

In the next part , the four sets of mathematical models for calculation the effect of atmospheric refraction are introduced .

i) Linear model (  $y = A + BX$  ).

ii) Logarithmic model (  $y = A + B \log x$  ).

iii) Exponential model (  $y = A e^{Bx}$  ).

iv) Power function model (  $y = A x^B$  ).

where:  $y$  is the atmospheric refraction effect in mm,  $x$  is the net solar radiation in  $\text{cal} / \text{cm}^2$  ,  $A$  and  $B$  are unknown coefficients obtained experimentally [HEER, 1985].

A FORTRAN computer program was designed to performs all computations of the field observations . It would be seen that measurements during the day time are divided into three parts from 6.0. to 11.0 , from 12 - 15.0 and from 16.0 to the end of observations . Tables (1) and (2) present the results of the four mathematical models using the field observations of the trigonometric levelling and the net solar radiation.

## 1. Introduction

The systematic atmospheric refraction correction in Trigonometric levelling observations is a very demanding aspect if an accuracy improvement and high precision are required. This accuracy can be only obtained, if refraction error is studied and avoided strictly. The difficulties in determining this error by using the classical meteorological parameters led to search about an alternative method.

The recent developments in electronics have led to accurate measuring of solar radiation for energy generation purposes. These devices are used in large areas all over the world. one of those stations now is existing in El-Mansoura university and data of solar radiation are available. Based on the available data of solar radiation it is possible to study the relation between the solar radiation and atmospheric refraction effect on trigonometric levelling.

The present study shows a very strong correlation between the solar radiation and atmospheric refraction which effect on trigonometric levelling. This relation is studied and an appropriate mathematical model is presented. Also the results of using predicted model are given and discussed.

## 2. Solar radiation

The amount of solar radiation on the earth's surface is attenuated by the terrestrial atmosphere. On clear days, it may reach the ground level after having been deflected up to 15% of its original value outside the atmosphere [ M. SAYEGH , 1983 ]. The attenuation is caused by some effects namely :-

- 1- scattering by molecules much smaller than the wave length of radiation .
- 2 - scattering by aerosols (Dust , smoke , pollen , etc. ) of size comparable to or larger than the wave length of radiation .
- 3 - scattering and absorption by cloud masses .
- 4 - selective absorption by gases present in the atmosphere and particularly by  $O_2$  ,  $O_3$  ,  $H_2O$  ,  $CO_2$  .

In this way, the solar radiation reaching the earth consists of direct and diffuse radiation. Owing to the importance of knowing accurately the solar radiation on the earth's surface and to the unreliability of available equipment, to measure a solar radiation several authors [STANTON,1975]

Table ( 1 ) Summary of the four models solution (date: 1993).

Date	Linear best fit $y = a + bx$				Logarithmic $y = a + b \ln x$			
	All day	1st part 6-10	2nd 10-2AM	3rd part 2-6, 00	All day	1st part	2nd part	3rd part
8-3	1.54+3.76x	0.87-2.70x	3.99+2.17x	1.99-4.48x	5.46+1.71lnx	3.08+0.87lnx	6.33+2.21lnx	6.22+1.72lnx
9-3	1.36+2.71x	3.45-1.01x	1.34+3.15x	0.13+4.96x	3.96+1.03lnx	2.56-0.36lnx	4.66+2.83lnx	3.49+1.03lnx
15-3	3.42+0.43x	3.04+1.68x	3.46+0.11x	3.08+0.99x	4.06+0.25lnx	4.66+0.77lnx	3.55+0.17lnx	4.72+0.97lnx
16-3	3.76+0.29x	2.53+0.83x	4.37-0.27x	3.59+2.24x	4.22+0.42lnx	3.23+0.19lnx	4.07-0.3lnx	6.45+1.72lnx
average	2.52+1.80x	2.47+1.05x	3.29+1.29x	2.2+3.17x	4.43+0.92lnx	3.38+0.37lnx	4.65+1.23lnx	5.22+1.36lnx
7-6	0.39+1.99x	-0.28+3.03x	1.39+0.96x	0.26+2.18x	2.73+1.06lnx	2.92+3.29lnx	2.1+1.80lnx	2.19+0.77lnx
8-6	2.48+0.01x	1.87+0.74x	7.58-3.75x	2.28+0.05x	2.52+0.11lnx	2.68+0.67lnx	3.76+4.64lnx	2.53+0.19lnx
27-6	1.52+0.3x	3.92-1.33x	6.38-3.29x	0.12+1.56x	1.93+0.29lnx	2.49-0.78lnx	3.16-4.19lnx	1.27+0.4lnx
14-7	1.15+0.81x	4.0+1.77x	0.79+1.11x	0.48+2.26x	2.1+0.62lnx	2.18-1.87lnx	1.78+1.7lnx	2.51+0.99lnx
15-7	0.87+1.17x	2.24-0.68x	-0.96-2.75x	0.85+1.19x	2.12+0.55lnx	1.50-0.61lnx	1.95+2.9lnx	1.92+0.44lnx
average	1.28+0.86x	2.36-0.01x	3.04-45x	0.8+1.45x	2.28+0.53lnx	2.35+0.14lnx	2.55-0.4lnx	2.08+0.56lnx
2-9	1.79+1.08x	0.99-1.56x	14.84-8.04x	0.92+3.9x	3.05+0.72lnx	2.6-1.03lnx	7.13-10.73lnx	4.22+1.32lnx
3-9	1.70+0.43x	2.27-0.14x	-10.53+8.8x	2.32-2.1x	2.13+0.08lnx	2.77+0.22lnx	2.15+1.95lnx	0.63+0.61lnx
4-10	1.75+0.75x	1.93+0.71x	2.16+0.38x	1.26+0.9x	2.59+0.40lnx	2.69+0.63lnx	2.49+0.61lnx	2.43+0.36lnx
average	1.85+1.14x	2.25+0.88x	2.83-1.22x	1.54+1.84x	3.1+0.62lnx	2.83+0.38lnx	3.23+0.45lnx	3.24+0.76lnx
Date	Exponential $y = a e^{bx}$				Power $y = a x^b$			
	All day	1 St. part	2nd part	3 rd-part	All day	1 St. part	2nd part	3 rd part
8-3	failed	$1.18e^{1x}$	$3.99e^{-39x}$	failed	failed	$2.65x^{-31}$	$6.09x^{-4}$	failed
9-3	failed	$3.2e^{-0.36x}$	$1.13e^{1.2x}$	failed	failed	$2.29x^{-0.15}$	$4.19x^{1.14}$	failed
15-3	failed	$2.98e^{-1.14x}$	$3.48e^{-0.01x}$	failed	failed	$4.42x^{18}$	$3.48x^{-0.01}$	failed
16-3	failed	$2.57e^{0.14x}$	$3.15e^{-15x}$	failed	failed	$2.84x^{0.01}$	$3.2x^{0.2}$	failed
average	failed			failed	failed			failed
7-6	failed	$0.89e^{-28x}$	$0.52e^{1.0x}$	failed	failed	$2.49x^{1.08}$	$1.25x^{1.10}$	failed
8-6	failed	$2.05e^{0.1x}$	$12.59e^{-1.3x}$	failed	failed	$2.41x^{1.3}$	$3.34x^{-1.62}$	failed
27-6	failed	$1.97e^{-2.5x}$	$26.76e^{-1.9x}$	failed	failed	$2.23x^{3.2}$	$3.82x^{-2.54}$	failed
14-7	failed	$4.63e^{0.8x}$	$0.77e^{-6x}$	failed	failed	$1.93x^{-0.87}$	$1.41x^{98}$	failed
15-7	failed	$2.18e^{-.4x}$	$1.63e^{1.5x}$	failed	failed	$1.39x^{-0.39}$	$1.63x^{1.51}$	failed
average	failed			failed	failed			failed
2-9	failed	$1.53e^{-38x}$	$4.04e^{-6x}$	failed	failed	$2.25x^{19}$	$10.5x^{-3.5}$	failed
3-9	failed	$2.54e^{-0.02x}$	$0.01e^{-3.5x}$	failed	failed	$2.52x^{10}$	$0.17x^{6.33}$	failed
4-10	failed	$3.56e^{-.8x}$	$2.14e^{-3.1x}$	failed	failed	$1.82x^6$	$3.6x^{-1.2}$	failed
average	failed			failed	failed			failed

Table ( 2 ) summary of the four models solution (date: 1994).

Date	Linear best fit $y = a + bx$				Logarithmic $y = a + b \ln x$			
	All day	1st part 6-10	2nd 10-2AM	3rd part 2-6,00	All day	1st part	2nd part	3rd part
11-2	2.52+1.8x	2.47+1.05x	1.05 + 4.1x	0.16+4.8x	3.78+0.48lnx	4.0+0.4lnx	3.73 -0.6lnx	3.21+0.84lnx
12-2	1.62+3.15x	2.7+0.92x	3.46+ 0.11x	1.66+1.5x	1.82+0.94lnx	5.3+1.4lnx	2.16+0.2lnx	2.48+0.4lnx
7-3	3.42+0.45x	3.04+1.7x	2.43+0.13x	3.0+1.0x	2.12+0.36lnx	0.07+0.8lnx	2.4+1.1lnx	1.46+0.75lnx
7-3	2.87+1.6x	2.1+0.98x	4.21+0.32x	0.47+3.78x	4.01+0.84lnx	0.11+0.7lnx	4.6+1.6lnx	3.14+0.02lnx
average	3.36+1.75x	2.7+1.16x	2.71+11.6 x	1.32+3.77x	2.18+0.65 lnx	2.37+0.62lnx	2.47+0.59lnx	2.32+0.49lnx
18-6	2.1+1.9x	3.4+1.2x	1.4+3.12x	1.88+3.8x	4.64+0.52lnx	2.14+0.34lnx	1.94+0.54lnx	4.06+1.06lnx
7-7	3.0+0.36x	0.85 + 2.8 x	4.21+0.32x	2.3+1.7x	1.0+0.14lnx	2.79+0.88lnx	1.88+0.58lnx	3.01+0.84lnx
8-7	2.48+0.12x	3.04 + 2.51x	4.17-1.6x	1.99+0.99x	3.51+0.4lnx	2.14+0.93lnx	0.88+0.34lnx	1.31+0.21lnx
3-8	3.32+0.48x	2.53 + 2.82x	1.34-0.27x	4.04+0.22x	0.64+0.83lnx	4.11+1.1lnx	3.14+1.06 lnx	2.44+0.75lnx
average	2.72+0.71x	2.45 + 1.73 x	2.78+0.57x	2.55-2.87x	2.79+0.47lnx	1.82+0.36lnx	2.02+0.46lnx	2.7+0.7lnx
11-9	2.6+1.02x	1.54 + 4.1x	3.04+2.1x	0.41+3.6x	2.06+0.09lnx	3.11+0.6lnx	1.6+0.62lnx	1.92+0.34lnx
2-10	2.82+3.2x	2.31 + 0.43x	0.82+2.1x	2.1+3.1x	3.61+1.6lnx	2.34+0.8lnx	6.0+0.18lnx	2.18+0.8lnx
3-10	1.16+0.43x	0.87 + 1.4x	-0.71+2.7x	1.4+0.81x	4.0+1.1lnx	4.06+0.04lnx	2.8+0.84lnx	0.93+0.33lnx
average	2.19+1.55x	1.74 + 1.9 x	1.05+2.3x	1.3+2.5x	3.2+0.93lnx	2.17+0.45lnx	3.13+0.54lnx	1.67+0.52lnx
Date	Exponential $y = a e^{bx}$				Power $y = a x^b$			
	All day	1st part	2nd part	3rd-part	All day	1st part	2nd part	3rd part
11-2	failed	$4.12e^{0.11x}$	$2.75e^{-.4x}$	failed	failed	$2.64x^{0.96}$	$5.5x^{0.51}$	failed
12-2	failed	$1.08e^{-0.2x}$	$4.11e^{-.98x}$	failed	failed	$9.84x^{-0.14}$	$4.12x^{-0.02}$	failed
7-3	failed	$3.34e^{2.1x}$	$1.07e^{-0.4x}$	failed	failed	$5.29x^{0.6}$	$4.56x^{-1.06}$	failed
7-3	failed	$4.86e^{0.1x}$	$3.35e^{2.11x}$	failed	failed	$4.19x^{0.89}$	$1.73x^{-0.24}$	failed
average	failed			failed	failed			failed
18-6	failed	$2.54e^{2.1x}$	$5.1e^{-1.9x}$	failed	failed	$4.08x^{0.14}$	$1.34x^{0.23}$	failed
7-7	failed	$4.36e^{1.1x}$	$4.01e^{-1.4x}$	failed	failed	$3.81x^{.24}$	$3.91x^{0.92}$	failed
8-7	failed	$4.81e^{-7.4x}$	$3.91e^{1.1x}$	failed	failed	$4.12x^{.74}$	$4.01x^{-2.31}$	failed
3-8	failed	$5.14e^{0.3x}$	$4.12e^{-1.1x}$	failed	failed	$2.18x^{-.91}$	$3.97x^{-.46}$	failed
average	failed			failed	failed			failed
11-9	failed	$2.14e^{.83x}$	$1.07e^{-.43x}$	failed	failed	$3.87x^{-.48}$	$2.48x^{.48}$	failed
2-10	failed	$2.76e^{.22x}$	$4.8e^{-1.2x}$	failed	failed	$4.12x^{-.14}$	$3.12x^{.77}$	failed
3-10	failed	$1.13e^{1.4x}$	$4.39e^{-.65x}$	failed	failed	$2.39x^{.06}$	$4.42x^{.73}$	failed
average	failed			failed	failed			failed

Date: 15. 7. 1993.

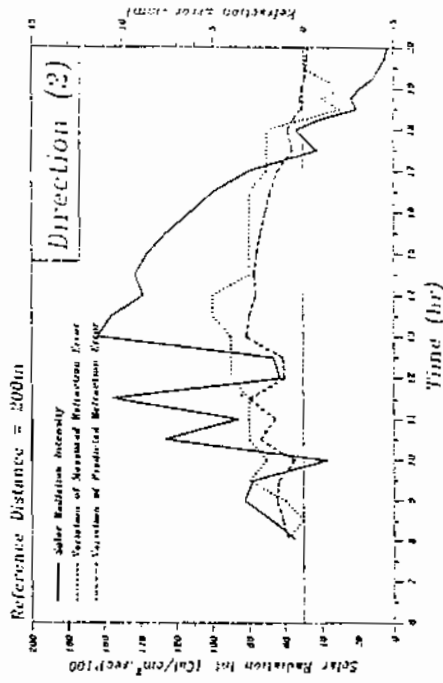
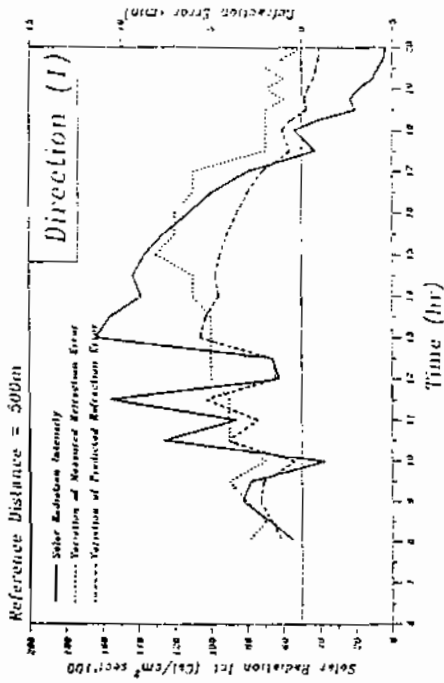


Fig. (1- a,b) Variation of Solar Radiation, Measured Refraction Correction and Predicted Refraction Correction

Time	Direction (1)			Direction (2)			Meteorological Parameters			
	Observed Distance (m)	Refraction Error (mm)	U (mm)	Observed Distance (m)	Refraction Error (mm)	U (mm)	Temperature (°C)	Pressure (mmbar)	Wind Velocity (km/h)	
0:00	490.215	01	+3	109.499	03	+1	10.6	990.15	0.53012	
0:30	490.214	01	+2	109.499	04	00	+1.1	990.32	0.6700	
1:00	490.215	01	+3	109.499	03	+1	11.2	990.45	0.8200	
1:30	490.216	03	+4	109.501	05	+1	11.3	990.32	0.7800	
2:00	490.214	00	+2	109.500	05	+2	10.5	990.32	0.3705	
2:30	490.215	01	+4	109.501	03	+3	+2.1	987.35	1.2675	
3:00	490.216	01	+4	109.501	00	+4	+1.5	988.59	0.1600	
3:30	490.216	01	+4	109.501	04	+3	+3.0	988	989.25	1.5600
4:00	490.217	01	+5	109.502	07	+4	+1.0	989.25	0.8240	
4:30	490.217	00	+5	109.502	04	+4	+1.1	989.25	0.6670	
5:00	490.217	00	+5	109.502	05	+4	+3.2	989.25	1.6300	
5:30	490.217	01	+5	109.503	06	+5	+3.0	989.25	1.5600	
6:00	490.218	02	+6	109.503	04	+5	+2.0	989.25	1.3045	
6:30	490.218	00	+6	109.501	00	+3	+2.7	989.12	1.4313	
7:00	490.220	01	+6	109.501	03	+3	+2.6	988.59	1.3650	
7:30	490.219	02	+7	109.501	04	+3	+2.1	988.59	1.2597	
8:00	490.218	00	+7	109.501	04	+2	+2.1	988.59	1.1370	
8:30	490.218	01	+6	109.501	05	+2	+1.0	988.59	0.9945	
9:00	490.218	01	+6	109.500	03	+2	+1.4	988.59	0.7995	
9:30	490.214	01	+2	109.499	02	+2	+0.6	988.59	0.4290	
10:00	490.214	01	+2	109.499	03	+2	+0.6	987.92	0.5121	
10:30	490.214	00	+2	109.499	01	00	+0.6	987.92	0.4895	
11:00	490.214	00	+2	109.497	03	-1	+0.2	988.19	0.2040	
11:30	490.214	01	+1	109.497	03	-1	+0.2	989.25	0.2310	
12:00	490.214	01	+2	109.496	01	-2	+0.1	989.92	0.1872	
12:30	490.213	00	+1	109.497	02	-1	-0.1	990.45	0.1492	
13:00	490.214	00	+2	109.498	01	00	-0.1	990.59	0.0780	
13:30	490.214	00	+1	109.498	00	00	-0.2	990.45	0.0480	
14:00	490.212	00	00	109.498	00	00	-0.2	990.99	0.0351	

Table (1-1) Observed Distances, Refraction Error and Meteorological Parameters

Date: 3. 9. 1993.

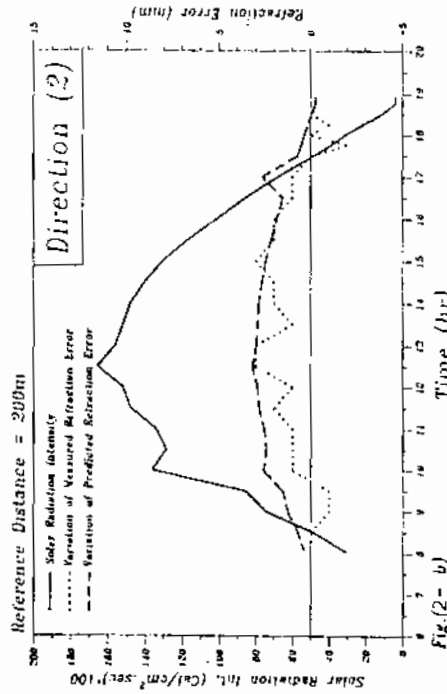
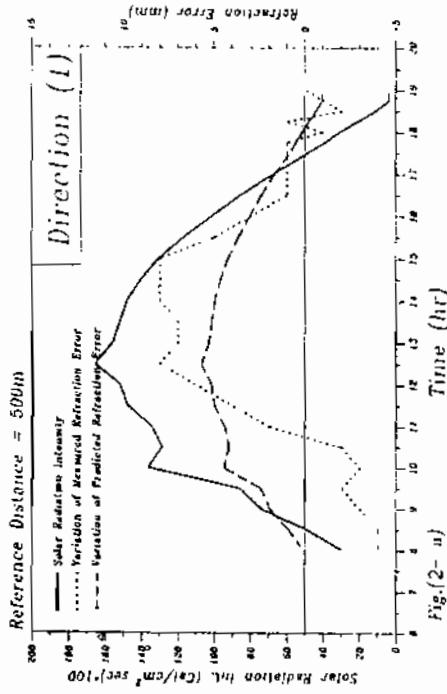


Fig. (2- a, b) Variation of Solar Radiation, Measured Refraction Correction and Predicted Refraction Correction

Time	Direction (1) Reference Distance = 500m			Direction (2) Reference Distance = 200m			Meteorological Parameters	
	Observed Distance (m)	Refraction Error (mm)	Observed Distance (ft)	Observed Distance (m)	Refraction Error (mm)	Observed Distance (ft)	Temperature (°C)	Pressure number
8:00	498.208 01	-4	189.498 00	00	+0.3	24.80	994.18	0.2925
8:30	498.208 01	-4	189.498 00	00	+0.7	25.70	994.18	0.1875
9:00	498.209 04	-3	189.497 00	-1	+1.3	27.50	993.25	0.7410
9:30	498.210 02	-2	189.497 02	-1	+1.5	27.70	991.92	0.0500
10:00	498.209 01	-3	189.499 03	+1	+2.0	28.00	991.25	1.3650
10:30	498.210 01	-2	189.499 00	+1	+2.4	30.00	993.25	1.3970
11:00	498.214 02	2	189.499 05	+1	+2.5	30.50	992.98	1.3455
11:30	498.216 01	1	189.500 03	+2	+2.8	31.30	992.92	1.4820
12:00	498.218 02	6	189.499 03	+1	+2.9	31.00	991.92	1.5210
12:30	498.220 00	0	189.501 05	+3	+3.2	33.20	991.55	1.6575
13:00	498.219 02	7	189.501 05	+3	+3.0	33.20	991.52	1.5600
13:30	498.219 01	7	189.499 02	+1	+2.9	33.70	990.92	1.5210
14:00	498.220 02	6	189.500 01	+2	+2.0	33.50	989.92	1.4820
14:30	498.220 01	8	189.500 01	+2	+2.7	33.50	989.92	1.4040
15:00	498.220 01	8	189.501 01	+3	+2.5	33.80	989.92	1.3065
15:30	498.217 00	5	189.500 02	+2	+2.2	33.00	989.92	1.1700
16:00	498.215 02	3	189.500 02	+2	+1.8	32.30	990.19	1.0140
16:30	498.213 02	1	189.499 02	+1	+1.5	32.30	990.19	0.8500
17:00	498.213 01	1	189.499 00	+1	+1.1	34.80	989.92	0.6825
17:30	498.213 00	1	189.498 01	00	+0.7	31.20	989.52	0.4875
18:00	498.211 00	-1	189.498 01	00	+0.5	30.80	989.52	0.3900
18:30	498.213 00	1	189.497 00	-1	+0.2	29.50	990.59	0.2145
19:00	498.210 01	-2	189.498 01	00	-0.1	29.20	990.59	0.1170
19:30	498.211 00	-1	189.498 00	00	-0.2	28.50	990.59	0.0390
19:00	498.213 00	0	189.498 00	00	-0.2	28.00	990.59	0.0390

Table (3-2) Observed Distances, Refraction Error and Meteorological Parameters



Table ( 1 ) provides a summary of the four model solutions . From this table , it is clear that the logarithmic and linear models may give a reliable solution . Where the most important is that the exponential and power function are failed to give a reasonable solution . Moreover , the strong correlation between the net solar radiation and atmospheric refraction effect on trigonometric leveling observations are valid .

As shown from the table the results obtained from the linear mathematical model is considered quit bigger than that of the logarithmic form solution . Based on the above results , in order to obtain the appropriate mathematical solutions for calculating the refraction effects on trigonometric levelling by using solar radiation data, the logarithmic model solution should be taken . On the other hand , the expected model from the author's point of view , to have the following logarithmic form as :

$$y = A + B \text{ Log } x$$

Where A and B values are determined and illustrated in the above Figures (1-a) ,(1-b) ,(2-a) ,(2-b) and table (3-a & b) present the samples of measuring refraction and predicted refraction error by using logarithmic model.

#### **4. Conclusions**

It is clear that , logarithmic form provides better results than other forms . This study show the atmospheric refraction correction on trigonometric levelling can be obtained efficiently by using solar radiation. In order to obtain the practical form of such correction, it is necessary to investigate a lot of trigonometric levelling observations including additional information about the net radiation.

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