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NODAL LINE FINITE DIFFERENCE METHOD FOR THE ANALYSIS OF RECTANGULAR PLATES WITH ABRUPT CHANGE IN THICKNESS

طريقة الفروق المحددة لخطوط التقسيم لتحليل الألواح المستطيلة ذات التغير الفجائي في السمك

BY

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الخلاصة — يقدم هذا البحث تطبيقاً جديداً لطريقة الفروق المحددة لخطوط التقسيم التي ابتكرها الباحث nodal line finite difference method وطبقها بنجاح في أبحاثه السابقة لتحليل الألواح المستطيلة المرنة ، ويتناول هذا البحث تحليل الألواح المرنة المستطيلة ذات التغير الفجائي في السمك في اتجاه واحد باستخدام الطريقة المقترحة ونظراً لصعوبة تطبيق الطريقة المقترحة عند خط التغير الفجائي في السمك للألواح المستطيلة قام الباحث باشتقاق معادلة للفروق difference equation خاصة بخط التغير الفجائي في السمك وذلك باعتبار شروط الاتزان equilibrium والاتصال compatibility على يمين وشمال خط التغير الفجائي في السمك مباشرة . وقدم الباحث أمثلة لبعض النماذج من الألواح المستطيلة ذات التغير الفجائي في السمك والتي ترتكز ارتكازاً بسيطاً simply supported عند حوافها . ولقد قورنت النتائج التي تم الحصول عليها في هذا البحث بالنتائج المستخلصة من الحل التحليلي لنفس هذه النماذج من الألواح بالمزج المتوازن لدى الباحث، وأظهرت تطابقاً كبيراً مما يدل على دقة وكفاءة هذه الطريقة وإمكانية استخدامها في تحليل مشاكل أخرى في هذا المجال .

ABSTRACT: *A semi-analytical procedure by means of the nodal line finite difference method, early developed by the Author, is presented for bending analysis of rectangular plates with abrupt change in thickness in one direction. To overcome the difficulty arised from the discontinuity at the line of abrupt change in thickness, a new form of the difference equation has been developed and incorporated in the technique presented. Numerical results obtained by the application of the proposed method are compared with those available from another analytical solution. The comparison demonstrates a close agreement, which indicates the applicability and versatility of the proposed method as a new technique for solving a class of plate bending problems.*

INTRODUCTION

The mathematical complexity encountered with the analytical solutions of two and three dimensional structural problems have prompted the development of numerical methods of analysis. These numerical methods, while powerful and versatile, have some drawbacks in that usually a large number of simultaneous algebric equations has to be solved. So, for certain class of problems, it is worthwhile to develop simplified semi-analytical methods of which the finite strip is one. The finite strip method is now well established as a special finite element technique for solving a class of two and three dimensional structural problems. It involves the use of basic functions to express the displacement variation in the longitudinal direction of the strip. The earliest formulation and the subsequent generalization of this method was developed by CHEUNG [1,2,3] for the analysis of static and dynamic problems. The Author [5,6] developed lately a finite strip solution with iteration procedure, in order to overcome the coupling property which occurs when using basic functions other than trigonometric series.

Recently, a new semi-analytical procedure named "The nodal line finite difference method" (N.L.F.D) pioneered by the Author, was developed for the analysis of plate bending problems. In this method the plate is divided into a mesh of fictitious parallel nodal lines in one direction. Basic functions which fit the boundary conditions at the two opposite ends perpendicular to the nodal lines are used to express the displacement variation along the nodal lines. This method is similar in approach to the finite strip method since both methods call for the use of basic functions in one direction and reduce two dimensional problems to one dimensional problems. The most commonly used basic functions in both methods are the eigenfunctions which are derived from the solution of the differential equation of the beam vibration [4]. The earliest formulation of the nodal line finite difference method was developed by the Author [7] who used a trigonometric series as a basic function in the analysis of elastic rectangular plates with two opposite simply supported ends. A basic function other than trigonometric series, is used by the Author [8] to analyze elastic rectangular plates with two clamped opposite ends. In this analysis the nodal line finite difference method is used in conjunction with an iteration procedure to overcome the coupling property of the static equilibrium equations. The method has been also extended by the Author [9] to solve the bending problem of rectangular plates with variable flexural rigidity in one direction.

Plates with abrupt change in thickness have different applications in structural engineering. Therefore, the bending analysis of such plates has been the subject of considerable research interest. The available pertinent work on this topic was presented by BUCHHOLZ [10], who used an analytical approach in the analysis.

The purpose of the present work is to extend the application of the nodal line finite difference method for the analysis of rectangular plates with abrupt change in thickness in one direction. The discontinuity of the plate at the line of abrupt change in thickness precludes the application of the method and necessitates derivation of special difference equation at that line. Herein, a new form of the difference equation has been developed by considering the equilibrium and compatibility conditions just to the left and just to the right of the line of the abrupt change in thickness. The derivation of this equation is similar in concept to that of the beam bending with sudden change in depth worked out in reference [11]. Illustrative examples are given to demonstrate the validity and the applicability of the present technique, where the obtained results have shown good agreement with those given by BUCHHOLZ [10].

METHOD OF ANALYSIS

In the present study, we shall restrict our attention to the bending analysis of elastic isotropic plates having abrupt change in thickness in one direction. It is assumed that the middle surface of the plate is plane and coincides with x-y plane of the cartisian coordinate system. The plate is considered as an assembly of plate portions joined together at their edges. The thickness within each plate portion is assumed constant. Fig. 1 shows examples for cross-sections of plates having abrupt change in thickness and their idealization with respect to the middle surface.

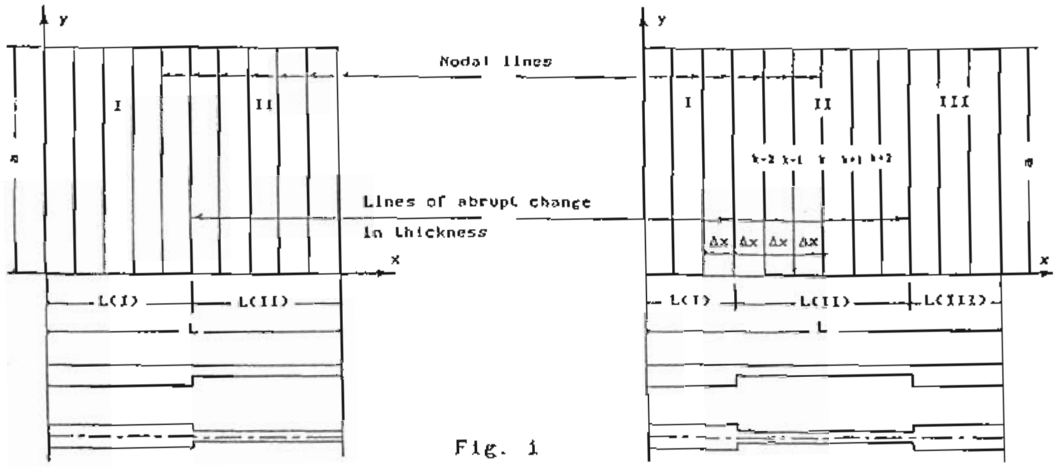


Fig. 1

a) Nodal Line Difference Equation

The differential equation relating the deflection to the surface load of the plates is governed by

$$B\,(W^{\cdot\cdot\cdot\cdot} + 2\,W^{\cdot\cdot\prime\prime} + W^{\prime\prime\prime\prime}) = q \qquad (1)$$

where $(\quad)^{\cdot} = \frac{\partial}{\partial x}$, $(\quad)^{\prime} = \frac{\partial}{\partial y}$ and

$B = \frac{E\,t^3}{12(1-\nu^2)}$ is the flexural rigidity of the plate

In order to apply the nodal line finite difference technique in the present study, the plate is divided into a mesh of fictitious nodal lines parallel to the lines of abrupt change in thickness. The displacement variation along the nodal lines is controlled by the choice of a basic function satisfying a priori the boundary conditions at the ends of the nodal lines. The displacement function at any nodal line is expressed as a summation of the chosen basic function terms multiplied by a single variable functions termed by the nodal line parameters. The displacement function at any nodal line labelled k shown in Fig. 1 can be written in the form

$$W_k = \sum_{m=1}^{r} F_{m,k}(x)\; Y_m(y) \qquad (2)$$

For the sake of simplicity, a trigonometric series fitting the boundary conditions of two opposite simply supported ends, has been chosen

$$Y_m(y) = \sin \frac{m\pi}{a}\, y = \sin k_m y \qquad (3)$$

Resolving the load into series similar to that of the chosen basic function and substituting equations (2),(3) into equation (1) gives

$$B \sum_{m=1}^{r} [\, F^{\cdot\cdot\cdot\cdot}_{m,k} - 2\,k_m^2\, F^{\cdot\cdot}_{m,k} + k_m^4\, F_{m,k} \,]\, \sin k_m y = \sum_{m=1}^{r} q_{m,k} \sin k_m y \qquad (4)$$

Equation (4) can be written for each term of the basic function as

$$D \left[F^{''''}_{m,k} - 2 k_m^2 F^{''}_{m,k} + k_m^4 F_{m,k} \right] = q_{m,k} \tag{5}$$

This ordinary differential equation can be transformed into a nodal line difference equation by applying the central finite difference technique in the x direction as follows

$$\left[1 \quad C_m^1 \quad C_m^2 \quad C_m^1 \quad 1 \right] \left\{ F_{m,k-2} \; F_{m,k-1} \; F_{m,k} \; F_{m,k+1} \; F_{m,k+2} \right\}^T = \frac{a^4}{D\lambda^4} q_{m,k} \tag{6}$$

where $C_m^1 = -(4+2\psi_m^2)$, $C_m^2 = (6+4\psi_m^2+\psi_m^4)$, $\psi_m = \frac{k_m a}{\lambda}$ and $\lambda = \frac{a}{\Delta x}$

Equation (6) represents the central nodal line difference equation for the elastic isotropic plates having a constant thickness.

b) Internal Forces

The forces per unit length at any point of an elastic isotropic plate are connected to the displacement through the following relations

$$\left.\begin{aligned}
M_x &= -D\,(W^{''} + \upsilon W'') \\
M_y &= -D\,(W'' + \upsilon W^{''}) \\
M_{xy} &= -M_{yx} = D\,(1 - \upsilon)\, W^{'\prime} \\
Q_x &= -D\,(W^{'''} + W^{'\prime\prime}) \\
Q_y &= -D\,(W^{''\prime} + W''') \\
\bar{Q}_x &= -D\,[W^{'''} + (2-\upsilon)\, W^{'\prime\prime}] = (Q_x - \frac{\partial M_{xy}}{\partial y})
\end{aligned}\right\} \tag{7}$$

he magnitude of the internal forces at any point can then be obtained as a roduct of the nodal line parameters and of the basic function terms at the iven point. By applying the central nodal line finite difference technique, he internal forces at any nodal line labelled K can be written as follows

$$\left.\begin{aligned}
M_{x,k} &= -\frac{D\lambda^2}{a^2} \sum_{m=1}^{r} \sin k_m y \; (F_{m,k-1} - C_m^3 F_{m,k} + F_{m,k+1}) \\
M_{y,k} &= -\frac{D\lambda^2}{a^2} \sum_{m=1}^{r} \sin k_m y \; (\upsilon F_{m,k-1} - C_m^4 F_{m,k} + \upsilon F_{m,k+1}) \\
M_{xy,k} &= \frac{D\lambda^2}{2a^2} (1-\upsilon) \sum_{m=1}^{r} \psi_m \cos k_m y \; (- F_{m,k-1} + F_{m,k+1}) \\
Q_{x,k} &= -\frac{D\lambda^3}{2a^3} \sum_{m=1}^{r} \sin k_m y \; (-F_{m,k-2} + C_m^5 F_{m,k-1} - C_m^5 F_{m,k+1} + F_{m,k+2}) \\
Q_{y,k} &= -\frac{D\lambda^3}{a^3} \sum_{m=1}^{r} \psi_m \cos k_m y \; (F_{m,k-1} - C_m^5 F_{m,k} + F_{m,k+1}) \\
\bar{Q}_{x,k} &= -\frac{D\lambda^3}{2a^3} \sum_{m=1}^{r} \sin k_m y \; (-F_{m,k-2} + C_m^6 F_{m,k-1} - C_m^6 F_{m,k+1} + F_{m,k+2})
\end{aligned}\right\} \tag{8}$$

where $C_m^3 = (2 + \upsilon\psi_m^2)$, $C_m^4 = (2\upsilon + \psi_m^2)$,

$C_m^5 = (2 + \psi_m^2)$ and $C_m^6 = [2 + (2-\upsilon)\psi_m^2]$

a) Equilibrium and Compatibility Conditions at the Line of Abrupt Change in Thickness

The Proposed technique requires the application of the nodal line difference equation (6) at each nodal line of the plate. The application of this technique at the nodal lines within each plate portion is quite simple and straightforward. This is due to the constant thickness, which gives a specified value for the flexural rigidity at any nodal line. The application of the proposed technique at the nodal lines joining the different plate portions faced with the difficulty arised from the abrupt change in thickness. This abrupt change gives two different values for the flexural rigidity just to the left and just to the right of these nodal lines. this difficulty can be exceeded by introducing a specified effective flexural rigidity and developing a special nodal line difference equation at the line of abrupt change in thickness. This can be achieved by considering the compatibility and the equilibrium conditions at this line.

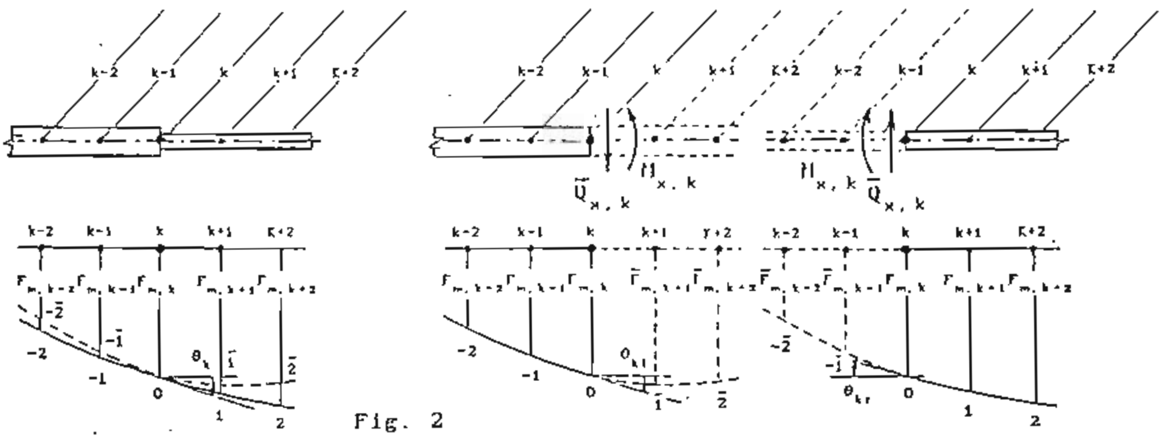


Fig. 2

Consider the plate cross-section shown in Fig. 2, which has an abrupt change in thickness at the nodal line labelled k. The flexural rigidities of the plate just to the left and just to the right of the nodal line k are denoted by Bkl and Bkr respectively. Let the nodal line displacement parameters at five equally spaced nodal lines be Fm,k-2 , Fm,k-1 , Fm,k , Fm,k+1 and Fm,k+2. Extend the two parts $0, \bar{1}, 2$ and $\bar{0}, -\bar{1}, -2$ of the curve respectively to the fictitious points $(-\bar{1}, -\bar{2})$ and $(\bar{1}, \bar{2})$ whose ordinates are ($\bar{F}$m,k-1, $\bar{F}$m,k-2) and ($\bar{F}$m,k+1, $\bar{F}$m,k+2). These fictitious nodal line parameters ($\bar{F}$m,k-2, $\bar{F}$m,k-1, $\bar{F}$m,k+1, $\bar{F}$m,k+2) can be determined in terms of the nodal parameters Fm,k-2, Fm,k-1, Fm,k, Fm,k+1 and Fm,k+2 as follows mpatibility Conditions

For compatibility, the deflection and the slope of the two curves at the dal line k must be the same, thus

$$\theta_{kl} = \theta_{kr} \tag{9}$$

r each term m, we get

$$\frac{1}{2\Delta x}\left(-F_{m,k-1} + \bar{F}_{m,k+1}\right) = \frac{1}{2\Delta x}\left(-\bar{F}_{m,k-1} + F_{m,k+1}\right) \tag{10}$$

uilibrium Conditions

For equilibrium, the bending moment Mx and the modified shearing force just to the left and just to the right of the nodal line k must be the ne, that is

$$\left.\begin{aligned} M_{x,kl} &= M_{x,kr} = M_{x,k} \\ \bar{Q}_{x,kl} &= \bar{Q}_{x,kr} = \bar{Q}_{x,k} \end{aligned}\right\} \tag{11}$$

For each term m, we obtain

$$\frac{B_{kl}}{\Delta x^2}\left(F_{m,k-1} - C_m^3 F_{m,k} + \bar{F}_{m,k+1}\right) = \frac{B_{kr}}{\Delta x^2}\left(\bar{F}_{m,k-1} - C_m^3 F_{m,k} + F_{m,k+1}\right) \tag{12}$$

$$\begin{aligned} &\frac{B_{kl}}{\Delta x^3}\left(-F_{m,k-2} + C_m^6 F_{m,k-1} - C_m^6 \bar{F}_{m,k+1} + \bar{F}_{m,k+2}\right) \\ &\qquad = \frac{B_{kr}}{2\Delta x^3}\left(-\bar{F}_{m,k-2} + C_m^6 \bar{F}_{m,k-1} - C_m^6 F_{m,k+1} + F_{m,k+2}\right) \end{aligned} \tag{13}$$

From equations (10) and (12), we get

$$\left.\begin{aligned} \bar{F}_{m,k-1} &= \frac{1}{1+\varkappa}\left[2\varkappa F_{m,k-1} + (1-\varkappa) C_m^3 F_{m,k} - (1-\varkappa) F_{m,k+1}\right] \\ \bar{F}_{m,k+1} &= \frac{1}{1+\varkappa}\left[(1-\varkappa) F_{m,k-1} - (1-\varkappa) C_m^3 F_{m,k} + 2 F_{m,k+1}\right] \end{aligned}\right\} \tag{14}$$

where $\varkappa = \dfrac{B_{kl}}{B_{kr}}$

Substitution of equation (14) into equation (12) gives

$$\begin{aligned} M_{x,k} &= -\frac{2}{1+\varkappa}\frac{B_{kl}}{\Delta x^2}\left(F_{m,k-1} - C_m^3 F_{m,k} + F_{m,k+1}\right) \\ &= -\frac{2\varkappa}{1+\varkappa}\frac{B_{kr}}{\Delta x^2}\left(F_{m,k-1} - C_m^3 F_{m,k} + F_{m,k+1}\right) \\ &= -\frac{B_{ke}}{\Delta x^2}\left(F_{m,k-1} - C_m^3 F_{m,k} + F_{m,k+1}\right) \end{aligned} \tag{15}$$

Where Bke is the effective flexural rigidity of the plate at the nodal line k of abrupt change in thickness

$$B_{ke} = \frac{2}{1+\varkappa} B_{kl} = \frac{2\varkappa}{1+\varkappa} B_{kr} = \frac{2B_{kl} \cdot B_{kr}}{B_{kl} + B_{kr}} \tag{16}$$

In order to determine the other two fictitious nodal line parameters $\bar{F}m,k-2$ and $\bar{F}m,k+2$, another equation is needed. This equation can be formulated from the application of the nodal line difference equations (6) just to the left and just to the right of nodal line k as follows

$$B_{kl} (F_{m,k-2} + C_m^1 F_{m,k-1} + C_m^2 \; F_{m,k} + C_m^1 \bar{F}_{m,k+1} + \bar{F}_{m,k+2}) =$$

$$B_{kr} (\bar{F}_{m,k-2} + C_m^1 \bar{F}_{m,k-1} + C_m^2 F_{m,k} + C_m^1 F_{m,k+1} + F_{m,k+2}) \tag{17}$$

From equations (13) , (17) and by considering equation (14), we get

$$\left.\begin{aligned} \bar{F}_{m,k-2} &= \frac{1}{1+\varkappa} [\, \varkappa (1+\varkappa) F_{m,k-2} + \varkappa (1-\varkappa) C_m^6 F_{m,k-1} - \frac{1-\varkappa}{2} ((1+\varkappa) \\ &\qquad (C_m^2 - C_m^3 C_m^3) - 2 C_m^3 C_m^6) F_{m,k} - (1-\varkappa) C_m^6 F_{m,k+1} \,] \\ \bar{F}_{m,k+2} &= \frac{1}{1+\varkappa} [(1-\varkappa) C_m^6 F_{m,k-1} + \frac{1-\varkappa}{2\varkappa} ((1+\varkappa)(C_m^2 - C_m^3 C_m^3) \\ &\qquad - 2 \varkappa C_m^3 C_m^6) F_{m,k} - \frac{1-\varkappa}{\varkappa} C_m^6 F_{m,k+1} + \frac{1+\varkappa}{\varkappa} F_{m,k+2} \,] \end{aligned}\right\} \tag{18}$$

d) Nodal Line Difference Equation at the Line of Abrupt Change in Thickness

The nodal line difference equation at the line of abrupt change in thickness can be now determined by elimination of the fictitious nodal line parameters $\bar{F}m,k-2$, $\bar{F}m,k-1$, $\bar{F}m,k+1$ and $\bar{F}m,k+2$ from equation (17). This can be done by substitution of equations (14) and (18) into equation (17). The nodal line difference equation at the line of abrupt change in thickness can be written in a matrix form as

$$\left[\bar{C}_m^1 \quad \bar{C}_m^2 \quad \bar{C}_m^3 \quad \bar{C}_m^4 \quad \bar{C}_m^5 \right] \left\{ F_{m,k-2} \; F_{m,k-1} \; F_{m,k} \quad F_{m,k+1} \; F_{m,k+2} \right\}^T$$

$$= \frac{a^4}{B_{ke}\lambda^4} \; q_{m,k} \tag{19}$$

where $\bar{C}_m^1 = \frac{1+\varkappa}{2}$, $\bar{C}_m^5 = \frac{1+\varkappa}{2\varkappa}$, $\bar{C}_m^2 = \frac{1-\varkappa}{2} C_m^6 - 2C_m^5$,

$\bar{C}_m^3 = \frac{1}{4\varkappa} [(1+\varkappa)^2 C_m^2 - (1-\varkappa)^2 C_m^3 C_m^3]$ and $\bar{C}_m^4 = - \frac{1-\varkappa}{2\varkappa} C_m^6 - 2C_m^5$

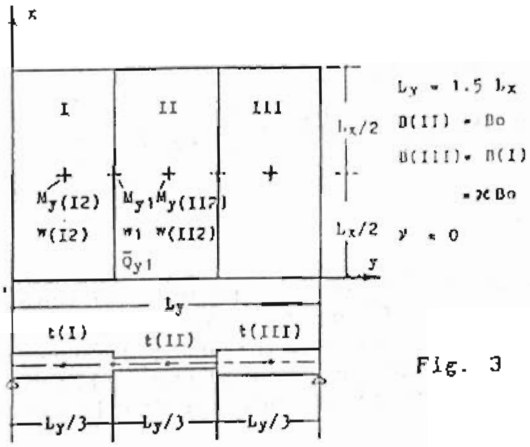
Application of the nodal line difference equation (6) within the plate portions and the nodal line difference equation (19) at the lines of abrupt change in thickness leads to a system of simultaneous linear equations. The final matrix of these equations has a nice property of banded matrices with small band width equals to 5. This reduces drastically the core storage and the computer time for execution.

NUMERICAL EXAMPLES

Two examples of rectangular plates with abrupt change in thickness have been analyzed by the proposed technique. Information regarding plate dimensions, boundary conditions and type of loading are taken the same as those for rectangular plates solved analytically by BOCHHOLZ. Due to the absence of the numerical values of the BOCHHOLZ's results, the comparison is restricted only to the plotted results. For the sake of comparison, the Author found it useful to represent the obtained results of the proposed technique in a manner similar to that used by BOCHHOLZ [10]. The close agreement of both results precludes the representation of the results on the same diagrams. Therefore, the results from the proposed technique were plotted separately. For comparison a copy of the plotted BOCHHOLZ's results is represented in APPENDIX II.

1. Analysis of rectangular plate simply supported on four sides subjected to uniformly distributed load of intensity q (Fig. 3).

Rectangular plate with abrupt change in thickness at one third of the medium position of the length in y direction. the plate has a ratio of rectangularty equals to Ly/Lx = 1.5. Due to symmetry in y direction, only half of the plate is used in the analysis, and divided into a mesh of thirty one nodal lines i.e Δx = Ly/60. The analysis was carried out for the first term of the basic function only. The results were obtained for selected values of $\varkappa$ and plotted as shown in Fig. 3 (a,b,c).



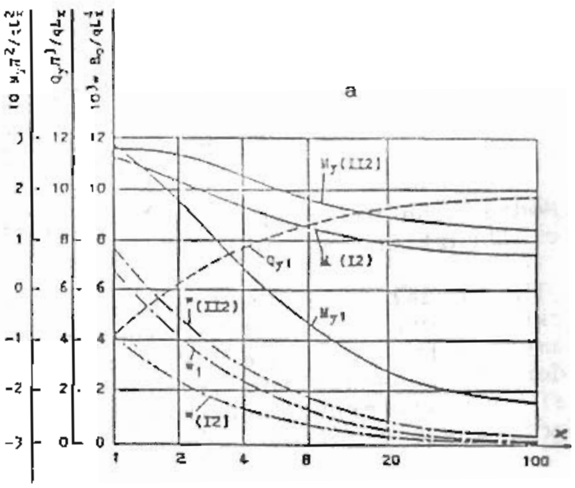
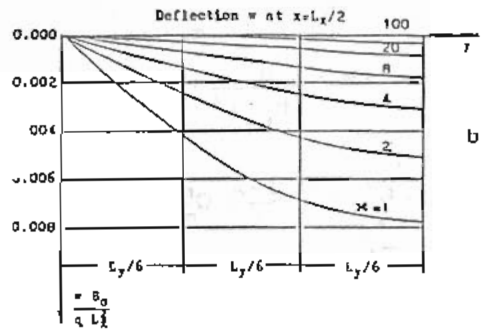
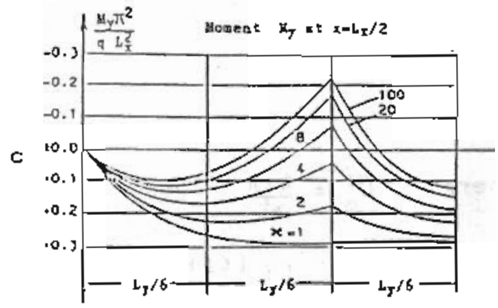


Fig. 3





Analysis of square plate simply supported on four sides subjected uniformly distributed load of intensity q (Fig. 4)..

Square plate with abrupt change in thickness at the middle of the len direction. The plate was divided into a mesh of forty one nodal l: $\Delta x = Ly/40$. Due to the symmetry in x direction, only odd terms of sic function (1,3,5,7,9,11,13) contribute to the results. The analysis rried out for a selected values of $\varkappa$. The results were obtained otted as shown in Fig. 4 (a,b,c,d).

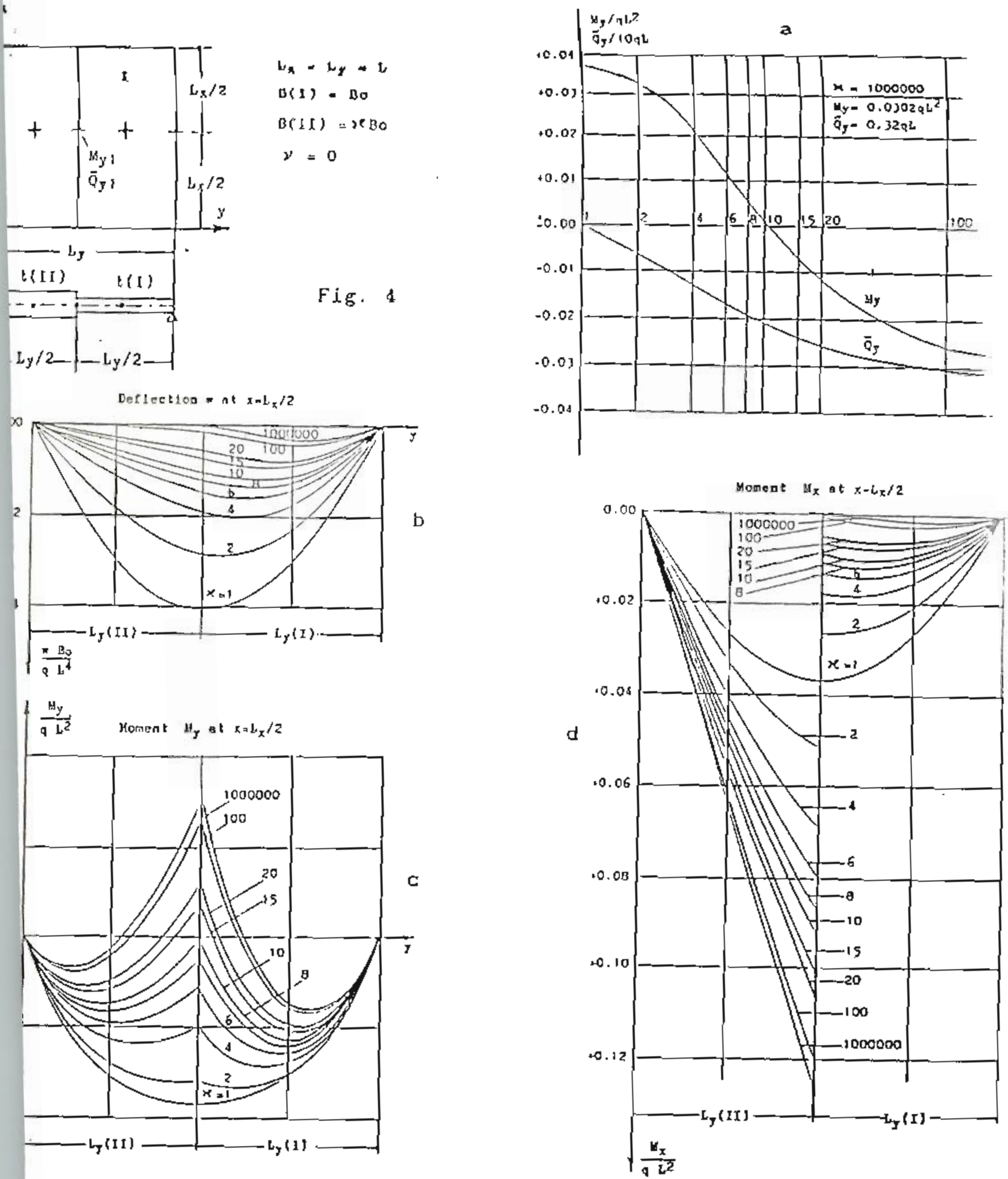


Fig. 4

CONCLUSION

In the present work, rectangular plates with abrupt change in thickness have been analyzed using the nodal line finite difference method developed early by the Author. Due to the discontinuity of the plate at the line of abrupt change in thickness, the nodal line difference equation, derived directly from the differential equation, is not applicable. To overcome this difficulty, a new form of the nodal line difference equation derived from the equilibrium and the compatibility conditions on the line of abrupt change in thickness has been developed. A comparison of the results obtained by the proposed technique with those obtained from the another analytical solution shows a close agreement and gives the proposed method the validity and power for subsequent applications in the field of two and three dimensional structural problems.

APPENDIX I

NOTATIONS

W = transverse deflection.

a = length of the nodal lines.

L_x , L_y = dimensions of plate in x and y directions.

Δx = constant distance between the nodal lines.

E = modulus of elasticity.

t = thickness within a plate portion.

ν = poisson's ratio.

D = flexural rigidity within a plate portion.

D_{kl} , D_{kr} = flexural rigidities just to the left and to the right of abrupt change in thickness.

D_{ke} = effective flexural rigidity at the line of abrupt change in thickness.

$\varkappa$ = ratio between the flexural rigidities just to the left and to the right of the line of abrupt change in thickness.

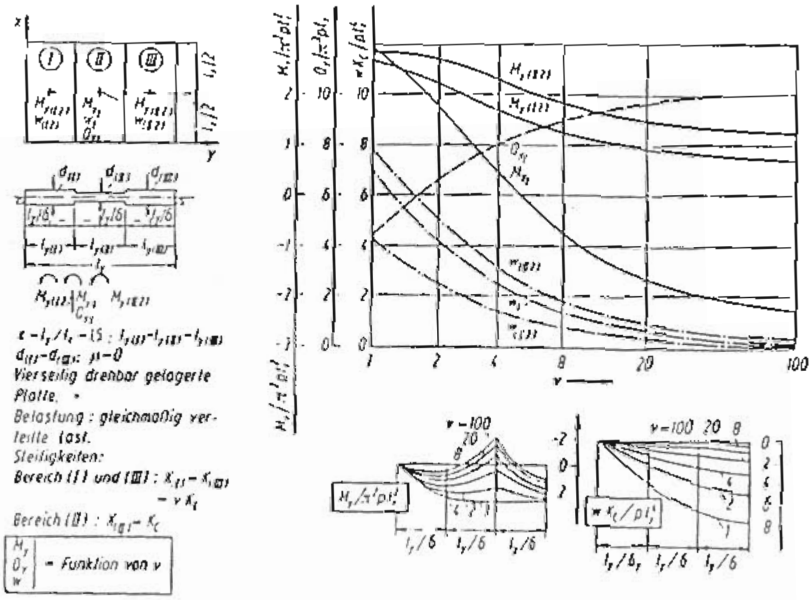
$F_{m,k}$ = nodal line parameters.

Y_m = basic function.

q = load intensity.

PPENDIX II

The plotted results obtained by BUCHHOLZ [10]



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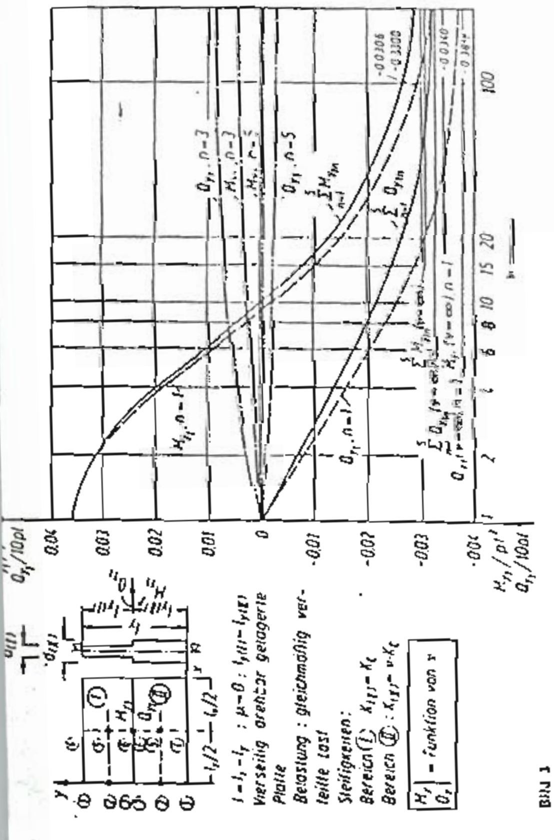

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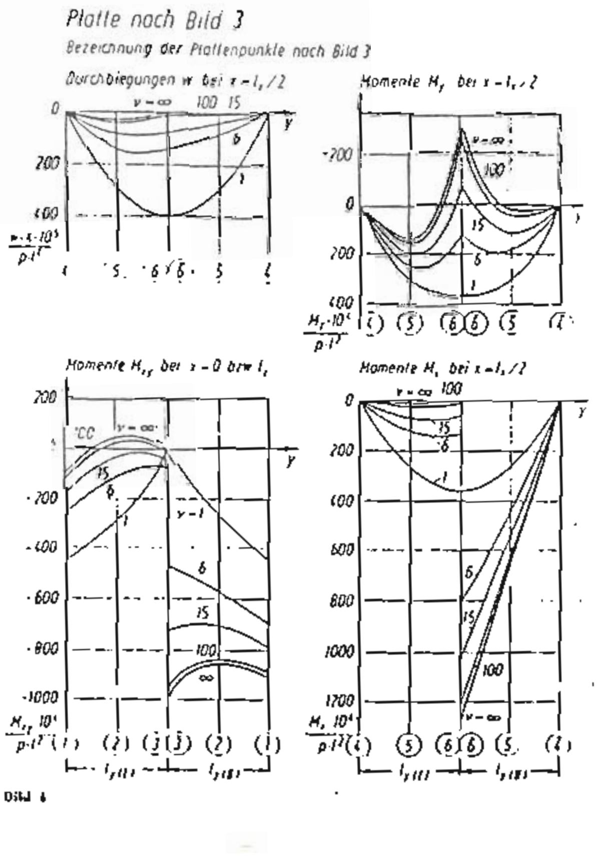

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