Mansoura Engineering Jurnal Vol. 18, No. 2, June, 1993.

INFLUNCE OF INSUFFICENT PERVIOUS LENGTH DOWNSTREAM OF HYDRAULIC STRUCTURES

تأثـــير عدم كفايه الطول المنفــذ خلــف المنشآت الهيدروليكــه

BY

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الخلاصه :

يهدف هذا البحث الى دراسه تأثير عدم كفايه طول الجزء المنفذ للمياه خلف منشآت الحجز الهيدروليكيه وذلك باستخدام نظريه العناصر المحيطه . فمن خلال هذه النظريه تم استخدام العنصر الخطي لتحديد تأثير كلا من :ـ طول الجزء المنفذ خلف منشآت الحجز وسمك طبقه التربه المنفذه للمياه اسفل فرشه افقيه على خصائص التسرب (قـــوه التعويم ـ كميه المياه المتسربه اسفل الفرشه ـ الميل الهيدروليكي عند المخرج) .

وقد اوضحت نتائج الدراسه ان الطول المنفذ خلف المنشات الهيدروليكيه له تأثير عكسي على قوه التعويم اسفـــل الفرشه بدرجه فعاله وايضا بالنسبه للميل الهيدروليكي عند المخرج ويؤثر تأثيرا طرديا على كميــه المياه المتسربه.

اوضحت نتائج الدراسه ايضا ان العمق النسبي لسمك الطبقه المنفذه يؤثر تأثيرا طرديا على كـــــــل الخصائص الســـــــابقه.

ABSTRACT

The stability of heading-up hydraulic structures will be greatly affected by the formation of the downstream pervious length that lies adjacent to the end of the solid floor. The boundary element technique using linear elements, is used to analyse the uplift pressure underneath a hydraulic structure of a simple flat floor as well as the seepage flow and the exit gradients. Ten cases of the downstream pervious portions have been considered with six thicknesses of the permeable layer under the structure. The results indicate that substantial increase in the uplift pressure and exit gradients may develop due to the insufficient length of the downstream pervious portion and the bigger thicknesses of the soil layer.

INTRODUCTION

To safeguard heading-up structures erected on a permeable soil, the most important precaution is to use an inverted filter covered with a pervious layer downstream of their solid floors, to let the percolating water leave the domain safely. If the existed length of the pervious portion is not sufficient, substantial increase in the uplift pressure and the exit gradients may develop.

Numerous analytical and experimental studies have been carried out to evaluate the uplift pressure distribution and the exit gradients for different boundary conditions and floor configurations, for infinite upstream (U.S) and downstream (D.S) seepage surfaces [8, 10]. For insufficient permeable portions D.S the structures, the downstream seepage surface should ideally diminish and a continuous impervious bed exists. It is very important for the designer to take sufficient permeable length D.S the structure to be sure that the acting uplift pressure will not exceed the considered value.

Gary and Chawla [7] and Chawla [4] derived analytical solutions using the conformal mapping technique for a floor founded on a permeable soil of infinite and finite depths, respectively, with finite pervious inlet and outlet zones and a cut-off at any general position along the floor. The analysis, however, did not cover the case of very narrow permeable portions, relative to the maximum acting seepage head, which are considered to be the practical case. The exit gradients were calculated at one point only which may be insufficient for design purpose.

In the present study, the Boundary Element Method (BEM) is used to analyse the practical problem of seepage under a simple flat floor constructed on a permeable soil, with relative pervious length ($l = L_2/H = 0.25 - 5.0$), and different permeable thicknesses under the floor ($m = T/L_1 = 0.5 - 2.0$).

Figure (1) shows the general layout of the considered problem. The considered length of the inlet seepage surface is two times the floor length L_1.

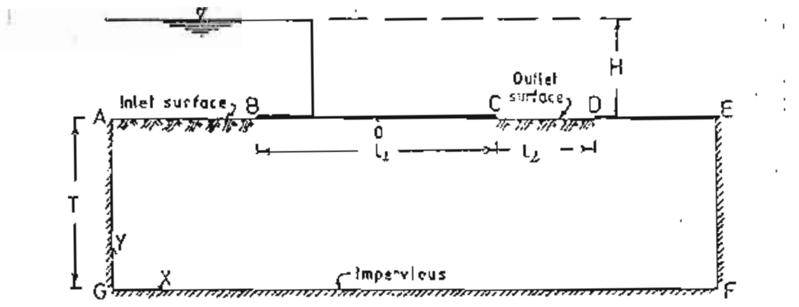

Fig.1- The general layout of the problem.

The maximum difference between the U.S and D.S water levels equals H, the length of the solid floor L_1, the length of the downstream pervious portion L_2. The permeable soil beneath the floor has a thickness T and is assumed to be homogeneous and isotropic with constant permeability coefficient K.

In a dimensionless forms, the considered parameters may be written as

$$l = L_2/H, \qquad m = T/L_1 \quad \text{and} \qquad h_r = h_e/h_o$$

where $h_e = h_i - h_o$, h_i is the computed uplift pressure at point o at the middle of the floor, corresponding to the considered L_2. h_o is the uplift pressure at o corresponding to m and l equal infinity ($h_o = H/2.$).

MATHEMATICAL IDEALIZATION OF THE PROBLEM

A typical two-dimensional problem of flow through the fully saturated porous media is shown in Fig.(2). Using principles of continuity of incompressible flow and Darcy's law, the governing equation of seepage in a two-dimensional flow domain R can be described by the Laplace equation [1] :

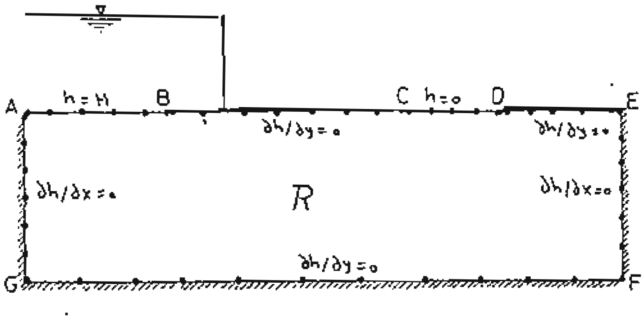

Fig.2- Idealization of the problem.

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \quad \text{in } R \tag{1}$$

where $h = (p/\gamma + z)$, the potential head. p/γ is the pressure head and z is the elevation head.

In general, for a confined seepage flow domain there are two types of boundaries;
1) A prescribed head on the upstream and downstream permeable beds (A-B and C-D),
2) A prescribed flux on boundaries B-C, D-E, E-F, F-G, and G-A.
The boundary conditions on A-B and C-D can be described by the following set of equations:

$$h = H \quad \text{on } A-B \tag{2-a}$$
$$h = 0 \quad \text{on } C-D \text{ (If D.S is dry)} \tag{2-b}$$

The boundary conditions for the impervious boundaries can be described as

$$\partial h / \partial n = 0 \tag{3}$$

where $\partial h / \partial n$ is the hydraulic gradient in the direction perpendicular to the boundary surface.

BOUNDARY ELEMENT FORMULATION

In the boundary element method, the boundary of a flow region is discretized into several linear segments connected by nodal points as shown in Fig.(2). The heads and the gradients on the boundary nodes are denoted as {h} and {∂h/∂n}, respectively. The value of head or gradient at any point on a line segment between two nodes can be obtained by using interpolation function <N> and {M} as follows :

$$h = \langle N \rangle \{h\} \tag{4}$$

$$\partial h/\partial n = \langle M \rangle \{\partial h/\partial n\} \tag{5}$$

where < > and { } represent a raw vector and a column vector, respectively. In this presentation, <N> is chosen to be as <M>, both being linear functions.

Using Green's theorem, the volume integral of the Laplace equation can be reduced to a boundary integral:

$$\alpha(x)\ h(x) \int_{\Gamma} \{\ G(\xi, x) \frac{\partial h}{\partial n}(\xi) - F(\xi, x)\ h(\xi)\}\ d\Gamma(\xi) \qquad (6)$$

$h(x)$ = the potential head at x : $G(\xi, x)$ and $F(\xi, x)$ represent the potential head and gradient at field point ξ due to a unit concentrated source at source point x (i.e, the fundamental solution), respectively. For two-dimensional problems:

$$G(\xi, x) = \frac{1}{2\pi} Ln\left(\frac{1}{r}\right); \qquad r = |\ \xi - x\ | \qquad (7)$$

$$F(\xi, x) = \frac{1}{2\pi} \frac{\partial r}{\partial n} \qquad (8)$$

Using the rigid-body analogy [1], the value of α can be evaluated by

$$\alpha(x) = \begin{cases} 1;\ x \text{ in } R \\ -\int F(\xi, x)\ d\Gamma(\xi);\ x \text{ on } \Gamma \\ 0.5:\ x \text{ on } \Gamma \text{ and } \Gamma \text{ is smooth} \end{cases} \qquad (9)$$

Substituting the interpolation functions into Eq.(6), the relationship between heads and gradients at the boundary nodal points of a given domain is given by

$$[H]\ \{h\} = [G]\ \{\partial h/\partial n\} \qquad (10)$$

In the above, the matrices [H] and [G] are obtained from

$$[H] = [\delta]\ \{\alpha(X^m)\} - \int \{F(x^m, \xi)\}\ \langle N(\xi)\rangle\ d\Gamma(\xi) \qquad (11)$$

$$[G] = \int_{\Gamma} \{G(x^m, \xi)\}\ \langle N(\xi)\rangle\ d\Gamma(\xi) \qquad (12)$$

where δ = the Kronecker delta ; x^m = the point of node m; ξ = the field point on the boundary surface Γ : and $\langle N(\xi)\rangle$ = the interpolation function.

In a boundary value problem, either gradient or potential head is known for a given node on the boundary. Therefore, Eq.(10) gives a set of simultaneous equations that can be solved for the unknown variables. The boundary element method that has been outlined for a homogeneous flow domain is fairly standard and can be found in the literatures [1.2. 9. and 11].

Using linear elements on the boundary lead to a problem at corner points, which can have two values for the normal derivative $\partial h/\partial n$ depending on the side under consideration. At these points, it is essential to select which of the two variables h or $\partial h/\partial n$ will be prescribed. As $\partial h/\partial n$ can not be uniquely defined, one generally will choose to prescribe h. This however, does not produce a very accurate computed value for the derivatives at the corners. This problem does not occur in finite elements due to the way in which the natural boundary conditions are prescribed and the fact that the solution is also approximated in the domain, i.e, errors tend to be more distributed.

A simple way to avoid the corner problem is to assume that, there are two points very near to each other but which belong to different sides (Fig.2). This empirical solution appears to give good results as shown in Fig.(3). At one node the h condition is prescribed and at the other the $\partial h/\partial n$.

RESULTS AND DISCUSSION

To illustrate the influence of the downstream pervious portion adjacent to the end of the floor, the relative length l has been considered from 0.25 to 5. and the relative thickness of the permeable layer m from 0.5 to 2.0.

The accuracy of the written program can be verified as shown in Fig.(3). This figure illustrates the comparison between the closed-form solution obtained by Pavlovsky [5], finite element results [6], experimental data obtained by others, and the present solution using BEM. It is clear that the present results considering m and l equal 2. and 5., respectively, are in a good agreement with the closed-form solution for m and l equal infinity.

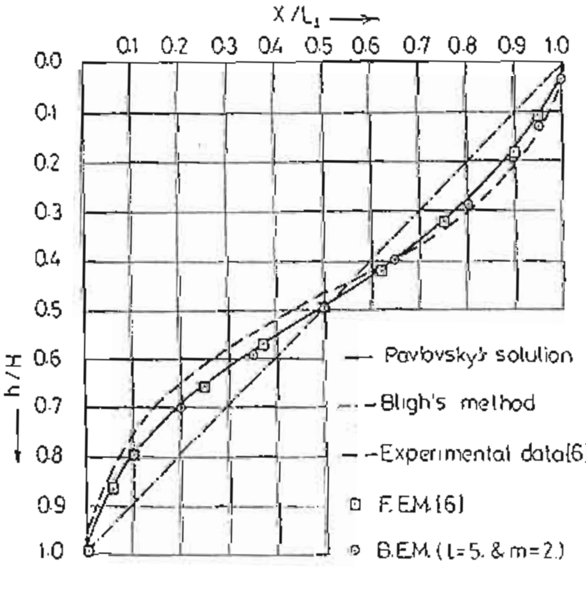

Fig.3- Comparison between the present solution using BEM and other solutions

Figures (4, 5 and 6) show the effect of the relative length of the pervious portion l on the uplift pressure for three different values of the relative thickness m. From these figures it is clear that the uplift pressure increases as a result of decreasing the relative length l downstream of the structure, and for the bigger values of m. Also, it is observed that smaller values of l give a bigger uplift pressure in the rear part of the floor than the front part.

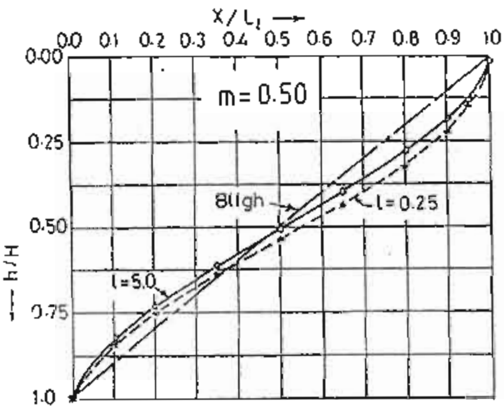

Fig.4- Effect of l on the uplift pressure for m = 0.5

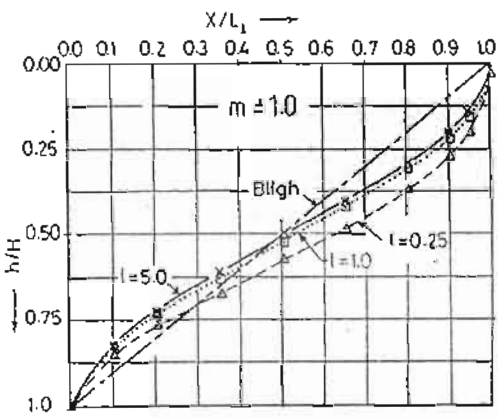

Fig.5- Effect of l on the uplift pressure for m = 1.

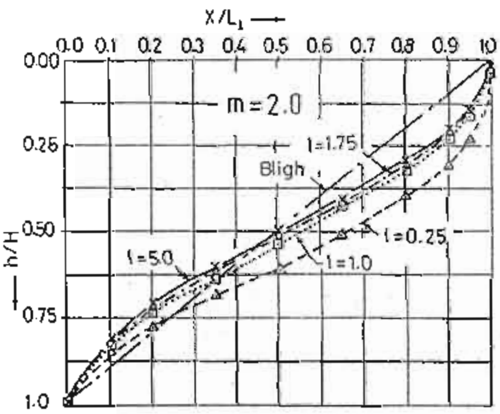

Fig. 6- Effect of l on the uplift pressure for m = 2.

Figure (7) shows the variation of the relative length of the down-stream permeable portion with the corresponding effect in the uplift pressure at the middle of the floor. point o in Fig.(1). for three values of m (m = 0.5, 1. and 2.).

In this figure the effect on the uplift pressure is illustrated in a percentage form where h_e is the excess in the uplift pressure at the middle of the floor corresponding to the considered l and h_o is the pressure at the same location for l = ∞.

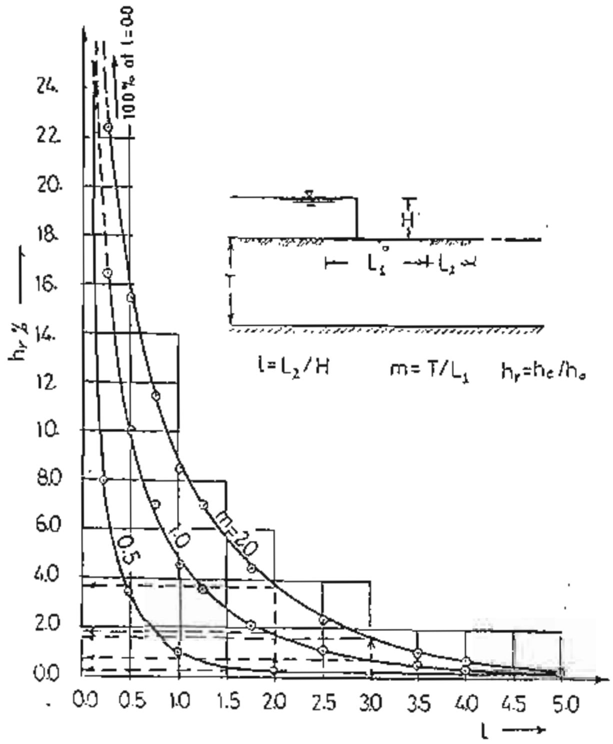

Fig.7- Variation of the relative error of the uplift pressure at the middle of the floor with l

One can observe that, if the existing length of the permeable portion L_2 is about two times the seepage head H. the corresspoding excess pressure will not exceed 0.2. 1.8 and 3.6 % for m equals 0.5, 1. and 2., respectively, of the pressure for L_2 = ∞.

These values are 0.2, 0.8 and 1.6 for L_2 = 3 H. This means that. in the range between l =2.- 3., the excess pressure for m equals 2 is about two times the corresponding pressure for m equals 1.

To illustrate the influence of the relative length l. and the relative thickness of the soil layer m. on the exit gradient I_t Figures (8, 9 and 10) show the variation of I with l_t/H for m = 0.5, 1.0 and 2.0, respectively.

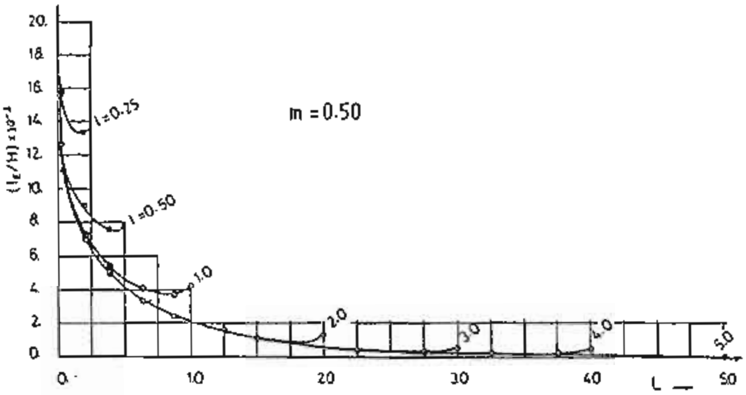

Fig.8-Variation of I_E/H with l for m = 0.5

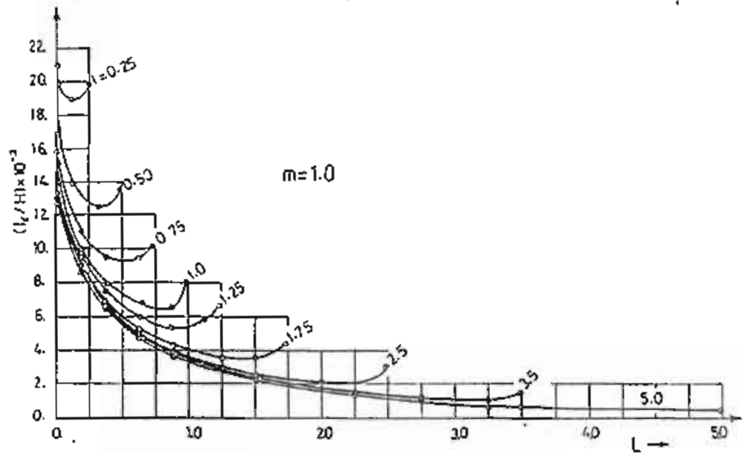

Fig.9- Variation of I_E/H with l for m = 1.

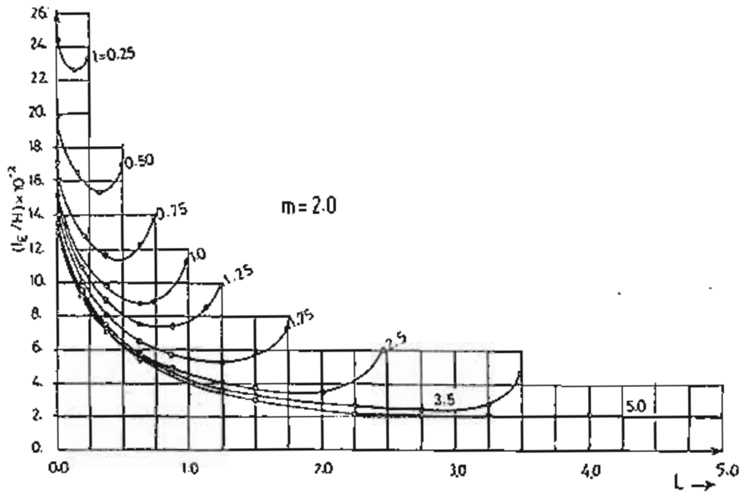

Fig.10- Variation of I_E/H with l for m = 2.

From these figures, it is clear that the relative length l and the relative thickness m have a very significant influence on the exit gradients. For example, for m= 1., the relative exit gradient I_r /H has been decreased with about 78% due to increasing l from 0.5 to 2. also, the relative exit gradient has been decreased as a result of decreasing the relative thickness of the soil layer m.

Finally, to illustrate the effect of the relative length l and the relative thickness m on the seepage flow. Fig.(11) shows the variation of l with Q/K.H for three different values of m (0.5, 1. and 2.), where K is the permeability of the soil layer.

It is clear that the relative length l and the relative thickness m have a significant effect on the seepage flow. The relative length l has no effect on the seepage flow for l greater than 2.5. The rate of seepage increases with about 46% due to increasing m from 0.5 to 1.0 and 34% for the change from 1.0 to 2.0.

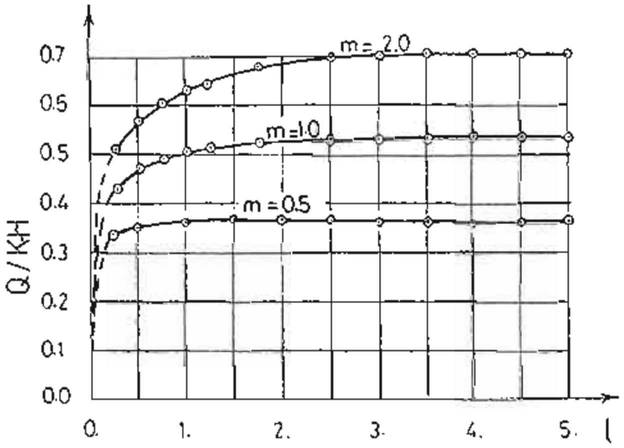

Fig.11- Variation of Q/K.H with l for three cases of the relative thickness m.

CONCLUSIONS

The effect of the relative length of the permeable portion existing downstream a hydraulic structure with simple flat floor rests on a finite permeable layer has been numerically analysed. The boundary element method has been applied to study the case under consideration. A FORTRAN -IV computer program has been written and calibrated through a comparison with other results. The present study clearly indicates that:

1- Substantial increase in the uplift pressures on the floor may result from short permeable portions in the downstream side of the structure.
2- The maximum increase of the uplift pressure, at the middle of the floor is about 3.6% if the existed length of the D.S permeable portion equals two times the seepage head, and 1.6 for three times of the same head corresponding to the relative thickness of the soil layer equals 2.
3- Both the uplift pressure, exit gradients, and seepage flow are reduced for smaller thicknesses of the permeable layer underneath the structure.

4- Increasing the thickness of the permeable layer increases the rate of seepage flow with 48. and 34% due to increasing this thickness from 0.5 to one and from one to two times of the floor length. respectively.
5- The effect of increasing the length of the downstream pervious portion on the seepage flow is completely vanished for the lengths greater than 2.5 of the seepage head.

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