يتألف الإختبار من 4 أسئلة في ورقتين. برجاء بدء إجابة كل فرع من إحدى نهايتي ورقة الإجابة.

[1]-(a) [15 pts] Solve by any method

1. $(\sin x \cosh y)\,dx - (\cos x \sinh y)\,dy = 0, \quad y(0) = 0,$
2. $\dfrac{dy}{dx} = \dfrac{y}{x + xy + x^2 y},$
3. $x^2 y'' - 2x y' + 2y = x^3 \sin x.$

(b) [5 pts] Find a general solution for the homogeneous differential equation with constant coefficients whose auxiliary equation is

$$(r-1)^3 (r-2)\,\left(r^2 + r + 1\right)\,\left(r^2 + 6r + 10\right)^3 = 0.$$

(c) [5 pts] Find the orthogonal trajectories of

$$y^2 (2C - x) = x^3.$$

(d) [5 pts] Determine the proper form of $y_p(x)$ for

$$\left(D^2 - 3D\right) y = 6e^{3x} - 5 \sin x$$

but do not solve for the undetermined coefficients.

[2]-(a) [6 pts] Find the Laplace transform of the following functions

$$f_1(t) = \begin{cases} e^t, & 0 < t < 2\pi, \\ e^t + \cos t, & t > 2\pi, \end{cases} \quad , \quad f_2(t) = t e^{3t} \sinh(2t) \sin(4t).$$

(b) [6 pts] Find the inverse Laplace transform of

$$F_1(s) = \ln\left(\frac{s+3}{s^2+4}\right), \qquad F_2(s) = \frac{2s^2 + 10s}{(s^2 - 2s + 5)(s+1)}.$$

(c) [8 pts] Evaluate

1. $\displaystyle\int_0^\infty \frac{\cos(3t) - \cos(6t)}{t}\,dt,$
2. $1 \star 1 \star 1.$

(d) [5 pts] Solve the initial value problem

$$y'' - 2y' - 8y = f(t), \quad y(0) = 1, \quad y'(0) = 0.$$

Assoc. Prof. Dr. El-Gamel

[3] (a) **[6 pts]** The domain of Ali's garden is describe by the domain of the function

$$f(x,y) = \sqrt{x - |y|} + \sqrt{4x - x^2 - y^2}$$

i) Find the domain of Ali's garden and sketch

ii) Using double integration prove that area of garden $= 2\pi + 4$

(b) **[9 pts]** If z, u, v are three **positive** real numbers satisfy equation

$z^2u - uvz + u^2 + v^2 = 8v$ where $u = (x+1)e^y$ and $v = (y+1)\cos(x)$

prove that $z_x = -1.6$ and $z_y = 0.2$ at $(x = 0$ and $y = 0)$

(c) **[8 pts]** Suppose that the elevation z of a hill is given by

$$z = f(x,y) = 39 + 10x - x^2 + 12y - y^2$$

i) If a small stone moves from site (6,8) to (9,12) . Find the rate of change of elevation in that direction

ii) Using second-order approximation, find the elevation z at point (6.01,8.02) use $[x_0 = 6 \; and \; y_0 = 8]$

[4] (a) **[7 pts]** Find $I = \int_0^1 \int_0^1 \frac{1}{1-(xy)^2} dx\, dy$ using transformation

$x = \frac{\sin(u)}{\cos(v)}$ and $y = \frac{\sin(v)}{\cos(u)}$ (note : this transformation transform square $0 \le x \le 1$, $0 \le y \le 1$ into triangle $0 \le u \le \frac{\pi}{2} - v$, $0 \le v \le \frac{\pi}{2}$)

(b) **[7 pts]** Find the center of mass of the lamina for the shape inside curve $r = 2 - 2\sin(\theta)$ shown in figure 1.

If the mass density given by $\rho(x,y) = 1$

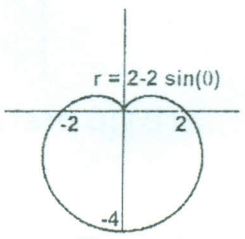

Figure 1

(c) **[4 pts]** For $\vec{F} = (x^2y)\,\hat{\imath} + (3x - yz)\,\hat{\jmath} + (z^3)\,\hat{k}$. Find Curl $\vec{F}$ and Div $\vec{F}$

(d) **[7 pts]** Compute the work done by the force field $F(x,y) = (y)\,\hat{\imath} + (-x)\,\hat{\jmath}$ acting on object as it moves along parabola $y = x^2 - 1$ from (1,0) to (−2,3)

(e) **[7 pts]** Ali has a tent its volume is similar to the volume of the solid bounded by $z = 4 - y^2, x + z = 4, x = 0 \; and \; z = 0$. Find the volume inside tent